

Technical report 13-006

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*If you want to cite this report, please use the following reference instead:*

L. Li, R.R. Negenborn, and B. De Schutter, "A general framework for modeling intermodal transport networks," *Proceedings of the 10th IEEE International Conference on Networking, Sensing and Control*, Paris, France, 7 pp., Apr. 2013. Paper FrA02.3.

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# A general framework for modeling intermodal transport networks

L. Li, R.R. Negenborn, B. De Schutter

**Abstract**—Intermodal transport is receiving increased attention due to the increasing demand for container transport in national and international trade, concerns on the sustainable development of the economy and the environment, and the recurring road congestion problems. In this paper, we first introduce intermodal transport networks and analyze the approaches for modeling intermodal transport networks in the literature. Next, a general framework for modeling intermodal transport networks is presented. In particular, a generic intermodal transport network model is formulated for the evolution of container flows in nodes and links taking into account time dependencies. The optimal route choices for container transport over the network are determined by solving a linear programming problem. Simulation experiments illustrate the properties of the proposed model.

**Index Terms**—Intermodal freight transport, intermodal transport networks, container flows

## I. INTRODUCTION

Due to the increasing demand for container transport in national and international trade, transport systems in logistic chains are facing great challenges. One crucial problem is to provide reliable and sufficient transport services in a cost-efficient way while using the current transport infrastructures. The sustainable development of the economy and the environment also requires major changes and developments in transport systems, such as using geographic information systems technology for managing freight movements, improving the market shares of railway networks and waterway networks in freight transport, etc. This requirement makes the problem even worse. One of the most promising approaches to handle this problem is to adopt the concept of intermodal freight transport. As a consequence, research interest in intermodal freight transport problems is growing steadily [1]–[5].

Intermodal freight transport is the movement of goods in one and the same loading unit or vehicle by successive modes of transport without handling of the goods themselves when changing modes [6]. By integrating and coordinating the use of different transport modes available in intermodal transport networks, intermodal freight transport provides the opportunity to obtain an optimal use of the physical infrastructure so as to provide cost and energy efficient transport services. Especially, with the assistance of information providers so-called synchronodal transport can be achieved by making modality choices according to the latest logistics information, e.g., transport demands, traffic information, etc.

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Containers are used widely for transporting cargo in modern transport systems. For determining the optimal routing of container transport over the network, a suitable model to represent the characteristic behaviors of an intermodal transport network is needed. In this paper, we propose a general framework for modeling intermodal transport networks.

This paper is organized as follows. In Section II, we first briefly introduce intermodal transport networks and modeling approaches existing in the literature. Next, our proposed modeling framework for intermodal transport networks is given in Section III. In Section IV, simulation experiments are conducted to validate the model. Finally, Section V concludes the paper and presents future research directions.

## II. INTERMODAL TRANSPORT NETWORKS

### A. Intermodal transport networks

Intermodal transport networks integrate different types of transport networks that are involved in the transport process of commodities in transport systems. Intermodal freight transport is typically focused on surface transport [2], [7], [8]. In addition, some work has been done to make air transport an alternative option in intermodal freight transport [9], [10].

An intermodal transport network can be modeled as a directed graph using two types of interconnected components, nodes and links [11]. Nodes represent entities like deepsea ports, inland ports, and terminals in the hinterlands. Containerized commodities are handled (unloaded from vehicles and loaded into other vehicles, or stored) at the network nodes and transported to other connected nodes (which could be their final destination or intermediate terminals on the way to their final destination) through corresponding links. Links represent entities like roads, railways, waterways, etc. Figure 1 illustrates an intermodal transport network consisting of three individual transport networks. The road network, the railway network, and the waterway network are indicated by different colors and line styles. The structure of the graph, which is the representation of specific physical infrastructures, is determined by the connectivity of the nodes within the network through links.

### B. Approaches for modeling intermodal transport networks

We now briefly present a review of modeling approaches for intermodal transport networks existing in the literature. An intermodal transport network model is used to deal with the problem of optimally positioning rail/road terminals for freight transport in [7]. This model gives the basic formulation of an intermodal transport network model but the time dependence

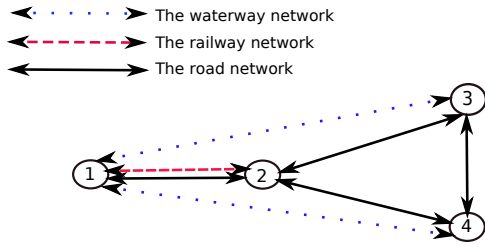


Fig. 1. An intermodal transport network. Each doubled-headed arc in the figure represents two directed links with opposite directions.

of the route choice is not taken into account. In [12], the multiple node method is used to represent each city by more than one node when the city has different modes of transport. Similarly, the multiple node method can be adopted for modeling intermodal terminals within intermodal transport networks. Meanwhile, the transport times and costs for links and mode transfers are also considered in [12]. The models in [7], [12], however, are static models.

Some work has been done on modeling *dynamics* of intermodal transport networks, especially on modeling the time dependence of route choice, but the presented models have some limitations. In [13], dynamic link transport times and switching delays are considered for transport networks with multiple transport modes. In [14], a parallel algorithm is proposed for solving the time-dependent transport problem with a model considering time-dependent link traveling times/costs, and mode transferring costs. However, the dynamic behavior (e.g., unloading/loading containers, storing containers, etc.) of nodes in the network is not taken into account in [13], [14]. Moreover, little work has been done involving constraints on the capacities of links and nodes. In this paper, we will investigate not only dynamics of container transport/transfer time and cost in links, but also dynamics of unloading, loading and storing containers in nodes. We will also consider the container entering and transport capacity of links and the container storing capacity of nodes in the network.

### III. A GENERAL FRAMEWORK FOR MODELING INTERMODAL TRANSPORT NETWORKS

Different types of transport networks show some common behaviors, such as container handling operations in nodes, transport times for crossing links, etc. At the same time, they also show distinguishing behaviors because of the particular physical nature of each type of transport infrastructure and the corresponding management strategies in intermodal transport networks [8], [15], [16]. For example, freight transport over waterway networks is slower but more environmentally friendly than freight transport over road networks; by implementing a pre-scheduled timetable, the corresponding transport times of railway networks are much more reliable than those of road networks or waterway networks. In order to develop a general framework for modeling dynamics of an intermodal transport network, it is, therefore, reasonable to first get the generic model of the intermodal transport network with a general dynamic model for various connections. Next, the generic model can be extended to capture the individual

dynamic behaviors of different transport connections based on their individual characteristics. Below, a generic intermodal transport network model will be proposed first.

In an intermodal transport network, intermodal terminals function as the container handling points where containers can be loaded, unloaded, stored, and transferred. We use the multiple node method to model intermodal terminals in the network. Due to the large-scale nature of intermodal transport networks, it is impractical to consider the movement of each individual container within the network due to the high computational complexity. A similar approach as in [17] for modeling the movement of automated vehicles in intelligent vehicle highway systems is to adopt a more aggregate model for the movement of containers in intermodal transport networks so as to obtain a trade-off between model accuracy and computational complexity of the model. Therefore, we will model the movement of containers as a flow. From the container flow perspective, the behavior of nodes and links in the network is identified by the incoming container flows and the outgoing container flows. In this case, the time step size for the discrete-time model of the intermodal transport network should be chosen large enough (e.g., an hour) in order to capture the evolution of container flows in the network while neglecting the details, e.g., considering the average container flows during each time step size.

#### A. A generic intermodal transport network model

In this paper, intermodal transport networks are considered to be the integration of road networks, railway networks, and waterway networks. Nevertheless, the generic intermodal transport network model that will be proposed here can be straightforwardly extended to include other types of transport, e.g., air transport, too.

Each single-mode transport network is represented by a directed graph  $\mathcal{G}_m(\mathcal{V}_m, \mathcal{E}_m)$ ,  $m \in \{\text{truck, train, barge}\}$  where  $\mathcal{V}_m$  is a finite nonempty node set, and  $\mathcal{E}_m \subseteq \mathcal{V}_m \times \mathcal{V}_m$  is the link set of all available connections among nodes within this transport network. The corresponding intermodal transport network can be represented as one directed graph  $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{M})$ . The node set  $\mathcal{V} = \mathcal{V}_{\text{truck}} \cup \mathcal{V}_{\text{train}} \cup \mathcal{V}_{\text{barge}} \cup \mathcal{V}_{\text{store}}$  is a finite nonempty set with the storage node set,  $\mathcal{V}_{\text{store}}$ , representing storage yards shared by different single-model terminals inside each intermodal terminal of the network. The set  $\mathcal{M} = \mathcal{M}_1 \cup \mathcal{M}_2$  represents transport modes and mode transfer types in the network with  $\mathcal{M}_1 = \{\text{truck, train, barge, store}\}$  and  $\mathcal{M}_2 = \{m_1 \rightarrow m_2 | m_1, m_2 \in \mathcal{M}_1 \text{ and } m_1 \neq m_2\}$ . The link set  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} \times \mathcal{M}$  represents all available connections among nodes. There are two kinds of links in  $\mathcal{E}$ , transport links and transfer links:

- A transport link  $l_{i,j}^m, i, j \in \mathcal{V}_m$  and  $i \neq j$ , in the link set  $\mathcal{E}$  denotes that a transport connection, using transport mode  $m \in \mathcal{M}_1 \setminus \{\text{store}\}$ , from node  $i$  to node  $j$  exists.
- A transfer link  $l_{i,j}^m, i, j \in \mathcal{V}, i \neq j$ , and  $m \in \mathcal{M}_2$  in the link set  $\mathcal{E}$  denotes that a mode transfer connection with transfer type  $m$  exists in node  $i$ .

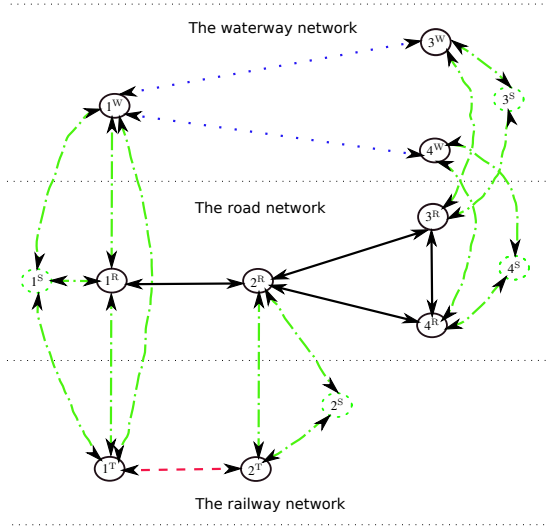


Fig. 2. An intermodal transport network model. The dotted blue arcs, the solid black arcs, the dashed red arcs, and the dash-dotted green arcs indicate 4 transport links of the waterway network, 8 transport links of the road network, 2 transport links of the railway network, and 30 transfer links among three different types of transport modes (barges, trucks and trains) in nodes of the intermodal transport network, respectively. The dashed green nodes indicate the storage nodes. Each doubled-headed arc in the figure represents two directed links with opposite directions.

Figure 2 shows the corresponding intermodal transport network model of the intermodal transport network illustrated in Figure 1. This model consists of 13 nodes, 14 transport links, and 30 transfer links. There are four storage nodes,  $\mathcal{V}_{\text{store}} = \{1^S, 2^S, 3^S, 4^S\}$  indicated by the dashed green nodes, and three types of individual transport networks or modes of transport available in the network:

- The waterway network  $\mathcal{G}_{\text{barge}}(\mathcal{V}_{\text{barge}}, \mathcal{E}_{\text{barge}})$  consists of the node set  $\mathcal{V}_{\text{barge}} \in \{1^W, 3^W, 4^W\}$  and the link set  $\mathcal{E}_{\text{barge}}$  with 4 links indicated by the dotted blue arcs in the figure.
- The road network  $\mathcal{G}_{\text{truck}}(\mathcal{V}_{\text{truck}}, \mathcal{E}_{\text{truck}})$  consists of the node set  $\mathcal{V}_{\text{truck}} \in \{1^R, 2^R, 3^R, 4^R\}$  and the link set  $\mathcal{E}_{\text{truck}}$  with 8 links indicated by the solid black arcs in the figure.
- The railway network  $\mathcal{G}_{\text{train}}(\mathcal{V}_{\text{train}}, \mathcal{E}_{\text{train}})$  consists of the node set  $\mathcal{V}_{\text{train}} \in \{1^T, 2^T\}$  and the link set  $\mathcal{E}_{\text{train}}$  with 2 links indicated by the dashed red arcs in the figure.

There are also 30 transfer links, indicated by the dash-dotted green arcs in the figure connecting nodes, which physically locate inside each intermodal terminal of the network.

A transport demand is defined as a group of containers sharing the origin node and the final destination node within an intermodal transport network. The evolution of each transport demand,  $d_{o,d}$ , over time implies the movement of a certain number of containers from their origin node  $o$  to their final destination node  $d$ , where  $(o, d)$  belongs to the set  $\mathcal{O}_{od} \subseteq \mathcal{V} \times \mathcal{V}$ , which is the set of all origin-destination pairs.

Dynamics of intermodal transport networks consist of three parts: dynamics of nodes, dynamics of links, and dynamics of the interconnections among the nodes and the links within the network. These dynamics will be modeled in more detail below. With the dynamics of an intermodal transport network,

the prediction of the behavior of the network and the optimization of route choices become possible by using on-line optimization and real-time route control.

### B. Nodes in the intermodal transport network

Nodes in the intermodal transport network can be categorized into three types based on the roles that they play in container transport: origin nodes, transfer nodes, and destination nodes. For different transport demands, a given node in the intermodal transport network might function as a different type of node at the same time. Therefore, the node model should capture these three different behaviors at the same time.

*Node dynamics:* We consider a discrete-time model with  $T_s$  (hour) as the time step size. For each transport demand  $d_{o,d}$ ,  $(o, d) \in \mathcal{O}_{od}$ , the dynamics of transport demand  $d_{o,d}$  in node  $i$  can be formulated as

$$\begin{aligned}
 x_{i,o,d}(k+1) &= x_{i,o,d}(k) + \sum_{(j,m) \in \mathcal{N}_i^{\text{in}}} u_{j,i,o,d}^m(k) T_s \\
 &\quad - \sum_{(j,m) \in \mathcal{N}_i^{\text{out}}} y_{i,j,o,d}^m(k) T_s + d_{i,o,d}^{\text{in}}(k) T_s \\
 &\quad - d_{i,o,d}^{\text{out}}(k) T_s
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 x_{i,o,d}(k), u_{j,i,o,d}^m(k), y_{i,j,o,d}^m(k), d_{i,o,d}^{\text{in}}(k), d_{i,o,d}^{\text{out}}(k) &\geq 0 \\
 \forall (o, d) \in \mathcal{O}_{od}, \forall i, j \in \mathcal{V}, \forall m \in \mathcal{M}, \forall k, &\tag{2}
 \end{aligned}$$

where

- The value of  $x_{i,o,d}(k)$  (TEUs) is the number of containers corresponding to transport demand  $d_{o,d}$  and staying at node  $i$  at time step  $k$ .
- The value of  $u_{j,i,o,d}^m(k)$  (TEUs/hour) is the container flow corresponding to transport demand  $d_{o,d}$  and entering node  $i$  through link  $l_{j,i}^m$ ,  $(j, m) \in \mathcal{N}_i^{\text{in}}$  at time step  $k$ . The set  $\mathcal{N}_i^{\text{in}}$  is defined as

$$\mathcal{N}_i^{\text{in}} = \{(j, m) \mid l_{j,i}^m \text{ is an incoming link for node } i\}.$$

The value of  $u_{j,i,o,d}^m(k)$  equals zero when  $i = o$  (which implies that node  $i$  is actually the origin node  $o$  of the transport demand  $d_{o,d}$ ).

- The value of  $y_{i,j,o,d}^m(k)$  (TEUs/hour) is the container flow corresponding to transport demand  $d_{o,d}$  and leaving node  $i$  through link  $l_{i,j}^m$ ,  $(j, m) \in \mathcal{N}_i^{\text{out}}$  at time step  $k$ . The set  $\mathcal{N}_i^{\text{out}}$  is defined as

$$\mathcal{N}_i^{\text{out}} = \{(j, m) \mid l_{i,j}^m \text{ is an outgoing link for node } i\}.$$

The value of  $y_{i,j,o,d}^m(k)$  equals zero when  $i = d$  (which implies that node  $i$  is actually the final destination node  $d$  of the transport demand  $d_{o,d}$ ).

- The value of  $d_{i,o,d}^{\text{in}}(k)$  (TEUs/hour) is the container flow corresponding to transport demand  $d_{o,d}$  and entering node  $i$  from the outside of the network at time step  $k$ . The value of  $d_{i,o,d}^{\text{in}}(k)$  equals  $d_{o,d}(k)$  when  $i = o$ , and otherwise it is zero.
- The value of  $d_{i,o,d}^{\text{out}}(k)$  (TEUs/hour) is the container flow corresponding to transport demand  $d_{o,d}$  and arriving at

the final destination node  $i$  at time step  $k$ . The value of  $d_{i,o,d}^{\text{out}}(k)$  equals  $\sum_{(j,m) \in \mathcal{N}_i^{\text{in}}} u_{j,i,o,d}^m(k)$  when  $i = d$  (here, we assume that containers coming from each transport demand will immediately leave the network once they arrive their destination), and otherwise it is zero.

*Node properties and constraints:* Each node has several properties that arise from the physical infrastructure. These properties can be modeled as parameters and constraints associated with the node in the intermodal transport network model. Properties of the nodes include:

- The handling capacity of the equipment to unload and load containers, denoted by  $h_i^{\text{in}}$  (TEUs/hour) and  $h_i^{\text{out}}$  (TEUs/hour), respectively.
- The storage capacity to store containers in the node,  $S_i$  (TEUs).
- The cost associated with storing containers in the node at time step  $k$ ,  $C_{i,\text{store}}(k)$  (€/TEU/hour).

The corresponding constraints in node  $i$  can be formulated as:

$$\sum_{(o,d) \in \mathcal{O}_{\text{od}}} \sum_{(j,m) \in \mathcal{N}_i^{\text{in}}} u_{j,i,o,d}^m(k) \leq h_i^{\text{in}} \quad (3)$$

$$\sum_{(o,d) \in \mathcal{O}_{\text{od}}} x_{i,o,d}(k) \leq S_i \quad (4)$$

$$\sum_{(o,d) \in \mathcal{O}_{\text{od}}} \sum_{(j,m) \in \mathcal{N}_i^{\text{out}}} y_{i,j,o,d}^m(k) \leq h_i^{\text{out}}. \quad (5)$$

### C. Links in the intermodal transport network

Each link connects two nodes in the network and provides transport services (transporting containers between two nodes using the same mode of transport or transferring containers between two nodes with different modalities inside one intermodal terminal). It takes a certain period of time, called transport time, for containers to cross the link. For a given link, the transport time might be fixed or vary according to the different management strategies. Moreover, the current operating conditions of physical infrastructures, such as the level of traffic density in the link, also influence the transport time.

One of the basic requirements for container transport is to deliver containers to their destination at the stipulated time. Therefore, transport time is one crucial element that should be taken into account when analyzing the behavior of a link and also transport demands evolving over transport networks. In general, the transport time of a given link is influenced not only by the traffic flows corresponding to the container transport but also by the external traffic flows in that link (e.g., the traffic flows corresponding to private cars, buses, and other trucks in a link of road networks). However, in this generic model, we make the following two basic assumptions:

- The external traffic flows in a link are assumed to be dominant and determine the transport time on this link.
- The transport time for a given link is determined when containers enter that link and it is assumed to be fixed for this container for the remaining time that is used to get to the end of the link.

When containers enter link  $l_{i,j}^m$  at time step  $k$ , a certain period of transport time  $T_{i,j}^m(k)$  is taken to cross the link:

$$\begin{aligned} T_{i,j}^m(k) &= t_{i,j}^m(k) T_s \\ t_{i,j}^m(k) &\in \mathbb{N} \setminus \{0\} \\ t_{i,j}^m(k) &\leq t_{i,j}^{m,\text{max}}, \end{aligned}$$

where the maximum transport time of link  $l_{i,j}^m$  is  $t_{i,j}^{m,\text{max}} T_s$ .

*Link dynamics:* The dynamics of each transport demand  $d_{o,d}$ ,  $(o,d) \in \mathcal{O}_{\text{od}}$  in link  $l_{i,j}^m$  can now be formulated as

$$\begin{aligned} q_{i,j,o,d}^{m,\text{out}}(k) &= \sum_{\substack{k_e = k - t_{i,j}^{m,\text{max}} \\ k_e + t_{i,j}^m(k_e) = k}}^{k-1} q_{i,j,o,d}^{m,\text{in}}(k_e) \quad (6) \\ x_{i,j,o,d}^m(k+1) &= x_{i,j,o,d}^m(k) + \left( q_{i,j,o,d}^{m,\text{in}}(k) - q_{i,j,o,d}^{m,\text{out}}(k) \right) T_s, \quad (7) \end{aligned}$$

where

- The value of  $q_{i,j,o,d}^{m,\text{out}}(k)$  (TEUs/hour) is the container flow corresponding to transport demand  $d_{o,d}$  and leaving link  $l_{i,j}^m$  at time step  $k$ .
- The value of  $q_{i,j,o,d}^{m,\text{in}}(k)$  (TEUs/hour) is the container flow corresponding to transport demand  $d_{o,d}$  and entering link  $l_{i,j}^m$  at time step  $k$ .
- The value of  $x_{i,j,o,d}^m(k)$  (TEUs) is the number of containers corresponding to transport demand  $d_{o,d}$  and presenting in link  $l_{i,j}^m$  at time step  $k$ .

*Link properties and constraints:* There are some properties associated with each link in an intermodal transport network. These properties are:

- Transport or transfer capacity, the maximum number of containers that can stay within a link,  $C_{i,j}^m$  (TEUs).
- Transport or transfer cost, the cost that has to be paid concerning the use of a link to transport or transfer containers at time step  $k$ ,  $C_{i,j,\text{Tran}}^m(k)$  (€/TEU/hour).
- Entering capacity, the maximum container flow that can enter a link,  $C_{i,j}^{m,\text{in}}$  (TEUs/hour).

The corresponding constraints on each link  $l_{i,j}^m$  of the network  $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{M})$  can be formulated as:

$$\sum_{(o,d) \in \mathcal{O}_{\text{od}}} x_{i,j,o,d}^m(k) \leq C_{i,j}^m \quad (8)$$

$$\sum_{(o,d) \in \mathcal{O}_{\text{od}}} q_{i,j,o,d}^{m,\text{in}}(k) \leq C_{i,j}^{m,\text{in}}. \quad (9)$$

### D. Dynamics of the complete intermodal transport network

The evolutions of container flows on the incoming and outgoing links of the nodes in  $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{M})$  are connected by

$$\begin{aligned} q_{i,j,o,d}^{m,\text{in}}(k) &= y_{i,j,o,d}^m(k) \quad \forall i \in \mathcal{V}, \forall (j,m) \in \mathcal{N}_i^{\text{out}} \\ &\quad \forall (o,d) \in \mathcal{O}_{\text{od}}, \forall k \quad (10) \end{aligned}$$

$$\begin{aligned} u_{i,j,o,d}^m(k) &= q_{i,j,o,d}^{m,\text{out}}(k) \quad \forall i \in \mathcal{V}, \forall (j,m) \in \mathcal{N}_i^{\text{in}}, \\ &\quad \forall (o,d) \in \mathcal{O}_{\text{od}}, \forall k, \quad (11) \end{aligned}$$

where

- (10) connects node  $i$  to each outgoing link  $l_{i,j}^m$  by requiring that the value of the container flow going out node  $i$  through link  $l_{i,j}^m$  is equal to the value of the container flow entering the link  $l_{i,j}^m$  at time step  $k$ .
- (11) connects each incoming link  $l_{i,j}^m$  to node  $j$  by requiring that the value of the container flow leaving link  $l_{i,j}^m$  and entering into the node  $j$  is equal to the value of the container flow entering the node  $j$  at time step  $k$  through link  $l_{i,j}^m$ .

The dynamics of the complete intermodal transport network  $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{M})$  are driven by the transport demands  $\mathcal{O}_{od}$  and the routing choices for each transport demand  $d_{o,d}$  at each node of the network. In our model, these routing choices are determined by minimizing the total transport time and the total delivery cost (that is the sum of the transport/transfer cost, and the storage cost.) of transport demands in the network. Therefore, the objective of this routing choice problem is defined as below (12), in which the terms  $J_1, J_3$  are the total transport time and the total delivery cost of transport demands  $\mathcal{O}_{od}$  and the terms  $J_2, J_4$  are the penalties on the unfinished transport demands at the end of the planning horizon:

$$J = \alpha(J_1 + J_2) + J_3 + J_4 \quad (12)$$

with

$$J_1 = \sum_{(o,d) \in \mathcal{O}_{od}} w_{o,d} \left[ \sum_{k=1}^{N-1} \left[ \sum_{i \in \mathcal{V}} x_{i,o,d}(k) T_s \right. \right. \\ \left. \left. + \sum_{(i,j,m) \in \mathcal{E}} x_{i,j,o,d}^m(k) T_s \right] \right] \quad (13)$$

$$J_2 = \sum_{(o,d) \in \mathcal{O}_{od}} w_{o,d} \left[ \sum_{i \in \mathcal{V}} x_{i,o,d}(N) r_{i,d} \right. \\ \left. + \sum_{(i,j,m) \in \mathcal{E}} x_{i,j,o,d}^m(N) r_{i,j}^m \right] \quad (14)$$

$$J_3 = \sum_{(o,d) \in \mathcal{O}_{od}} w_{o,d} \left[ \sum_{k=1}^{N-1} \left[ \sum_{i \in \mathcal{V}} x_{i,o,d}(k) T_s C_{i,\text{store}}(k) \right. \right. \\ \left. \left. + \sum_{(i,j,m) \in \mathcal{E}} x_{i,j,o,d}^m(k) T_s C_{i,j,\text{Tran}}^m(k) \right] \right] \quad (15)$$

$$J_4 = \sum_{(o,d) \in \mathcal{O}_{od}} w_{o,d} \left[ \sum_{i \in \mathcal{V}} x_{i,o,d}(N) c_{i,d} \right. \\ \left. + \sum_{(i,j,m) \in \mathcal{E}} x_{i,j,o,d}^m(N) c_{i,j}^m \right], \quad (16)$$

where

- The value of  $w_{o,d} \in (0, 1]$  indicates the relative priority of the transport demand  $d_{o,d}$ . The relation  $\sum_{(o,d) \in \mathcal{O}_{od}} w_{o,d} = 1$  always holds.

- The average transport time and the average delivery cost for containers being transported from node  $i$  to destination node  $d$  are  $r_{i,d}$  and  $c_{i,d}$ , respectively. They can be determined from statistical data.
- The average transport time and the average delivery cost for containers being transported from link  $l_{i,j}^m$  to destination node  $d$  is  $r_{i,j}^{m,d}$  and  $c_{i,j}^{m,d}$ , respectively. They can be determined from statistical data.
- The parameter  $\alpha$  (€/hour) is the conversion factor for converting the transport time to the equivalent cost.
- The planning horizon  $N \in \mathbb{N} \setminus \{0\}$  is a multiple of  $T_s$ .

*Network dynamics:* Dynamics of the complete intermodal transport network  $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{M})$  can be formulated as an optimization problem by denoting:

$$\min_{\tilde{x}_1, \tilde{x}_2, \tilde{y}, \tilde{u}} J(\tilde{x}_1, \tilde{x}_2, \tilde{y}, \tilde{u}) \quad (17)$$

subject to (1) – (11),

where

- $\tilde{x}_1$  contains all  $x_{i,o,d}(k)$ , for  $i \in \mathcal{V}, (o,d) \in \mathcal{O}_{od}, k = 1, \dots, N$ .
- $\tilde{x}_2$  contains all  $x_{i,j,o,d}^m(k)$ , for  $(i,j,m) \in \mathcal{E}, (o,d) \in \mathcal{O}_{od}, k = 1, \dots, N$ .
- $\tilde{y}$  contains all  $y_{i,j,o,d}^m(k)$ , for  $i \in \mathcal{V}, (j,m) \in \mathcal{N}_i^{\text{out}}, (o,d) \in \mathcal{O}_{od}, k = 1, \dots, N$ .
- $\tilde{u}$  contains all  $u_{j,i,o,d}^m(k)$ , for  $i \in \mathcal{V}, (j,m) \in \mathcal{N}_i^{\text{in}}, (o,d) \in \mathcal{O}_{od}, k = 1, \dots, N$ .

This problem (17) is a linear programming problem, which can be solved very efficiently using state-of-the-art solvers [18].

#### IV. SIMULATION EXPERIMENTS

In this section, we present a simple simulation for routing container transport over an intermodal transport network to illustrate the behavior of the generic intermodal transport network model proposed in this paper. First, the network set-up is introduced. Next, we analyze the simulation results.

##### A. Scenario

We consider a simple intermodal transport network, consisting of three different types of transport networks that are connected at three intermodal terminals. The set-up of the corresponding intermodal transport network model is shown in Figure 3. The network model comprises 10 nodes and 32 links. The corresponding link transport/transfer time and transport cost parameters are shown as labels of each link in Figure 3. For example, labels "4/10" for the transport link from node  $1^R$  to node  $2^R$  represents that the transport time to cross this link is 4 hours and the transport cost is 10 €/TEUs/hour. Note that, for simplicity, the transport/transfer times of links are considered to be constant. The corresponding capacity parameters for nodes and links are given in Tables I & II.

The simulation time step,  $T_s$ , is chosen to be one hour. We simulate the network for a period of 24 hours. We consider a piecewise constant transport demand in the simulation period from node  $1^R$  to node  $3^R$  as given in Table III. The average

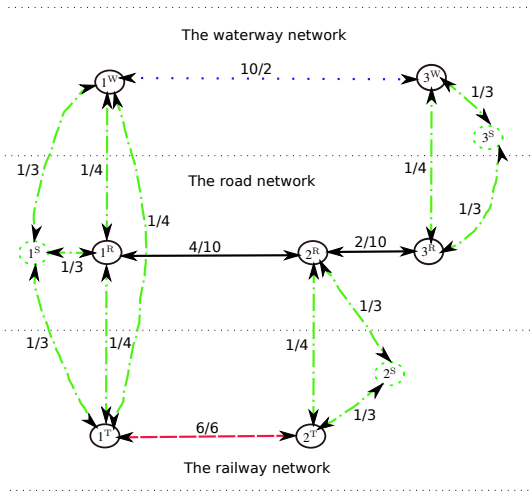


Fig. 3. The set-up of the simulation network model. Each doubled-headed arc in the figure represents two directed links with opposite directions.

TABLE I  
NODE PARAMETERS

Nodes	$h_i^{\text{in}}$ (TEUs/hour)	$h_i^{\text{out}}$ (TEUs/hour)	$S_i$ (TEUs)
$1^R, 2^R, 3^R, 1^T, 2^T$	10000	10000	500
$1^W, 2^W$	10000	10000	500
$1^S, 2^S, 3^S$	10000	10000	100000

TABLE II  
LINK PARAMETERS

Links	Road link (TEUs/hour)	Rail link (TEUs/hour)	Water link (TEUs/hour)	Transfer link (TEUs/hour)
$C_{i,j}^{m,\text{in}}$	100	200	350	1000

TABLE III  
TRANSPORT DEMAND

Period (hours)	0 – 5	5 – 24
$d_{1^R, 3^R}$ (TEUs/hour)	500	0

TABLE IV  
THE AVERAGE TRANSPORT TIME  $r_{i,d}$  (HOURS)

	$1^S$	$1^W$	$1^R$	$1^T$	$2^R$	$2^T$	$2^S$	$3^W$	$3^R$	$3^S$
$1^S$	0	1	1	1	6	8	10	18	21	20
$1^W$	1	0	2	2	7	9	10	16	19	18
$1^R$	1	2	0	2	4	9	10	10	8	9
$1^T$	1	2	2	0	9	6	8	15	13	14
$2^R$	6	7	4	9	0	2	1	5	2	4
$2^T$	8	9	9	6	2	0	1	8	5	7
$2^S$	10	10	10	8	1	1	0	7	4	6
$3^W$	18	16	10	15	5	8	7	0	2	1
$3^R$	21	19	8	13	2	2	4	2	0	1
$3^S$	20	18	9	14	4	7	6	1	1	0

transport times and the average delivery costs between any pair of nodes of the network are given in Table IV and Table V, respectively. In particular, the average transport times for links are chosen to be the maximum of the corresponding average transport times of the two nodes of this link. The same approach is used for the calculation of the average delivery costs for links in the network. For the proposed scenario, the initial state of the network is taken to be empty (e.g.,  $x_{i,o,d}(k) = 0$  and  $x_{i,j,o,d}^m(k) = 0$  for  $\forall(o, d) \in \mathcal{O}_{od}, k \leq 0$ ).

TABLE V  
THE AVERAGE TRANSPORT COST  $c_{i,d}$  (EUROS)

	$1^S$	$1^W$	$1^R$	$1^T$	$2^R$	$2^T$	$2^S$	$3^W$	$3^R$	$3^S$
$1^S$	0	3	3	3	44	40	45	24	29	28
$1^W$	3	0	4	4	45	41	46	20	25	24
$1^R$	3	4	0	4	40	41	45	25	37	41
$1^T$	3	4	4	0	41	36	40	25	30	29
$2^R$	44	45	40	41	0	4	3	25	20	24
$2^T$	40	41	41	36	4	0	3	30	25	29
$2^S$	45	46	45	40	3	3	0	29	24	28
$3^W$	24	20	25	25	25	30	29	0	4	3
$3^R$	29	25	37	30	20	25	24	4	0	3
$3^S$	28	24	41	29	24	29	28	3	3	0

The value of the conversion factor  $\alpha$  influences the solution of the optimization problem (17) and accordingly the selection of the optimal routes for transporting containers. A low conversion factor means that the total delivery cost has a large influence on the optimal route choice. This implies that the routes with cheap total delivery costs will be selected even though long transport times might be required to finish these routes. Instead, for a high conversion factor the total transport time makes a great impact on the optimal route choice. Accordingly, the routes with short transport times will be chosen even though there are high transport costs associated with these routes. Therefore, we consider two different cases:

- Case A: A low conversion factor  $\alpha = 0.1$  (€/hour).
- Case B: A high conversion factor  $\alpha = 10$  (€/hour).

## B. Results and analysis

We use the Matlab Optimization Toolbox to conduct the simulation experiment. The optimal routing choices are obtained by solving (17) for both case A and case B. The evolution of the number of containers in nodes and links of the optimal routes for these two cases is illustrated in Figures 4–5. In case A, the cost terms  $J_3$  and  $J_4$  contribute to a larger part of the optimization criterion. Therefore, the cheaper routes are more likely to be selected. As been confirmed by the dashed red lines in Figures 4 and 5, the optimal route of the given transport demand is  $1^R \rightarrow 1^W \rightarrow 3^W \rightarrow 3^R$ . The container flow corresponding to the given transport demand enters origin node  $1^R$ , transfers to node  $1^W$ , and enters transport link  $1^W \rightarrow 3^W$ . Because the entering capacity of transport link  $1^W \rightarrow 3^W$  is smaller than the entering capacity of transfer link  $1^R \rightarrow 1^W$ , a certain part of the container flow will be stored in node  $1^R$  and node  $1^W$  and waits for being transferred/transported latter. Next, the container flow goes through link  $1^W \rightarrow 3^W$ , enters nodes  $3^W$ , and transfers to the destination node  $3^R$ . For the difference in entering capacity, the container flow, once leaving link  $1^R \rightarrow 1^W$  and entering node  $3^W$ , will immediately transfer to node  $3^R$  through link  $3^W \rightarrow 3^R$ . Therefore, there will be no container flow storing in node  $3^W$  and node  $3^R$ . The delivery process happens over the cheapest waterway network.

When the conversion factor  $\alpha$  increases in case B, the contribution of the equivalent cost of  $J_1$  and  $J_2$  to the optimization criterion will also increase. This implies that the faster routes will be chosen in this case. The solid blue lines



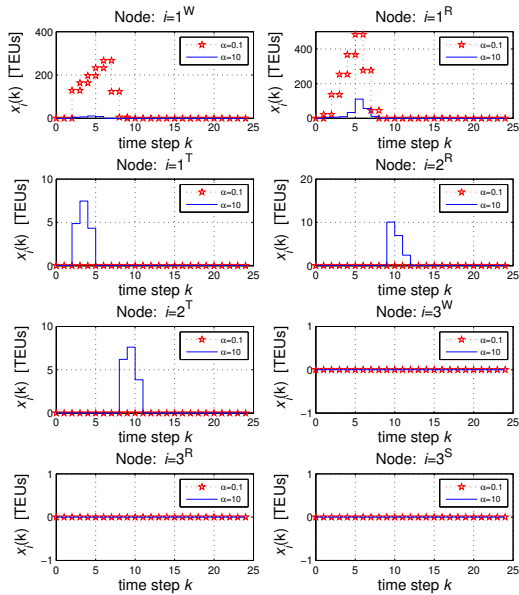


Fig. 4. The evolution of the number of containers in nodes of the optimal route for both Case A and Case B. In other nodes of the network, the number of containers is zero.

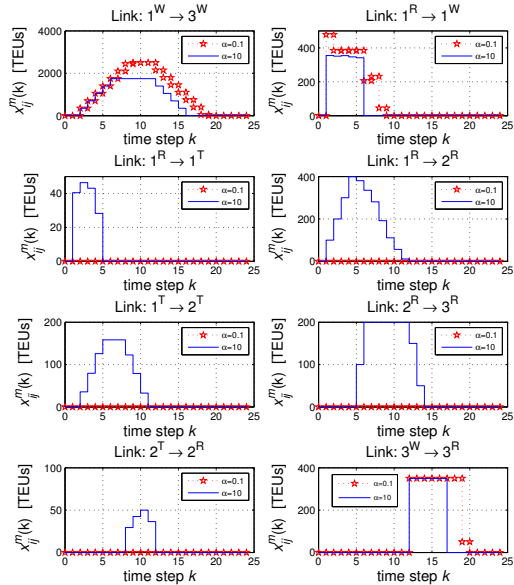


Fig. 5. The evolution of the number of containers in links of the optimal route for both Case A and Case B. In other links of the network, the number of containers is zero.

in Figures 4 and 5 demonstrate that three routes are chosen:  $1^R \rightarrow 1^W \rightarrow 3^W \rightarrow 3^R$ ;  $1^R \rightarrow 1^T \rightarrow 2^T \rightarrow 2^R \rightarrow 3^R$ ;  $1^R \rightarrow 2^R \rightarrow 3^R$ . The latter two routes confirm that the fast road and railway networks are also selected to deliver the transport demand in case B.

## V. CONCLUSIONS AND FUTURE WORK

By considering the common and distinguishing behaviors of different types of transport connections, we have proposed a general framework for modeling intermodal transport networks. Simulation experiments illustrate the behavior of the proposed model. In our future work, we will investigate the dynamic behavior of links with different transport modes and the

implementation of so-called synchomodal transport, in which container transport can easily switch between various modes of transport at terminals when necessary. The current route choice control will be embedded in a multi-level control framework and that we will also investigate efficient algorithms for the other levels as well as the interfacing of the different levels.

## ACKNOWLEDGMENTS

This research is supported by the China Scholarship Council under Grant 2011629027 and the VENI project “Intelligent multi-agent control for flexible coordination of transport hubs” (project 11210) of the Dutch Technology Foundation STW, a subdivision of the Netherlands Organization for Scientific Research (NWO).

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