A model predictive approach for baggage handling systems

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Abstract—This paper proposes a new strategy for control of baggage handling systems. Three main control issues in baggage handling systems, namely, routing and scheduling problem, empty-cart management, and line balancing, are identified and a combined control approach based on model predictive control is proposed to tackle these issues in an optimal way. It is shown that the control approach can be formulated as a linear programming problem that can be solved efficiently, and hence can be extended to large-scale baggage handling systems. The applicability and performance of the proposed approach is illustrated by a case study.

I. INTRODUCTION

In the past decade, modern baggage handling systems have been implemented in large airports to accommodate the rising demand in air travel. Such baggage handling systems are controlled by state-of-the-art techniques that are mostly tailor-made for a specific layout. However, with increasing demand, it becomes necessary to increase the efficiency and reliability of the baggage handling systems by utilizing a systematically designed control approach. Such a control approach will optimize the performance of the baggage handling system in terms of reliability and cost. A modern baggage handling system is composed of the following components:

- Loading stations, where the baggage flow enters the system either from the check-in desks or from the transfer flights.
- Unloading stations, where the pieces of baggage leave the system, and wherefrom they are transported to the planes.
- Destination coded vehicles (DCVs), which are high-speed vehicles powered by linear induction motors, carrying the pieces of baggage from the loading stations to the unloading stations. Each DCV can carry only one piece of baggage.
- An early baggage storage (EBS) where the early pieces of baggage are stored until their boarding time.
- A Network of uni-directional tracks connecting the loading stations to the unloading stations via junctions of the network.
- A switch controller at each junction that determines the path of DCVs that pass through that junction.

When pieces of baggage enter the system at the loading stations, they are boarded on DCVs. The loaded DCVs then move to pre-assigned unloading stations or to the EBS. The EBS stores the loaded DCVs and can release them at a later time. Each DCV travels on the links of the network. At each junction of the network, a switch controller determines the next link to travel on, hence assigning a path for each DCV to its destination. The DCVs are unloaded at the unloading stations and the empty DCVs are then re-routed to the loading stations.

From a high-level control perspective, there are three main control challenges related to the baggage handling systems [1], namely, i) routing and scheduling of DCVs, ii) line balancing, and iii) empty-cart management. The routing problem is the problem of routing loaded DCVs from the loading stations to unloading stations or to the EBS and the problem of routing the DCVs from EBS to the unloading stations. Line balancing is the problem of dynamically assigning empty DCVs at the unloading stations to the loading stations. Closely related to the line balancing is empty-cart management, which is the problem of routing empty DCVs from the unloading stations, through the network, to their assigned loading stations. The state-of-the-art control method uses look-up tables to control the switches at the junctions. These look-up tables are computed offline for different scenarios of system operation. However, such control schemes cannot guarantee optimal performance of the system.

Recently, it has been indicated that systematically designed control systems can improve the overall performance of baggage handling systems [2]–[4]. The authors of [3] have proposed a multi-agent approach for conveyor-based baggage handling systems that uses the shortest path algorithm for dynamical route assignment of the bags on the conveyor belts. They have shown that their method outperforms the conventional control schemes, but they have not considered achieving the optimal performance. In [2], the authors have considered model-based control of DCV-based baggage handling systems, where they have proposed an automated way of learning routing rules. In comparison with [2], we explicitly consider time windows on DCVs arrival to the destinations as well as optimality of the solution.

A solution for the routing problem based on model predictive control (MPC) has been proposed in [1]. This approach guarantees optimal performance of the system, but it is computationally prohibitive for large-scale systems. A more computationally efficient MPC-based approach for the routing problem has been proposed in [5], which arrives at a mixed integer linear programming (MILP) formulation of the problem. In [6], it has been shown that the problem in [5] can in fact be recast as a linear programming (LP) problem, which can be solved efficiently for large-scale systems. The line balancing problem has been addressed in [7], where an MPC-based approach based on non-linear programming has been proposed. The authors also have provided a sub-optimal LP approach. Nevertheless, to the best of the authors’ knowledge, the previous works have only focused on a particular control issue of the baggage handling systems.
To achieve an overall optimal performance, in this paper, we propose a control scheme based on MPC that addresses the aforementioned control problems in one integrated approach. Moreover, based on our developed model, we show that the optimization problem can be formulated as an LP problem. There are two criteria for an effective baggage handling system. First, the pieces of baggage should be delivered to their destinations with minimal delay. More specifically, they should reach their assigned unloading stations within pre-specified time windows. Second, the cost of operating the system should be minimized. This includes the energy consumption due to dispatching the DCVs within the network and the cost of storing the pieces of baggage in the EBS.

II. DYNAMICAL MODEL

A. Notation and Assumptions

The baggage handling system network can be seen as a directed graph, where nodes of the graph are composed of loading stations, unloading stations, junctions, and the EBS, and where the links represent the tracks of the system. In our mathematical description of the system, we will replace physical loading stations, unloading stations, and the EBS to refer to their extended versions. For the sake of simplicity of exposition of the model, we propose the following assumptions regarding configuration of the network:

A1 The network layout is designed in such a way that each link of the network is at least on one directed path from some loading station \( o_l \in V_1 \) to some unloading station \( d_l \in V_3 \) or from some unloading station \( d_z \in V_3 \) to some loading station \( o_z \in V_1 \).

A2 For each \( d \in V_3 \), the EBS node \( v^* \) is on at least one directed path from some loading station \( o \in V_1 \) to \( d \).

A3 The movement of DCVs on the network is approximated by a continuous flow of DCVs. The baggage queues at the unloading stations are ignored. This is because we assume either destination nodes have unlimited capacity for baggage or the pieces baggage are immediately transported to the planes upon arrival. Assumptions A1 and A2 guarantee that there is no redundant link in the network. Assumption A3 is necessary for tractability of the control problem. Even though the number of DCVs is an integer in reality, for a fairly large number of DCVs, the movement of DCVs can be approximated by continuous flows. This is not very restrictive as the computed flows can then be realized as well as possible by a lower-level control loop that determines the optimal switching pattern for the switch controllers at the junctions. Assumption A4 allows us to arrive at a linear discrete-time model of the system. Assumptions A5 and A6 are restrictive as they only simplify exposition of the model.

For loading stations nodes \( o \in V_1 \), we associate a baggage stack and a small empty-DCV stack, where pieces of baggage and empty DCVs are stored in vertical queues. The pieces of baggage are sent toward the unloading station or toward the EBS after having been loaded on the DCVs. At the unloading stations, the DCVs are offloaded and stored in empty-DCV stacks in vertical queues from where they are dispatched to the loading stations. In EBS, the loaded DCVs will be stored in vertical queues in order to be released at a later time to continue their path toward the unloading stations. In this setup, we control the flows of the network. The flows in the network are indexed based on their destinations, enabling us to distinguish between loaded DCVs and empty DCVs, and also loaded (empty) DCVs with different destinations among themselves. The DCV flows with an index \( o \in V_1 \) refer empty-DCV flows whereas the DCV flows with an index \( d \in V_3 \) refer to loaded-DCV flows. Consequently, partial DCV queues associated with different destinations are shaped along the links of the network. The total DCV queue length along a link is then given as the sum of such partial queue lengths. The system is composed of baggage and empty-DCV vertical queues at the loading stations, empty-DCV vertical queues at the unloading stations, loaded-DCV queues at the EBS, and empty and loaded DCV queues along the links of the network. Note that certain links of the network may carry both empty and loaded DCV flows.

In our mathematical model, we make use of the following notation:

- For each node \( v \in V \), \( L^i_v = \{(w,v)|w \in V, (w,v) \in A\} \) is the set of incoming links of \( v \).
- For each node \( v \in V \), \( L^o_v = \{(v,w)|w \in V, (v,w) \in A\} \) is the set of outgoing links of \( v \).
- For each link \( l \in \{(v,w)|v,w \in (V_2 \times V_2) \cap A\} \) of the network, \( q_{l,p,z} \) refers to the flow sent from the end of link \( l \) to link \( p \in L^o_w \), with final destination \( z \in V_1 \cup V_3 \).
- For each destination \( z \in V_1 \cup V_3 \), \( L_z \) denotes the set of links that are on some directed path to \( z \).

B. Model Description

Now we will derive the dynamical model of baggage handling system in discrete-time under the assumptions A1-A6.

1) Loading Stations: For each loading station node \( o \in V_1 \), let \( L^i_o = (w_o,o) \) and \( L^o_o = (o,w_o) \) respectively be the virtual incoming link and virtual outgoing link of \( o \) for some \( w_o \in V_2 \). The control variables at each loading station are the outflows of loaded DCVs, \( q_{o,L^o_o,z} \), to the outgoing
The total inflow and outflow of the DCV stack are then described by

\[ F_o^\text{in} (k + k_{\text{pp}}) = \sum_{p \in L_o^{\text{pp}}} q_{p,o}^\text{in,0} (k), \]

and

\[ F_o^\text{out} (k) = \sum_{d \in V} q_{o,d}^\text{out,0} (k), \]

where \( k_{\text{pp}} \) is the number of time steps that is required to store empty DCVs in the DCV stack. This is equal to the number of travel time steps for DCVs on link \( l_{\text{pp}} \) given by

\[ k_{\text{pp}} = \left[ \frac{s_{\text{pp}}}{v_{\text{DCV},s_{\text{pp}},T_s}} \right]. \]

The total inflow and outflow of the DCV stack are described by

\[ x_o (k + 1) = x_o (k) + T_s (F_o^\text{in} (k) - F_o^\text{out} (k)), \]

with the constraint

\[ 0 \leq x_o (k) \leq x_{o,\text{max}}, \]

where \( x_{o,\text{max}} \) is the maximum capacity of DCV stack at loading station \( o \). In order to guarantee that there is no queue along the virtual outgoing link, \( l_{\text{out}}^{V} \), its inflow and outflow must be equal, or equivalently by

\[ q_{o,l}^\text{out,0} (k) = \sum_{p \in L_{o,l}} q_{o,p}^\text{in,0} (k + k_{\text{pp}}), \]

where \( k_{\text{pp}} = \left[ \frac{s_{\text{pp}}}{v_{\text{DCV},s_{\text{pp}},T_s}} \right] \) is the number of time steps required to load a piece of baggage onto the DCVs with \( s_{\text{pp}} \) being the length of link \( l_{\text{pp}} \). The evolution of baggage queue, with destination \( d \), at loading station \( o \) is given by

\[ l_{o,d}^\text{bag} (k + 1) = l_{o,d}^\text{bag} (k) + T_s (Q_o^\text{bag} (k) - q_{o,d}^\text{out,0}, (k)), \]

with the constraint

\[ l_{o,d}^\text{bag} (k) \geq 0. \]

The total baggage queue at node \( o \) is given by

\[ l_{o}^\text{bag} (k) = \sum_{d \in V} l_{o,d}^\text{bag} (k), \]

where \( Q_o^\text{bag} (k) \) is the baggage demand at loading station \( o \) that needs to be transported to destination \( d \).

\[ \text{We use the notation } \lceil x \rceil \text{ to denote the smallest integer that is bigger than or equal to } x. \]

2) **Unloading Stations:** For each unloading station node \( d \in V_3 \), let \( l_{\text{un}}^d = (w_d, d) \) and \( l_{\text{un}}^d = (d, w_d) \) respectively be the virtual incoming link and virtual outgoing link of \( d \). The control variables at each unloading station are the empty DCV outflows, \( q_{d,o}^\text{out,0} \), to the outgoing link of \( d \) with destination \( z \in V_1 \cup V_3 \), and the inflows of loaded DCVs, \( q_{p,z}^\text{in,0} \), from the links \( p \in L_{o}^{V} \) that originate from \( z \in V_1 \cup V_3 \). First, we impose the following constraints:

\[ q_{d,o}^\text{out,0} (k) = 0 \quad \text{if } z \not\in V_1, \]

and

\[ q_{d,o}^\text{out,0} (k) \geq 0 \quad \text{otherwise}, \]

and

\[ q_{p,z}^\text{in,0} (k) = 0 \quad \text{if } z \neq d \]

\[ q_{p,z}^\text{in,0} (k) = 0 \quad \text{otherwise}. \]

The total inflow and outflow of the DCV stack are respectively given by

\[ F_d^\text{in} (k + k_{\text{pp}}) = \sum_{p \in L_{d}^{V}} q_{p,o}^\text{in,0} (k), \]

and

\[ F_d^\text{out} (k) = \sum_{o \in V_1} q_{d,o}^\text{out,0} (k), \]

where \( k_{\text{pp}} = \left[ \frac{s_{\text{pp}}}{v_{\text{DCV},s_{\text{pp}},T_s}} \right] \) is the number of time steps that is required to unload and store the DCVs in the DCV stack at the unloading station with \( s_{\text{pp}} \) being defined as the length of \( l_{\text{pp}} \). The evolution of the DCV stack is given by

\[ x_d (k + 1) = x_d (k) + T_s (F_d^\text{in} (k) - F_d^\text{out} (k)), \]

with the constraint

\[ 0 \leq x_d (k) \leq x_{d,\text{max}}, \]

where \( x_{d,\text{max}} \) being defined as the maximum capacity of DCV stack at unloading station \( d \). Since no queues are allowed at the end of the virtual links, we also need to impose

\[ q_{d,o}^\text{out,0} (k) = \sum_{p \in L_{d}^{V}} q_{p,o}^\text{in,0} (k + k_{\text{pp}}), \]

where \( k_{\text{pp}} = \left[ \frac{s_{\text{pp}}}{v_{\text{DCV},s_{\text{pp}},T_s}} \right] \) is the number of time steps that is required release the DCVs stored in the DCV stack with \( s_{\text{pp}} \) being defined as the length of \( l_{\text{pp}} \).

3) **EBS:** Let \( l_{\text{un}}^V = (v^*, w^*) \) and \( l_{\text{un}}^V = (w^*, v^*) \) be the virtual outgoing and incoming links of EBS, respectively. For the EBS node \( v^* \) and for each \( z \in V_1 \cup V_3 \), the control variables are the outflows of loaded DCVs, \( q_{v^*,z}^\text{out,0} \), with destination \( z \), and the inflows of loaded DCVs, \( q_{p,v^*,z}^\text{in,0} \), from the links \( p \in L_{o}^{V} \) whose final destination is \( z \in V_1 \cup V_3 \). We first introduce the following constraints:

\[ q_{v^*,z}^\text{out,0} (k) = 0 \quad \text{if } v^* \not\in L_z \text{ or } z \not\in V_3, \]

\[ q_{v^*,z}^\text{out,0} (k) \geq 0 \quad \text{otherwise}, \]

and

\[ q_{p,v^*,z}^\text{in,0} (k) = 0 \quad \text{if } p \not\in L_z \text{ or } v^* \not\in L_z \text{ or } z \not\in V_3 \]

\[ q_{p,v^*,z}^\text{in,0} (k) \geq 0 \quad \text{otherwise}. \]
The total inflow of DCV to the EBS with final destination $d \in V_3$ is then given as

$$F_{l \rightarrow d}^{in}(k) = \sum_{p \in I_{d}^{in}} q_{p,l \rightarrow d}(k),$$

(20)

where $k_{\text{in}} = \left\lfloor \frac{s_{\text{in}}}{v_{\text{DCV} T_{s}}} \right\rfloor$ is the number of time steps that is required to store loaded DCVs in the EBS with $s_{\text{in}}$ being defined as the length of $l_{\text{in}}$. Moreover, to guarantee no queues are shaped along the link $l$, we have

$$x_{l \rightarrow d}(k + 1) = x_{l \rightarrow d}(k) + T_{s}(F_{l \rightarrow d}^{in}(k) - q_{l \rightarrow d}(k))$$

(21)

where $x_{l \rightarrow d}(k)$ is partial DCV queue at EBS with destination $d$. Since $x_{l \rightarrow d}(k)$ cannot take negative values, we impose the constraint

$$x_{l \rightarrow d}(k) \geq 0$$

(22)

Moreover, to guarantee no queues are shaped along the virtual link, we need to impose the constraint

$$q_{l \rightarrow d}(k) = \sum_{p \in I_{d}^{out}} q_{p,l \rightarrow d}(k + k_{\text{out}}),$$

(23)

where $k_{\text{out}} = \left\lfloor \frac{s_{\text{out}}}{v_{\text{DCV} T_{s}}} \right\rfloor$ is the number of time steps that is required to release loaded DCVs from the EBS with $s_{\text{out}}$ being the length of $l_{\text{out}}$. The total DCV queue length at the EBS is given by

$$x_{l \rightarrow d}(k) = \sum_{d \in V_3} x_{l \rightarrow d}(k),$$

(24)

where we impose the constraint

$$x_{l \rightarrow d}(k) \leq x_{l \rightarrow d, \text{max}},$$

(25)

with $x_{l \rightarrow d, \text{max}}$ being the maximum capacity of EBS.

4) Links: For each $l = (v,w) \in \{(v_1,v_2) | (v_1,v_2) \in (V_2 \times V_2) \cap A\}$ and for each $z \in V_1 \cup V_3$, the control variables are the empty and loaded DCV flows, $q_{l \rightarrow z}$, from the link $l$ to its outgoing links $p \in I_{l \rightarrow z}^{out}$ whose final destination is $z$. For each destination node and each link, the flows must satisfy

$$q_{l \rightarrow z}(k) = 0, \quad \text{if} \quad p \notin I_{l \rightarrow z}^{out}$$

$$q_{l \rightarrow z}(k) \geq 0, \quad \text{otherwise}$$

(26)

The DCV inflow and outflow of link $l$ with destination $z$ are therefore described by

$$F_{l \rightarrow z}^{in}(k) = \sum_{p \in I_{l}^{in}} q_{p,l \rightarrow z}(k),$$

(27)

and

$$F_{l \rightarrow z}^{out}(k) = \sum_{p \in I_{l}^{out}} q_{l \rightarrow z}(k),$$

(28)

where $k_{i}(k)$ is the number of travel time steps for DCV on the link given by

$$k_{i}(k) = \left\lfloor \frac{s_{l} - x_{l}(k) T_{s}}{v_{\text{DCV} T_{s}}} \right\rfloor$$

(29)

with $s_{l}$ and $l_{\text{DCV}}$ respectively being the length of link $l$, and the DCV length. Moreover, $x_{l}(k)$ is the total DCV queue length along link $l$ described by

$$x_{l}(k) = \sum_{z \in V_1 \cup V_3} x_{l \rightarrow z}(k),$$

(30)

and

$$x_{l}(k) \leq x_{l, \text{max}}.$$

(31)

with $x_{l, \text{max}}$ being the maximum allowed queue length on link $l$ and $x_{l, z}(k)$ being the partial DCV queue length along link $l$ associated with destination $d$ described by

$$x_{l, z}(k + 1) = x_{l, z}(k) + T_{s}(F_{l \rightarrow z}^{in}(k) - F_{l \rightarrow z}^{out}(k)),$$

(32)

and

$$x_{l, z}(k) \geq 0.$$  

(33)

The equality and inequality constraints (2)-(33) define the set of feasible trajectories of the system.

III. MPC PROBLEM FORMULATION

In this section, we use the dynamic model introduced in Section II within the context of MPC. At every time step, based on the current state of the system and a future prediction of baggage demands, a constrained finite horizon optimization problem will be solved yielding a sequence of optimal controls. According to the receding horizon policy, only the first step of this sequence is applied to the system and this process is repeated at the next time step [8].

A. Prediction Model

We use the model introduced in Section II as the prediction model. However, in the beginning of the prediction horizon given the value of queue lengths at time step $k$, we will assume that the value of transfer delay given by (29) remains constant during the prediction horizon for time steps $k + 1, \ldots, k + N_{p} - 1$. This is necessary in order to arrive at a linear programming formulation of the optimization problem.

B. Objective Function

In line with the control objectives stated in Section I, we consider a cost function that is a weighted combination of penalty terms penalizing the queue lengths and the flows in such a way that the pieces of baggage arrive at their destination within their specified time windows and energy consumption is minimized. In this section, we make use of the shorthand notation $k_{v,z}^{1} = k_{v,z}^{\text{open}} - k_{v,z}^{\text{nom}}$, $k_{v,z}^{2} = k_{v,z}^{\text{close}} - k_{v,z}^{\text{nom}}$, where $k_{v,z}^{\text{nom}}$, for each $v \in V_2 \cup V_1 \cup \{v^1\}$ and each $z \in V_3$, refers to the number time steps the DCVs need to travel from $v$ to $z$ under nominal operating conditions$^2$ and where $k_{v,z}^{\text{open}}$, and $k_{v,z}^{\text{close}}$, respectively, denote the opening time step and closing time step of destination $d$, for each $z \in V_3$. Moreover, for $v \in V_1 \cup V_2 \cup \{v^1\}$ and $z \in V_3$, we define the weighting functions

$$C_{v,z}^{1}(k) = \begin{cases} 
0 & \text{if } k \leq k_{v,z}^{1} \\
 r_{0} + m_{1}(k - k_{v,z}^{1}) & \text{if } k_{v,z}^{1} < k \leq k_{v,z}^{2} \\
 r_{0} + m_{1}(k_{v,z}^{2} - k_{v,z}^{1}) + m_{2}(k - k_{v,z}^{2}) & \text{if } k > k_{v,z}^{2},
\end{cases}$$

(34)

and

$$C_{v,z}^{0}(k) = \begin{cases} 
0 & \text{if } k \leq k_{v,z}^{2} \\
 t_{0} + n_{2}(k - k_{v,z}^{2}) & \text{if } k > k_{v,z}^{2}.
\end{cases}$$

(35)

where $r_{0}$, $m_{1}$, $m_{2}$, $t_{0}$, and $n_{2}$ are strictly positive constants.

$^2$The nominal travel times can be obtained based on historical data collected for different demand scenarios.
For loading station \( o \in V_l \), the cost at time step \( k \) associated with the baggage queues introduced in Section II-B.1 is defined as:
\[
J_{\text{bag}}^{\text{LS}}(k) = \sum_{o \in V_l} \sum_{d \in L_o} C_{\text{bag}}(d) x_{o,d}(k), \quad (36)
\]
where \( C_{\text{bag}}(d) = C_{\text{bag}}^{e,0}(k) \). Note that, since the early pieces of baggage should be sent to the EBS, we associate a constant weight \( r_0 \) with them for \( k \leq k_{d}^{1} \). Within the time window of the destination, the weight increases with constant slope \( m_1 \). Since late baggage arrival at the destination is not desired, the slope for \( k \geq k_{d}^{2} \) is greater. The cost term associated with the loaded DCV flows is defined as:
\[
J_{\text{flow}}^{\text{LS}}(k) = \sum_{o \in V_l} \sum_{d \in L_o} C_{\text{flow}}(d) q_{o,d}^{\text{in},0}(k), \quad (37)
\]
where \( C_{\text{flow}}(d) = C_{\text{flow}}^{e,0}(k) \). To avoid indefinite flow circulations within the network, we assign a constant weight \( t_0 > 0 \) for \( k \leq k_{d}^{1} \). This allows loaded DCVs to travel to the EBS or the unloading stations before \( k \leq k_{d}^{2} \). The weight increases with slope \( n_2 \) for \( k \leq k_{d}^{2} \) since flows of late DCVs are not desired. The cost associated with the empty DCV flows is defined as:
\[
J_{\text{empty}}^{\text{LS}}(k) = t_0 \sum_{o \in V_l} \sum_{d \in L_o} q_{p,o}^{\text{in},0}(k). \quad (38)
\]
For links \( l = (v,w) \in A \) defined in Section II-B.4, we penalize the loaded DCV queues, loaded DCV flows, and empty DCV flows. Using the notation of Section II-B.4, the cost of loaded DCV queues at time step \( k \) is defined as:
\[
J_{l}^{\text{DCV}}(k) = \sum_{d \in L_v} \sum_{d \in L_v} C_{d}^{\text{DCV}}(d) x_{l,d}(k), \quad (39)
\]
where \( C_{d}^{\text{DCV}}(d) = C_{d}^{b}(k) \). The choice of this weighting function can be justified using the same argument set forth for the baggage queues.

The cost associated with loaded DCV flows, \( q_{l,p,d} \), is defined as:
\[
J_{l}^{\text{flow}}(k) = \sum_{d \in L_v} \sum_{d \in L_v} C_{d}^{\text{flow}}(d) \sum_{p \in L_{\text{us}}} q_{l,p,d}(k), \quad (40)
\]
where \( C_{d}^{\text{flow}}(k) = C_{d}^{b}(k) \). The cost of empty DCV flows, \( q_{l,p,o} \), is given by
\[
J_{l}^{\text{empty}}(k) = t_0 \sum_{o \in V_l} \sum_{d \in L_o} q_{p,o}(k). \quad (41)
\]
Please note that we associate a constant weight \( t_0 > 0 \) for empty DCV flows to avoid arbitrary empty DCV flow circulations.

For the EBS, we penalize the DCV queue lengths at the EBS and the loaded DCV flow to and from the EBS. Using the notation of Section II-B.3, the cost of loaded DCV queues in the EBS at time step \( k \) is defined as:
\[
J_{EBS}^{\text{DCV}}(k) = \sum_{d \in L} C_{d}^{\text{DCV}}(d) x_{d}(k), \quad (42)
\]
where \( C_{d}^{\text{DCV}}(d) = C_{d}^{b}(k) - r_0 \). Note that since \( C_{d}^{\text{DCV}} \) is zero for \( k \leq k_{d}^{1} \), the DCVs will stay in EBS until the beginning of the time window. During the time-window, \( C_{d}^{\text{DCV}} \) increases, forcing the loaded DCVs to leave the EBS.

The penalty term associated with the inflows of loaded DCVs to EBS is defined as:
\[
J_{EBS}^{\text{inflow}}(k) = \sum_{d \in V_l} \left( C_{d}^{\text{inflow}}(k) \sum_{p \in L_{\text{us}}} q_{p,o}^{\text{in},0}(k) \right), \quad (43)
\]
where \( C_{d}^{\text{inflow}}(k) = C_{d}^{b}(k) - t_0 \). The weight associated with the EBS inflow is zero for \( k \leq k_{d}^{2} \). This allows the loaded DCVs into the EBS initially. For \( k \leq k_{d}^{2} \), the weight increases with slope \( n_2 \), preventing the loaded DCVs from entering the EBS.

The penalty term associated with the outflows of loaded DCVs from EBS is defined as:
\[
J_{EBS}^{\text{outflow}}(k) = \sum_{d \in V_l} C_{d}^{\text{outflow}}(k) q_{d}^{\text{in},0}(k), \quad (44)
\]
where
\[
C_{d}^{\text{outflow}}(k) = \begin{cases} 
- n_2 (k - k_{d}^{1}) & \text{if } k \leq k_{d}^{2} \\ 0 & \text{if } k > k_{d}^{2} \end{cases} \quad (45)
\]
Since \( C_{d}^{\text{outflow}} > 0 \) for \( k \leq k_{d}^{2} \), the DCVs stored in the EBS are not released until the end of the time window is approaching.

For the unloading stations \( d \in V_l \), using the notation of Section II-B.2, we penalize the empty DCVs outflows, \( q_{d}^{\text{out},p,o} \), \( d \), and the loaded DCVs inflows, \( q_{d}^{\text{in},0} \), \( d \). The cost associated with loaded DCV flows is defined as:
\[
J_{US}^{\text{flow}}(k) = \sum_{d \in V_l} C_{d}^{\text{flow}}(k) \sum_{p \in L_{\text{us}}} q_{d}^{\text{in},0}(k), \quad (46)
\]
where
\[
C_{d}^{\text{flow}}(k) = \begin{cases} 
t_0 - n_1 (k - k_{d}^{1}) & \text{if } k \leq k_{d}^{1} \\ t_0 & \text{if } k_{d}^{1} < k \leq k_{d}^{2} \\ t_0 + n_2 (k - k_{d}^{2}) & \text{if } k > k_{d}^{2} \end{cases} \quad (47)
\]
and where \( n_1 > 0 \) is a constant. This particular shape of \( C_{\text{flow}} \), allows loaded DCV inflows to the unloading station during the time window associated with the unloading station as the early and late flows are associated with higher weights.

The cost associated with empty-DCV flows is defined as:
\[
J_{US}^{\text{empty}}(k) = t_0 \sum_{o \in V_l} \sum_{d \in L_o} q_{d}^{\text{out},0}(k), \quad (48)
\]
where we have associated a constant weight to the empty-DCV flows to prevent indefinite empty DCV circulations in the network; thus, minimizing energy consumption.

The total cost function at time step \( k \) is therefore given by
\[
J(k) = J_1(k) + \alpha_1 J_2(k) + \alpha_2 J_3(k), \quad (49)
\]
where \( \alpha_0 > 0, \alpha_0 > 0, \) and \( \alpha_0 > 0 \) are constants indicating the relative importance of the respective component of the objective function, and where
\[
J_1(k) = J_{\text{bag}}^{\text{LS}}(k) + J_{\text{flow}}^{\text{LS}}(k) + J_{\text{flow}}^{\text{LS}}(k) \quad J_{\text{flow}}^{\text{LS}}(k) \quad (50)
\]
is the total cost of baggage stacks and loaded DCV queues, and
\[
J_2(k) = J_{\text{flow}}^{\text{flow}}(k) + J_{\text{flow}}^{\text{flow}}(k) + J_{\text{flow}}^{\text{flow}}(k) + J_{\text{flow}}^{\text{flow}}(k) + J_{\text{flow}}^{\text{flow}}(k) \quad (51)
\]
is the total cost of loaded DCV flows, and
\[ J_2(k) = J_{1,L}(k) + J_{2,L}(k) \]
is the total cost of empty DCV flows.

Let \( u(k) \) be the control vector composed of all control variables in Section II-B and let \( x(k) \) be the state vector composed of all queue lengths and delayed values of the flows. At every time step \( k \), we solve the optimization problem
\[
\min_{u_{N_\ell}(k)} \sum_{j=0}^{N_\ell-1} J(k+j)
\]
subject to:
\[
A_{eq} u_{N_\ell}(k) = b_{eq} \\
A_{ineq} u_{N_\ell}(k) \leq b_{ineq} \\
0 \leq u_{N_\ell}(k) \leq u_{\text{max}}
\]
where \( u_{N_\ell}(k) = [u^T(k), \ldots, u^T(k+N_\ell-1)]^T \). The matrices \( A_{eq} \) and \( A_{ineq} \) and the vectors \( b_{eq} \) and \( b_{ineq} \) are obtained based on the equality and inequality constraints of Section II-B, the current state of the system, \( x(k) \), and the baggage demand. This is an LP problem, which can be solved efficiently.

IV. CASE STUDY

In order to evaluate the closed-loop performance of the proposed control approach, we consider the baggage handling system the layout of which is illustrated by Fig. 1. For simulation purposes, we exactly implement (2)-(33), thus taking into account the variation of transfer delay given by (29). The values of simulation and control parameters are listed in Table I. The baggage demand at loading stations is depicted in Fig. 2. Initially, there are empty DCVs in the loading stations and the unloading station \( U_1 \). One can observe from Fig. 3 that the DCVs are dispatched from the loading stations as soon as the associated demand arrives. Early DCVs end up in the EBS as seen in Fig. 2. At the same time, since there is not sufficient number of empty DCVs in the loading stations to handle the baggage demands, the DCV depot at \( U_1 \) dispatches additional empty DCVs to the loading stations (mostly to \( L_1 \)). This is shown in Fig. 4. From the total inflows of DCVs to the unloading stations shown in Fig. 4, we can observe that some of the pieces of baggage miss their time windows. This is more significant for the baggage demand at \( L_1 \) with destination \( U_2 \). The reason is that there are not sufficient empty DCVs in loading station \( L_1 \) hence additional DCVs are needed from the depot at \( U_1 \). However, those DCVs are first routed to \( L_2 \) since the time window associated with demand at \( L_2 \) approaches sooner. Therefore, the DCVs will need to make a round trip before they can be sent to \( L_1 \). One can also observe that once the pieces of baggage at loading stations are transported, no empty DCV is dispatched from the depot at \( U_1 \) or \( U_2 \). The associated CPU times for the scenario under consideration are listed in Table I as well. One can observe that for a relatively large number of problem size, the optimization problem can be solved within a reasonable amount of time.

V. CONCLUSIONS AND FUTURE WORK

In this paper, we have revisited the problem of dynamic routing and scheduling of DCVs in baggage handling systems. To the best of our knowledge, for the first time, we have jointly addressed the main control problems of modern baggage handling systems, namely, routing and scheduling of DCVs, line balancing, and empty cart management. We have derived a dynamical model of the system based flows. This model was then used as the prediction model within an MPC framework. The objective function was defined such as to penalize the deviation
TABLE I  
SIMULATION PARAMETERS AND CPU TIMES

<table>
<thead>
<tr>
<th>MPC Parameters</th>
<th>Closed-loop Simulation Parameters</th>
<th>Computational Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_p$</td>
<td>$\mu_{uu} \text{[DCV/s]}$</td>
<td>number of opt. variables</td>
</tr>
<tr>
<td>55</td>
<td>(60, 90), (80, 120)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.1)</td>
<td>Solver</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>CPLEX</td>
</tr>
<tr>
<td></td>
<td>(1, 1.2), (1, 1.2)</td>
<td>0.2028</td>
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<tr>
<td></td>
<td></td>
<td>1.8564</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.7202</td>
</tr>
</tbody>
</table>

We have formulated the MPC problem as a LP problem. We have shown via simulations that our proposed control scheme achieves a desirable performance with relatively small computational effort.

As future work, we will consider taking into account variation of queue lengths within the prediction horizon and comparing the performance of such approach with the LP-based approach in terms of closed-loop performance and computational burden. Additionally, we will compare the performance of our approach with the approach that solves the routing, empty-cart management, and line balancing individually (i.e., not in an integrated manner) for larger network layouts and multiple baggage demand scenarios.

REFERENCES


