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Real-Time Scheduling for Trains in Urban Rail Transit Systems Using Nonlinear Optimization

Yihui Wang, Bart De Schutter, Ton J.J. van den Boom, Bin Ning, and Tao Tang

Abstract— **The real-time train scheduling problem for urban rail transit systems is considered with the aim of minimizing the total travel time of passengers and the energy consumption of trains. Based on the passenger demand in urban rail transit systems, the optimal departure times, running times, and dwell times are obtained by solving the scheduling problem. Three solution approaches are proposed to solve the real-time scheduling problem for trains: a pattern search method, a mixed integer nonlinear programming (MINLP) approach, and a mixed integer linear programming (MILP) approach. The performance of these three approaches is compared via a case study based on the data of the Beijing Yizhuang line. The results show that the pattern search method provides a good trade-off between the control performance and the computational efficiency.**

I. Introduction

With the increasing passenger demand for urban rail transit systems, such as subway systems, the frequency of train operations is becoming very high, especially in large cities like Beijing, Shanghai, Tokyo, New York, and Paris, where trains arrive at a station every 3 to 5 minutes. The planning process for the urban rail transit systems becomes more important for reducing the operation costs of railway operators and for guaranteeing passenger satisfaction, which can be characterized by waiting times, on-board times, and transfers. This paper considers the real-time train scheduling problem for urban rail transit systems.

In contrast to interurban rail transit systems, where the rail infrastructure is a limited resource on which several lines compete, we consider the urban rail transit systems. In interurban rail transit systems, trains usually overtake or meet each other. However, the lines of the urban rail transit systems mentioned here are separated from each other, and each direction of the line has a separate route. Moreover, trains do not overtake and meet each other normally in these urban rail transit systems. For completeness, we review the research of the train scheduling planning problem for both interurban railway systems and urban rail transit systems.

1) Scheduling: The train schedule planning problem consists in optimizing the departure and arrival times for all trains at stations and the assignment of railway resources (i.e., tracks and platforms) so that a set of safety constraints is satisfied. The research on interurban rail transit systems has considered the noncyclic timetabling and cyclic timetabling [1]. In the noncyclic timetabling approaches, the train scheduling problem is generally formulated as a mixed integer linear programming (MILP) problem, which was solved using branch-and-bound techniques [2] and heuristic algorithms [3]. The periodic event scheduling problem framework introduced by Serafini and Ukovich [4] is the basis of the cyclic timetabling [5]. An advantage of cyclic timetables is that passengers can easily remember departure times. A drawback of cyclic timetables is the potential inefficiency since trains may have to be operated even in case of a lower passenger demand.

For urban rail transit systems, the timetables obtained in [6] and [7] are cyclic timetables, where cooperative approaches are proposed to adjust running times and dwell times to save energy consumption. However, the passenger satisfaction that can be characterized by passenger waiting time is not considered in [6] and [7]. Wong et al. [8] presented a nonperiodic timetable to synchronize the schedule of different lines in urban rail transit systems with the objective of minimizing the transfer waiting time of passengers. Vazquez-Abad and Zubieta [9] proposed a stochastic approximation approach to adjust the frequencies of different urban transit lines according to the observed variable passenger demand.

2) Rescheduling: The function of the timetable is often loosely connected to its execution in practice. For interurban rail transit systems, the principal task of rescheduling is identifying and resolving conflicts that may arise during actual operation. Mazarello and Ottaviani [10] proposed a conflict detection and resolution model based on the alternative graph formulation to create an optimized conflict-free timetable using a heuristic algorithm. D'Ariano et al. [11] presented a model including updated speed profiles based on alternative graphs and adopted a branch-and-bound algorithm to minimize the delays.

Due to the different rail infrastructure in urban rail transit systems, the rescheduling process of urban rail transit does not need to detect and resolve conflicts between different routes. Several rescheduling approaches are proposed in urban rail transit systems [12]: holding, zone scheduling, short turning, deadheading, and dynamic stop-skipping. Holding is used to regulate the headways by holding an early-arriving train, or a train with a relatively short leading headway. In zone scheduling, the whole line is divided into several zones, where the trains stop at all stations within a single zone and then run without stopping to the terminal station. The short-turn strategy consists of a system of two service patterns, in which the short-turn trip is entirely overlapped by the longer full-length trip. The deadheading strategy involves

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some trains to run empty through a number of stations at the beginning of their trips to reduce the headways at later stations. The dynamic stop-skipping strategy allows trains that are late and behind the schedule to skip certain low-demand stations in order to increase the running speed. However, these operation control strategies are based on an existing timetable or a fixed headway between trains. So the effect of these rescheduling strategies are limited.

3) Real-time scheduling: Therefore, we have proposed a real-time scheduling approach for trains based on the passenger demand in [13], where the capacity of the trains, the capacity of the stations, and the safety constraints caused by urban rail transit systems are included. In [13] the headways, the running times, and the dwell times of trains are optimized with respect to the travel time of passengers. In addition, the real-time train scheduling problem is solved by the sequential quadratic programming approach.

However, the train scheduling problem is essentially a multi-objective optimization problem because it should consider both the benefits of the railway operators and the passengers. More and more researchers are focusing on the multi-objective train scheduling problem, e.g. [2], [14], [15]. As different from [13], this paper considers a multi-objective optimization for the real-time train scheduling problem, where the energy consumption of the trains and the total travel time of passengers are minimized. Furthermore, in this paper we propose three approaches to solve the real-time scheduling problem: a pattern search method, a mixed integer nonlinear programming approach, and a mixed integer linear programming approach.

The rest of the paper is structured as follows. Section II formulates the dynamics of the trains' operation, the passenger demand characteristics, and the passenger/vehicle interaction. Section III describes the multi-objective cost function and the constraints of the real-time scheduling problem. Section IV proposes three solution approaches for the resulting nonlinear non-convex programming problem. Section V illustrates the performance of those three methods with a case study. Finally, Section VI concludes the paper.

II. Model formulation

A. Assumptions

This paper considers one direction of an urban transit line consisting of J stations, where station 1 is the origin station and station J is the final station of each trip. When formulating the real-time scheduling model, we make the following assumptions:

A1. Station j for $j \in \{2,3,\ldots,J-1\}$ can only accommodate one train at a time and no overtaking can occur at any point in the line.

A2. Passengers arrive uniformly at a constant rate λ_j at station j.

A3. The number of passengers alighting from trains at station j for $j \in \{1,2,\ldots,J\}$ is a fixed proportion ρ_j of its arrival load. At the final station J, all the passengers will get off trains, i.e., $\rho_J = 1$.

A4. The number of passengers waiting at station and the number of passengers on-board immediately after a train's departure are approximated by real numbers instead of integers.

Assumption A1 generally holds for most urban transit systems, which are usually operated in first-in first-out order from station 1 to J. Assumption A2 is consistent with observed uniformly passengers arrivals for short headway (less than 10 minutes) services [16]. An estimate of these passenger arrival rates at stations can be obtained by analyzing historical data of the passenger flow. Assumption A3 is made according to [17]. Similar as the passenger arrival rate, the passenger alighting proportion can be determined by analyzing historical data. For Assumption A4, if the number of passengers is high, then the error made by this assumption is small. Furthermore, this assumption simplifies the optimization problem later on.

B. The dynamics of train's operation

The literature on train scheduling ignores the detailed dynamics of trains and describes it by departure times, arrival times, running times, and dwell times. The departure time $d_{i,j}$ of train i at station j is equal to the sum of the arrival time $a_{i,j}$ and the dwell time $\tau_{i,j}$ of train i at station j, i.e.

$$d_{i,j} = a_{i,j} + \tau_{i,j}, \tag{1}$$

where $i \in \{1,2,\ldots,I\}$. In the literature, the dwell time is usually considered as a constant. However, in practice, it is influenced by the number of passengers boarding and alighting from the train. Therefore, we consider a variable dwell time, as will be explained in Section II-D. The arrival time $a_{i,j+1}$ of train i at station $j+1$ equals the sum of the departure time $d_{i,j}$ at station j and the running time $r_{i,j}$ on segment j (i.e., the track between station j and station $j+1$) for train i:

$$a_{i,j+1} = d_{i,j} + r_{i,j}. \tag{2}$$

The running time $r_{i,j}$ satisfies

$$r_{i,j} \in [r_{i,j,\min}, r_{i,j,\max}], \tag{3}$$

where $r_{i,j,\min}$ and $r_{i,j,\max}$ are the minimal and maximal running time for train i traversing segment j, respectively. The minimal running time can be calculated based on the detailed train dynamics, the speed limits, and the grade profiles along the line. The maximum running time should be chosen to ensure the passenger satisfaction.

The minimum headway is the minimum time interval between two successive trains so that they can enter and depart from a station safely. Followed from assumption A1, a train cannot enter a station until a minimum headway h_0 after the preceding train's departure, which can be formulated as

$$a_{i,j} - d_{i-1,j} \geq h_0. \tag{4}$$

In addition, we select the order in which the trains run such that vehicle $i-1$ always precedes train i for $i \in \{1,2,3,\ldots,I\}$ with I the total number of trains.

C. Passenger demand characteristic

The number of passengers still remaining at the station after the departure of train $i-1$ immediately after its departure at station j is defined as $w_{i-1,j}$. The number of passengers who want to get on train i at station j can then be formulated as $w_{i-1,j}+\lambda_j(d_{i,j}-d_{i-1,j})$, where $\lambda_j(d_{i,j}-d_{i-1,j})$ is number of the passengers that arrived during the departure of train $i-1$ and the departure of train i.

By defining the number of passengers on train i immediately after its departure at station $j-1$ as $n_{i,j-1}$, the remaining capacity of train i at station j immediately after the alighting process of the passengers is $C_{i,\max}-n_{i,j-1}(1-\rho_j)$, where $C_{i,\max}$ is the maximum capacity of train i, and $n_{i,j-1}(1-\rho_j)$ is the number of passengers remaining on train i immediately after all the passengers that wanted to leave the train have gotten off.

The number of passengers boarding train i at station j is equal to the minimum of the remaining capacity and the number of waiting passengers, i.e. $\min\big(C_{i,\max}-n_{i,j-1}(1-\rho_j), w_{i-1,j}+\lambda_j(d_{i,j}-d_{i-1,j})\big)$. The number of passengers at station j immediately after the departure of train i, i.e. the passengers who cannot get on train i, is then given by

$$\begin{aligned} w_{i,j} =& w_{i-1,j}+\lambda_j(d_{i,j}-d_{i-1,j})-\min\big(C_{i,\max} \\ &-n_{i,j-1}(1-\rho_j), w_{i-1,j}+\lambda_j(d_{i,j}-d_{i-1,j})\big) \end{aligned} \tag{5}$$

The number of passengers on train i when it departs from station j is equal to the sum of the passengers arriving but not getting off at station j and the passengers boarding on train i at station j, which can be formulated as

$$\begin{aligned} n_{i,j} =& n_{i,j-1}(1-\rho_j)+\min\big(C_{i,\max}-n_{i,j-1}(1-\rho_j), w_{i-1,j} \\ &+\lambda_j(d_{i,j}-d_{i-1,j})\big). \end{aligned} \tag{6}$$

D. Passenger/vehicle interaction

As mentioned before, the dwell time is influenced by the number of alighting and boarding passengers. We assume here that its minimum value can be described as a linear function of the alighting and boarding passengers [18]:

$$\begin{aligned} \tau_{i,j,\min} =& \alpha_{1,\mathrm{d}}+\alpha_{2,\mathrm{d}}n_{i,j-1}\rho_j+\alpha_{3,\mathrm{d}}\min\big(C_{i,\max}-n_{i,j-1}(1-\rho_j), \\ & w_{i-1,j}+\lambda_j(d_{i,j}-d_{i-1,j})\big), \end{aligned}$$

where $\alpha_{1,\mathrm{d}}$, $\alpha_{2,\mathrm{d}}$, and $\alpha_{3,\mathrm{d}}$ are coefficients that can be estimated based on historical data. The dwell time $\tau_{i,j}$ should be larger than the minimal dwell time $\tau_{i,j,\min}$ such that the passengers can get on and get off the train and less than a maximum dwell time $\tau_{i,j,\max}$ to ensure passengers' satisfaction, i.e.,

$$\tau_{i,j,\min} \geq \tau_{i,j} \geq \tau_{i,j,\max}. \tag{7}$$

III. The real-time scheduling problem

For the real-time train scheduling problem, we minimize the energy consumption caused by the operation of trains and the total travel time of all passengers using the weighted sum strategy. The energy consumption is proportional to the resistance [2], which includes the rolling resistance, air resistance, and grade resistance. The energy consumption of train i on segment j is a nonlinear function of the running time $r_{i,j}$ that can be described as $E_{i,j}=R_{i,j}(\bar{v}_{i,j})s_j\xi_i$, where s_j is the length of segment j, and ξ_i is the energy consumption per unit power output, $\bar{v}_{i,j}$ is the mean velocity of train i on segment j, which can be calculated by $\bar{v}_{i,j}=s_j/r_{i,j}$, and the resistance $R_{i,j}(v)$ is a function of train speed v, i.e., $R_{i,j}(v)=(m_{\mathrm{e},i}+n_{i,j}m_{\mathrm{p}})(k_{1i}+k_{2i}v+g\sin(\theta_j))+k_{3i}v^2$, where $m_{\mathrm{e},i}$ is the mass of train i itself, m_{p} is the mass of one passenger, $(m_{\mathrm{e},i}+n_{i,j}m_{\mathrm{p}})$ is the mass of train i and the passengers on-board of train i at station j, k_{1i},k_{2i}, and k_{3i} are the resistance coefficients of train i, and θ_j is the grade profile of segment j. The total energy consumption for all I trains can then be formulated as

$$E_{\text{total}}=\sum_{i=1}^{I}\sum_{j=1}^{J-1}E_{i,j}. \tag{8}$$

The total travel time is the sum of the passenger waiting time and the passenger in-vehicle time. The passenger waiting time $t_{\text{wait},i,j}$ at station j for train i includes the waiting time of both passengers left by the previous train $i-1$ and the newly arrived passengers, i.e. $t_{\text{wait},i,j}=w_{i-1,j}(d_{i,j}-d_{i-1,j})+\frac{1}{2}\lambda_j(d_{i,j}-d_{i-1,j})^2$, where the first term represents the waiting time of the passengers left by train $i-1$ at station j, and the second term represents the waiting time of uniformly arriving passengers between the departures of train $i-1$ and train i. The passenger in-vehicle time for train i running from station j to $j+1$ includes the running time for all passengers on train i after its departure form station j and the waiting time of the passengers who do not get off the train at station $j+1$, which can be formulated as $t_{\text{in-vehicle},i,j}=n_{i,j}r_{i,j}+n_{i,j}(1-\rho_{j+1})\tau_{i,j+1}$. The total passenger travel time for all I trains can then be formulated as

$$t_{\text{total}}=\sum_{i=1}^{I}\sum_{j=1}^{J-1}(t_{\text{wait},i,j}+t_{\text{in-vehicle},i,j}). \tag{9}$$

We apply the weighted sum strategy to solve the multi-objective optimization of the real-time scheduling problem, i.e. we write the total objective function as a weighted sum of the objectives E_{total} and t_{total}:

$$f_{\text{opt}}=\frac{E_{\text{total}}}{E_{\text{total,nom}}}+\lambda\frac{t_{\text{total}}}{t_{\text{total,nom}}}, \tag{10}$$

where λ is the non-negative weight, and the normalization factors $E_{\text{total,nom}}$ and $t_{\text{total,nom}}$ are the nominal values of the total energy consumption and the total travel time of passengers, respectively. The nominal values are determined by running trains using a feasible initial schedule.

The constraints of the real-time scheduling problem consist of the running time constraints, dwell time constraints, headway constraints, and capacity of trains, shown as (1)-(4), (5)-(7) in Section II.

IV. Solution approaches

The resulting real-time train scheduling problem is a non-smooth non-convex problem, where the non-smoothness is caused by the min function in (5) and (6), and the non-convexity is due to the nonlinear non-convex objective function. We propose three solution approaches to solve the real-time train scheduling problem: a pattern search method, a mixed-integer nonlinear programming (MINLP) approach, and a mixed-integer linear programming (MILP) approach.

A. Pattern search method

The pattern search method is a logical choice for the given problem since it is a gradient-free method and can handle non-smooth non-convex optimization problems. The pattern search method was first proposed by Hooke and Jeeves [19], and it has been proved particularly successful in locating minima on hypersurfaces which contain "sharp valleys" in practice. On such surfaces classical techniques behave badly and can only be induced to approach the minimum slowly. For the pattern search method, the variables of the real-time scheduling problem are the departure times $d_{i,j}$, the running times $r_{i,j}$, and the dwell times $\tau_{i,j}$. The other variables, such as the passengers waiting at stations $w_{i,j}$, the passengers on-board the trains $n_{i,j}$, the passenger waiting times $t_{\text{wait},i,j}$, and the passenger on-board times $t_{\text{in-vehicle},i,j}$, can be eliminated.

B. The MINLP approach

In the MINLP approach, we introduce binary variables and auxiliary variables to deal with the min function in (5) and (6). By introducing a binary variable $\delta_{i,j} \in \{0,1\}$ and defining $\tilde{f}_{i,j} = w_{i-1,j} + \lambda_j(d_{i,j} - d_{i-1,j}) - [C_{i,\max} - n_{i,j-1}(1-\rho_j)]$, the following equivalence holds [20]:

$$[\tilde{f}_{i,j} \leq 0] \Leftrightarrow [\delta = 1]$$

is true iff

$$\begin{cases} \tilde{f}_{i,j} \leq \tilde{M}_{i,j}(1-\delta_{i,j}) \\ \tilde{f}_{i,j} \geq \varepsilon + (\tilde{m}_{i,j} - \varepsilon)\delta_{i,j} \end{cases},$$

where ε is a small positive number that is introduced to transform a strict equality into a non-strict inequality, and $\tilde{M}_{i,j}$ and $\tilde{m}_{i,j}$ are the maximum value and the minimum value of $\tilde{f}_{i,j}$, respectively. The min function can now be rewritten as $\delta_{i,j}[w_{i-1,j} + \lambda_j(d_{i,j} - d_{i-1,j})] + (1-\delta_{i,j})[C_{i,\max} - n_{i,j-1}(1-\rho_j)]$. Four auxiliary variables are introduced to replace these four nonlinear terms: $\delta_{i,j}w_{i-1,j}$, $\delta_{i,j}d_{i,j}$, $\delta_{i,j}d_{i-1,j}$, $\delta_{i,j}n_{i,j-1}$.

We consider the departure times $d_{i,j}$, the running times $r_{i,j}$, and the dwell times $\tau_{i,j}$, the passengers waiting at stations $w_{i,j}$, and the passengers on-board the trains $n_{i,j}$ as variables of the real-time scheduling problem. After introducing binary variables and auxiliary variables, the real-time scheduling problem becomes an MINLP problem. A bi-level optimization method is proposed in this paper to solve the resulting MINLP problem. This method consists of two levels of optimization. The high-level optimization optimizes the binary variables, where a brute force approach can be used to find all the combinations for the binary variables in case the size of the problem is small. Otherwise, other integer programming approaches, such as genetic algorithms, can be applied in the high-level optimization. For each combination of binary variables, the nonlinear optimization problem in the lower level is now a smooth problem, which can be solved using gradient-based approaches, such as interior-point method.

TABLE I

INFORMATION OF THE YIZHUANG SUBWAY LINE

Station number	Distance to next station [m]	Passenger arrival rate [passenger/s]	Passenger alighting proportion	Minimum running time [s]
1	1332	2	0	87.700
2	1286	2	0.1	85.628
3	2086	2	0.3	121.664
4	2265	4	0.5	129.727
5	2331	4	0.3	132.700
6	1354	0	1	88.691

C. The MILP approach

In our earlier work [21], we have shown that the MILP approach can be very efficient for train trajectory planning problems. Therefore, we also apply it to the real-time train scheduling problem.

The MILP approach deals with the min function in (5) and (6) the same as the MINLP approach. In the resulting MINLP problem of Section IV-B, the constraints are linear, but the objective function is nonlinear and non-convex. Therefore, in order to solve the real-time scheduling problem as an MILP problem, we need to approximate the nonlinear terms as piecewise affine functions, such as $w_{i-1,j}d_{i,j}$, $n_{i,j}r_{i,j}$, and $n_{i,j}\tau_{i,j}$. For more information about the piecewise affine approximation see [20]. As MILP solver, we use CPLEX, implemented through the cplex interface function of the Matlab Tomlab toolbox.

V. CASE STUDY

In order to demonstrate the performance of the three solution approaches proposed in Section IV for the multi-objective real-time train scheduling problem, the train characteristics and a part of the line data of the Yizhuang subway line in Beijing are used as a test case study. There are 14 stations in the Yizhuang line, but we only consider the first 6 stations for the sake of compactness. The speed limit for the line is 80 km/h (i.e., 22.2 m/s). The detailed information about the 6 stations we consider is listed in Table I. The minimum running time in Table I is calculated by taking a fixed acceleration of 0.8 m/s^2 and a fixed deceleration of -0.8 m/s^2. Furthermore, the train is assumed to run at the maximum speed 22.2 m/s at the holding phase. The maximum running time is assumed as $r_{i,j,\max} = \zeta r_{i,j,\min}$, where ζ is larger than 1. We have chosen ζ as 1.5 to ensure that the passengers do not complain that the train is too slow.

The train mass is 1.99×10^5 kg and the mass of one passenger is 60 kg. The maximal dwell time is chosen as 150 s. Based on the dwell time research about Beijing subway stations [22], the values of the minimum dwell time coefficients $\alpha_{1,d}$, $\alpha_{2,d}$, and $\alpha_{3,d}$ are chosen as 4.002, 0.047, and 0.051, respectively. For the calculation of energy consumption, the coefficients of the resistance k_{1i}, k_{2i} and k_{3i} are chosen as 1.210×10^{-2}, 5.049×10^{-4}, and 8.521. The capacity of each train is 1468 passengers according to the

TABLE II

THE NOMINAL VALUES OF THE ENERGY CONSUMPTION AND THE PASSENGER TRAVEL TIME

I & J	Nominal passenger travel time [s]	Nominal energy consumption [J]
$I=2, J=3$	$3.721 \cdot 10^5$	$1.795 \cdot 10^8$
$I=3, J=3$	$5.270 \cdot 10^5$	$2.674 \cdot 10^8$
$I=3, J=4$	$1.146 \cdot 10^6$	$5.331 \cdot 10^8$
$I=3, J=5$	$2.134 \cdot 10^6$	$8.648 \cdot 10^8$
$I=4, J=5$	$2.723 \cdot 10^6$	$1.136 \cdot 10^9$
$I=4, J=6$	$4.485 \cdot 10^6$	$1.620 \cdot 10^9$

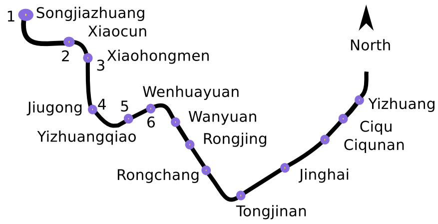

Fig. 1. The layout of the Yizhuang subway line

train characteristics of the Yizhuang line, and the minimum headway h_0 between two successive trains is 90 s.

This paper proposes three approaches to solve the real-time train scheduling problem: pattern search, a MINLP approach, and a MILP approach. For the pattern search method, we use the patternsearch function in the global optimization toolbox of Matlab. The high level of the bi-level optimization for the MINLP approach applies the brute force approach, and the low-level optimization uses the fmincon function implementing sequential quadratic programming method of the Matlab Tomlab toolbox as a nonlinear optimization solver. We use CPLEX, implemented through the cplex interface function of the Matlab Tomlab toolbox as the MILP solver. In order to compare the performance of these three approaches, we consider 6 cases with different problem sizes as shown in Table II, where the values of I and J are the number of trains and stations involved. For the cases with J less than 6, the passenger arrival rate and the passenger alighting rate at station J are changed to 0 and 1, respectively. The weight λ in the multi-objective function (10) is chosen as 1. In addition, the nominal values of the passenger travel time and the energy consumption for these 6 cases are shown in Table II.

Table III shows the computation time, the control performance (i.e., the weighted sum of the energy consumption and the passenger travel time in (10)), the level of suboptimality[1], the passenger travel time, and the energy consumption for the three solution approaches. The computation time of the pattern search method is an average value of 10 runs, where each run has 10 feasible random initial points. The computation time of the MINLP approach is an average value of 10 runs, where the fmincon function in the low-level optimization is executed for 10 feasible random initial points. For the MILP approach, only 1 feasible random initial point is needed to obtain the global minimum of the MILP problem. The passenger travel time and the energy consumption of the MILP approach are calculated using the original nonlinear objective function.

As shown in Table III, the computation time of the MILP approach is the shortest among these three approaches. For the MILP approach, the computation time increases slowly in the beginning, but the computation time for the case with $I=4$ and $J=6$ is about 8 times more than that with $I=4$ and $J=5$. The computation time of the pattern search method increases almost linearly with the size of the problem. The computation time of the MINLP approach for smaller sized problem is short, but it grows exponentially. For the problem with $I=4$ and $J=6$, the MINLP problem cannot be solved in 5 hours using the exhaustive bi-level optimization method. However, the control performance of the MINLP approach is the best among these three approaches for the other combinations of I and J. The control performance of the pattern search method is around 7% higher than the MINLP approach. For the small-sized problems, the control performance of the MILP approach is about 30% higher than the best control performance, but the relative difference becomes smaller when the size of the problem increases. For the cases with $I=3$, $J=5$ and $I=4$, $J=5$, the passenger travel time obtained by the MILP approach is even less than that of the MINLP approach, but the energy consumption of the MILP approach is much higher. This may caused by the approximation errors of the nonlinear terms in the MILP approach.

For the problem with smaller size, the MINLP approach using the exhaustive bi-level optimization is recommended. For larger sized problem, the pattern search method is most suitable since it has an acceptable control performance in a reasonable computation time.

VI. CONCLUSIONS

We have considered the real-time train scheduling problem for urban rail transit systems. The total travel time of passenger and the energy consumption of the operation of trains are minimized in the real-time train scheduling problem. We have proposed three solution approaches to solve the real-time scheduling problem: a pattern search method, a mixed-integer nonlinear programming (MINLP) approach, a mixed integer linear programming (MILP) approach. A comparison of these three approaches is illustrated via a case study. The simulation results show that the MINLP approach has the best control performance, but its computation time grows exponentially. The MILP has the best computational efficiency, but the control performance is much worse comparing to the MINLP approach and the pattern search method. The pattern search approach provides a good trade-off between the control performance and the computational efficiency for real-time applications.

[1]The level of optimality is the relative difference of the control performance of the current approach $f_{\text{opt,cur}}$ with that of the best case $f_{\text{opt,best}}$, and it is calculated by $f_{\text{relative}} = \frac{f_{\text{opt,cur}} - f_{\text{opt,best}}}{f_{\text{opt,best}}}$.

TABLE III

THE PERFORMANCE COMPARISON OF THE PATTERN SEARCH METHOD, THE MINLP APPROACH, AND THE MILP APPROACH

I & J	Solution method	Computation time [s]	Control performance	Level of suboptimality	Passenger travel time [s]	Energy consumption [J]
	Pattern search	28.850	2.167	7.3%	$4.332\cdot10^5$	$1.801\cdot10^8$
$I=2$	MINLP	7.628	2.019	0%	$3.840\cdot10^5$	$1.771\cdot10^8$
$J=3$	MILP	0.804	2.563	26.9%	$4.237\cdot10^5$	$2.556\cdot10^8$
	Pattern search	44.724	2.170	5.14%	$6.128\cdot10^5$	$2.692\cdot10^8$
$I=3$	MINLP	25.722	2.064	0%	$5.648\cdot10^5$	$2.654\cdot10^8$
$J=3$	MILP	0.871	2.736	32.6%	$6.944\cdot10^5$	$3.793\cdot10^8$
	Pattern search	115.424	2.144	6.6%	$1.307\cdot10^6$	$5.344\cdot10^8$
$I=3$	MINLP	248.231	2.012	0	$1.175\cdot10^6$	$5.262\cdot10^8$
$J=4$	MILP	1.206	2.443	21.4%	$1.183\cdot10^6$	$7.522\cdot10^8$
	Pattern search	204.979	2.126	8.8%	$2.414\cdot10^6$	$8.599\cdot10^8$
$I=3$	MINLP	$1.207\cdot10^3$	1.954	0%	$2.105\cdot10^6$	$8.370\cdot10^8$
$J=5$	MILP	2.320	2.262	15.8%	$2.096\cdot10^6$	$1.107\cdot10^9$
	Pattern search	268.815	2.145	7.3%	$3.099\cdot10^6$	$1.143\cdot10^9$
$I=4$	MINLP	$1.706\cdot10^4$	2.000	0%	$2.765\cdot10^6$	$1.118\cdot10^9$
$J=5$	MILP	7.526	2.366	18.3%	$2.745\cdot10^6$	$1.543\cdot10^9$
	Pattern search	410.030	2.118	0%	$5.373\cdot10^6$	$1.634\cdot10^9$
$I=4$	MINLP	>5 h	-	-	-	-
$J=6$	MILP	56.270	2.252	6.3%	$4.543\cdot10^5$	$2.008\cdot10^9$

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