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# Towards a Robust Multi-level Control Approach for Baggage Handling Systems

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***Abstract*— This paper revisits the routing problem in baggage handling systems. We propose a two-level control approach based on a model predictive controller at the top level and a constrained feedback controller at the bottom level that minimizes the $L_2$ gain of the closed-loop system. The model predictive control problem is recast as a linear programming problem and the constrained feedback controller design problem is formulated as minimization of a linear objective function subject to linear matrix inequalities. The effectiveness of the proposed method is illustrated by a case study.**

## I. INTRODUCTION

There has been a growing interest, in the last decade, in automated modern baggage handling systems for large airports. Such baggage handling systems have enabled big airports to achieve high throughput of passengers and cargo. The efficiency and reliability of baggage handling systems have improved over time by implementing more advanced control strategies. However, in order to meet the increasing demand for air travel and cargo shipment, we need more intelligent and reliable control methods than the currently available state-of-the-art methods. Modern baggage handling systems are composed of the following main components: i) loading stations, where the baggage demand originates. The pieces of baggage arrive at the loading stations either from a check-in desk or from a transfer flight, ii) unloading stations that are the final destination of the luggage and from where the pieces of baggage are boarded on to the planes, iii) a network of tracks that connect loading stations to unloading stations through junctions, iv) high-speed destination coded vehicles (DCV) that transport the pieces of baggage on the network form the loading stations to the unloading stations, v) switch controllers at the junctions that determine the route of DCVs. A complete description of the baggage handling system, the state-of-the-art control approaches, and the high-level control problems can be found in [1] and [2]. In this paper, building on the work of [3], we develop a new approach for dynamic routing of DCVs within the network such that the pieces of baggage arrive at their destination within a given time window with minimum energy consumption. We also improve the robustness of our approach against variations in the baggage demand.

The proposed control structure is composed of a controller based on model predictive control (MPC) at the top level and a controller based on $L_2$ gain optimization at the bottom level. The MPC controller computes the nominal control input based on nominal prediction of the baggage demand such that the pieces of baggage arrive at their destination within a specified time window with minimal energy consumption. The $L_2$ based controller then minimizes the deviation of system trajectories from the nominal behavior due to unpredicted variations in the nominal predicted baggage demand. Fig. 1 depicts a schematic overview of the proposed two-level control approach.

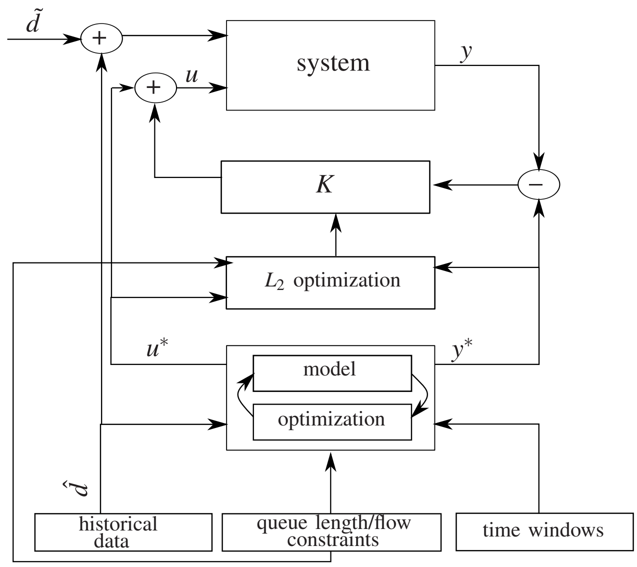


Fig. 1. Schematic overview of the proposed two-level control approach, where $u^*$ and $y^*$ are the nominal input generated by the MPC controller and the resulting output trajectory, respectively, $\hat{d}$ is the predicted baggage demand, and $\tilde{d}$ is the unpredicted deviation of baggage demand around $\hat{d}$

The rest of the paper is organized as follows. In Section II, we present the dynamical model of the baggage handling system used for our control purposes. Section III and Section IV describe the MPC approach and the $L_2$ optimization control approach, respectively. In Section V, we explain how to combine these two controller approaches into a two-level control structure. In Section VI, we present a case study illustrating the performance of our proposed control scheme and finally Section VII concludes the paper.

## II. DYNAMICAL MODEL

The baggage handling system network can be seen as a directed graph $G = (V, A)$, where $V = O \cup I \cup D$ is the set of nodes composed of origin nodes $O$ (i.e., loading stations), intermediate nodes $I$ (i.e., junctions), and destination nodes

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$D$ (i.e., unloading stations), and $A$ is the set of arcs composed of links (i.e., tracks) connecting the nodes. The queue lengths are associated with the nodes and the control variables are defined at each node as the flows of DCVs from that node to its neighbor nodes. In a similar manner to the model in [4], the flows are indexed by their destination, enabling us to distinguish between baggage with different destinations. This is important as the baggage must end up in the right destination. Accordingly, at each node $v \in V$ there is a partial queue of DCVs associated with each destination $d \in D$. The following assumptions are made in the derivation of the model:

A1 Each node in the network belongs to at least one directed path from an origin node (i.e., a loading station) to a destination node (i.e., an unloading station).

A2 A DCV is present at the loading station whenever a piece of baggage arrives.

A3 The movement of pieces of baggage on the network is approximated by a continuous flow of baggage.

A4 At each node $v$, with exception of destination nodes, the DCVs stack up in vertical queues according to their destination. The queue lengths at destination nodes are considered to be zero. This is because we assume either destination nodes have unlimited capacity or there is no restriction on the outflow of destination nodes so the baggage are immediately taken to the planes upon arrival.

A5 The DCV travel time on each link is an integer multiple of the sampling time $T_\mathrm{s}$.

Assumption A1 guarantees that there are no redundant nodes in the network. By Assumption A2, the pieces of baggage are immediately dispatched from the loading stations as they arrive. Therefore, we do not need to distinguish between baggage flows and DCV flows within the system. Otherwise, we would need to take into account the movement of empty DCVs from the unloading stations to the loading stations. Assumption A3 is necessary for tractability of the control problem. Although the number of DCVs is an integer in reality, for a fairly large number of DCVs, the movement of DCVs can be approximated by continuous flows. This is not very restrictive as the computed flows can then be realized as well as possible by a lower-level control loop that determines the optimal switching pattern for the switch controllers at the junctions. The actual time required to travel from a node to another one depends on the length of the DCV queue at the end of the link connecting these nodes. However, if the queue lengths are sufficiently small compared to the length of the links, the variation in the travel time is negligible. This is equivalent to having vertical queues at each node as stated in assumption A4. Assumption A5 allows us to arrive at a linear discrete-time model of the system.

We also make use of the following notation:

- The set of sending nodes of a node $v \in V$ defined as $V_v^\mathrm{send} = \{w \in V \mid (w,v) \in A\}$, is the set of nodes that can send flow to node $v$ .
- The set of receiving nodes of a node $v \in V$ defined as $V_v^\mathrm{recv} = \{w \in V \mid (v,w) \in A\}$, is the set of nodes that can receive flow from node $v$.
- The set of all nodes that are on some directed path to a destination node $d \in D$ is $V_d$.
- For each destination node $d \in D$ and for each origin node $v \in O \cap V_d$, $Q_{v,d}(k)$ is the baggage inflow (demand) at $v$ with destination $d$ during the time interval $[kT_\mathrm{s},(k+1)T_\mathrm{s})$.

For each destination $d \in D$ and each $v \in V_d$ and each $w \in V_v^\mathrm{recv} \cap V_d$, we define the control variable $q_{v,w,d}(k)$ that is the partial flow of DCVs with destination node $d$ from node $v$ to node $w$ during the time interval $[kT_\mathrm{s},(k+1)T_\mathrm{s})$. Accordingly, $x_{v,d}(k)$ denotes the vertical queue length at node $v$ associated with destination $d$. The set of feasible trajectories of the system is described by the following linear constraints in discrete time:

$$x_{v,d}(k+1) = x_{v,d}(k) + T_\mathrm{s}(F_{v,d}^\mathrm{in}(k) - F_{v,d}^\mathrm{out}(k)) \tag{1a}$$

$$x_{v,d}(k) \geq 0 \tag{1b}$$

$$q_{v,w,d}(k) \geq 0 \tag{1c}$$

where $F_{v,d}^\mathrm{in}(k)$ is the total inflow of DCVs to node $v$, associated with destination $d$, given by

$$F_{v,d}^\mathrm{in}(k) = \begin{cases} Q_{v,d}(k) + \sum\limits_{w \in V_v^\mathrm{send}} q_{w,v,d}(k - k_{w,v}) & \text{if } v \in V_d \cap O \\ \sum\limits_{w \in V_v^\mathrm{send}} q_{w,v,d}(k - k_{w,v}) & \text{if } v \in V_d \cap (D \cup I) \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

with $k_{w,v}T_\mathrm{s}$ being the travel time[1] on the link $(w,v)$, and $F_{v,d}^\mathrm{out}(k)$ is the total outflow of DCVs from node $v$ with destination $d$, given by

$$F_{v,d}^\mathrm{out}(k) = \begin{cases} F_{v,d}^\mathrm{in}(k) & \text{if } v \in V_d \cap D \\ \sum\limits_{w \in V_v^\mathrm{recv}} q_{v,w,d}(k) & \text{if } v \in V_d \cap (O \cup I) \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Equation (1a) describes the evolution of the queue lengths and (1b) constrains queue lengths to non-negative values. Likewise, (1c) guarantees non-negativity of the control variables (flows).

Let $x(k)$ be the state vector that includes all queue lengths $x_{v,d}(k)$ and *delayed* samples of $q_{v,w,d}(k)$ with delay$\geq 1$. Let $u(k)$ and $d(k)$ be the control input vector that includes all control variables $q_{v,w,d}(k)$, and the demand vector composed of all individual demands $Q_{v,d}(k)$, respectively. Then (1) can be expressed by a constrained discrete-time linear system as

$$x(k+1) = Ax(k) + B_1 d(k) + B_2 u(k) \tag{4a}$$

$$x(k) \geq 0 \tag{4b}$$

$$u(k) \geq 0 \tag{4c}$$

with properly defined matrices $A$, $B_1$, and $B_2$.

[1] Assuming a constant speed for DCVs $v_\mathrm{DCV}$, $k_{w,v}$ is given by $k_{w,v} = \frac{s_{w,v}}{T_\mathrm{s} v_\mathrm{DCV}}$, where $s_{w,v}$ is the length of link $(w,v)$.

## III. MPC Problem Formulation

The model presented in Section II is used as internal prediction model for the MPC approach. At time step $k$, given the current state of the system and an estimate of future baggage demand, this model is used to compute the trajectories of the system based on which a constrained optimal control problem is solved over a horizon yielding an optimal control sequence. The first element out of the optimal control sequence is then applied to the system according to the receding horizon policy and this process is then repeated at the next time step $k+1$ with new measurements [5].

The objective function must reflect the following performance criteria: i) the pieces of baggage assigned to a certain destination (unloading station) must reach the destination within a given time window, ii) the energy consumption of the system should be minimized. The time window represents the time duration in which the end point is ready to receive the luggage. It is undesirable to have the luggage arrive at the destination out of this time window. Indeed, if the pieces of luggage arrive too late, they will miss the flight. Too early arrival of the luggage at the destination point also might inflict a high storage cost on the operator. The energy consumption is associated with manipulating the actuators in the system and wear and tear inflicted on the actuators. There are two contributors to the energy consumption in the system: i) movements of DCVs in the system, which is related to the magnitude of DCV flows, and ii) variation in the DCV flows. This is particularly important when the DCV flows obtained here will be realized using switch controllers at each junction of the network. The variation in the flow then translates to switching frequency.

In order to achieve the aforementioned control objectives, we consider a cost function that is a weighted combination of four penalty terms that penalize the DCV queue lengths, DCV flows (control variables), and the variation of DCV flows. The cost associated with the DCV is defined as: The constrained linear model given in Section II cannot be used to determine the time instant at which a certain flow of baggage reaches to its destination explicitly. However, we can consider a cost function to indirectly penalize baggage arrival time deviation from a given time window. The cost function is composed of three penalty terms. The first penalty term penalizes the queue lengths being defined as

$$J_d^{\mathrm{tw}}(k) = \sum_{v \in V_d} C_{v,d}^{\mathrm{tw}}(k) x_{v,d}(k) \tag{5}$$

where $C_{v,d}^{\mathrm{tw}}(k)$ as illustrated in Fig. 2 is given as

$$C_{v,d}^{\mathrm{tw}}(k) = \begin{cases} 0 & \text{if } k + k_{v,d} \leq k_d^{\mathrm{open}} \\ c^{\mathrm{tw}}(k - k_d^{\mathrm{open}} + k_{v,d}) & \text{if } k_d^{\mathrm{open}} < k + k_{v,d} \leq k_d^{\mathrm{close}} \\ c^{\mathrm{tw}}(k_d^{\mathrm{close}} - k_d^{\mathrm{open}}) & \text{if } k + k_{v,d} > k_d^{\mathrm{close}} \end{cases} \tag{6}$$

where $k_d^{\mathrm{open}}$ and $k_d^{\mathrm{close}}$ are, respectively, the opening and the closing time steps of destination $d$ and $k_{v,d}T_{\mathrm{s}}$ is the expected travel time from node $v$ to destination $d$ under the current nominal operating conditions[2]. Note that since $C_{v,d}^{\mathrm{tw}}(k) = 0$ for $k \leq k_d^{\mathrm{open}} - k_{v,d}$, the queue lengths associated with destination $d$ are not penalized before the destination is open, taking into account the DCVs travel time from $v$ to $d$. During the time window of destination $d$, the weight associated with DCV queues increases linearly in time, hence, forcing the DCVs to move towards $d$. The penalty term associated with the DCV flows is defined as:

$$J_d^{\mathrm{flow}}(k) = \sum_{v \in V_d} \sum_{w \in V_v^{\mathrm{recv}} \cap V_d} C_{v,d}^{\mathrm{flow}}(k) q_{v,w,d}(k) \tag{7}$$

with $C_{v,d}^{\mathrm{flow}}(k)$ as depicted in Fig. 3 being

$$C_{v,d}^{\mathrm{flow}}(k) = \begin{cases} -c_1^{\mathrm{flow}}(k - k_d^{\mathrm{open}} + k_{v,d}) & \text{if } k + k_{v,d} \leq k_d^{\mathrm{open}} \\ 0 & \text{if } k_d^{\mathrm{open}} < k + k_{v,d} \leq k_d^{\mathrm{close}} \\ c_2^{\mathrm{flow}}(k - k_d^{\mathrm{close}} + k_{v,d}) & \text{if } k + k_{v,d} > k_d^{\mathrm{close}} \end{cases} \tag{8}$$

Note that $C_{v,d}^{\mathrm{flow}}(k)$ is chosen in such a way that DCV flows to destination $d$ are allowed during the time window of $d$. Higher values $C_{v,d}^{\mathrm{flow}}(k)$ outside of the time window prevent early or late DCV flows to the destination $d$. Moreover, in order to allow late DCVs to reach the destination, the slope of the third part of $C_{v,d}^{\mathrm{flow}}(k)$ is smaller than the slope of the first part. Now we will introduce the terms in the cost function that reflect the energy consumption in the network. We penalize all flows in the network in order to avoid indefinite circulation of DCVs throughout the network. Hence, we consider the following penalty term:

$$J^{\mathrm{e}}(k) = \sum_{d \in D} \sum_{v \in V_d} \sum_{w \in V_v^{\mathrm{recv}} \cap V_d} q_{v,w,d}(k) \tag{9}$$

In addition, we use the following penalty term to penalize the total variation of the control signal (i.e., flows), which reflects the wear and tear of the DCVs:

$$J^{\mathrm{sw}}(k) = \sum_{d \in D} \sum_{l \in V_d} \sum_{w \in V_v^{\mathrm{recv}} \cap V_d} |q_{v,w,d}(k) - q_{v,w,d}(k-1)| \tag{10}$$

The total cost at time step $k$ is therefore given as

$$J(k) = \sum_{d \in D} J_d^{\mathrm{tw}}(k) + \alpha_1 \sum_{d \in D} J_d^{\mathrm{flow}}(k) + \alpha_2 J^{\mathrm{e}}(k) + \alpha_3 J^{\mathrm{sw}}(k) \tag{11}$$

where $\alpha_i > 0$ is a weight factor indicating the relative importance of the associated term in the objective function. The MPC performance index over the prediction horizon of $N_{\mathrm{p}}$ step is thus given as

$$J(k, N_{\mathrm{p}}) = \sum_{i=k}^{k+N_{\mathrm{p}}-1} J(i) \tag{12}$$

Now we would like to highlight the following remarks:

R1 The plots of Fig. 2 and Fig. 3 show respectively coefficients of the penalty terms (5) and (7), not the penalty terms themselves. In fact, at the given time step $k$ and for a prediction horizon $N_{\mathrm{p}}$ the values of these coefficients are known for $k, \ldots, k+N_{\mathrm{p}}-1$. Therefore, these coefficients have fixed values and hence

[2]These can be obtained based on historical data for periods with similar conditions as the current one.

the associated penalty terms (5) and (7) are linear in the control variable.

R2 By introducing some dummy variables according to standard techniques in optimization [6], terms of the form (10) can be recast as a linear programming problem with linear constraints.

Consider $u(k)$, $x(k)$, and $d(k)$ as introduced in Section II. At every time step $k$ we solve the following optimization problem:

$$\begin{aligned}&\min_{\overline{u}(k)} F(k)\overline{u}(k)\\ &\text{subject to: } A_{\text{ineq}}(k)\overline{u}(k) \le b_{\text{ineq}}(k)\\ &\qquad\qquad A_{\text{eq}}(k)\overline{u}(k) = b_{\text{eq}}(k)\end{aligned} \tag{13}$$

where the vector $F(k)$ is defined based on the MPC objective function (11), and the vector $\overline{u}(k)$ includes the control inputs $u(k),\dots,u(k+N_{\text{p}}-1)$ and the dummy variables mentioned in Remark R2. Moreover, $A_{\text{ineq}}(k)$, and $A_{\text{eq}}(k)$ are determined based on the constraints, and $b_{\text{ineq}}(k)$, and $b_{\text{eq}}(k)$ are constant vectors that depend on the current state $x(k)$ and the demand values $d(k),\dots,d(k+N_{\text{p}}-1)$.

The optimization problem given by (13) is an LP problem, that can be solved efficiently with currently available solvers, e.g., MATLAB `linprog`.

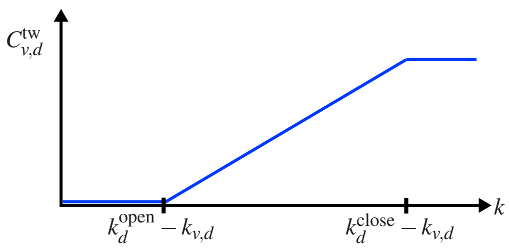


Fig. 2. The coefficient for the queue length penalty term.

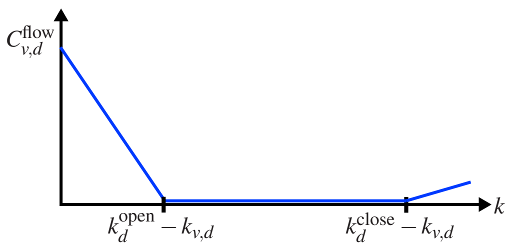


Fig. 3. The coefficient for the flow penalty term.

## IV. Feedback Control Problem Formulation

### A. Problem Setup

Consider a discrete-time linear system

$$x(k+1) = Ax(k) + B_1 d(k) + B_2 u(k) \tag{14a}$$
$$z(k) = C_1 x(k) + D_1 d(k) + D_2 u(k) \tag{14b}$$

with full state feedback

$$u(k) = Kx(k) \tag{15}$$

where the system matrices, $x \in \mathbb{R}^n$, $d \in \mathbb{R}^{n_d}$, and $u \in \mathbb{R}^m$ are those of (4a) and $z \in \mathbb{R}^{n_z}$ is the controlled output vector. Assume that $(A, B_2)$ is stabilizable and $K$ is a stabilizing feedback gain. The $L_2$ gain of the closed-loop system is bounded by $\gamma > 0$ (i.e., $\sup_{z\neq 0} \frac{\|z\|_2}{\|d\|_2} \le \gamma$) if and only if there exists a $P > 0$ such that [7], [8]

$$\begin{bmatrix} \mathscr{A}^{\mathrm{T}} P \mathscr{A} - P + \frac{1}{\gamma}\mathscr{C}^{\mathrm{T}}\mathscr{C} & \mathscr{A}^{\mathrm{T}} P \mathscr{B} + \frac{1}{\gamma}\mathscr{C}^{\mathrm{T}}\mathscr{D} \\ \mathscr{B}^{\mathrm{T}} P \mathscr{A} + \frac{1}{\gamma}\mathscr{D}^{\mathrm{T}}\mathscr{C} & \mathscr{B}^{\mathrm{T}} P \mathscr{B} + \frac{1}{\gamma}\mathscr{D}^{\mathrm{T}}\mathscr{D} - \gamma I \end{bmatrix} \le 0 \tag{16}$$

where

$$\begin{bmatrix} \mathscr{A} & \mathscr{B} \\ \mathscr{C} & \mathscr{D} \end{bmatrix} = \begin{bmatrix} A + B_2 K & B_1 \\ C_1 + D_2 K & D_1 \end{bmatrix} \tag{17}$$

or equivalently

$$\begin{bmatrix} -Q & \mathscr{A} Q & \mathscr{B} & 0 \\ Q\mathscr{A}^{\mathrm{T}} & -Q & 0 & Q\mathscr{C}^{\mathrm{T}} \\ \mathscr{B}^{\mathrm{T}} & 0 & -\gamma I & \mathscr{D}^{\mathrm{T}} \\ 0 & \mathscr{C} Q & \mathscr{D} & -\gamma I \end{bmatrix} \le 0 \tag{18}$$

with $Q = P^{-1} > 0$.

Consider the problem of determining a feedback gain $K$ that minimizes the $L_2$ gain of the closed-loop system. It is well-known [8] that with the transformation $Y = KQ$, the matrix inequality of (18) can be written as

$$\begin{bmatrix} -Q & AQ + B_2 Y & B_1 & 0 \\ QA^{\mathrm{T}} + Y^{\mathrm{T}} B_2^{\mathrm{T}} & -Q & 0 & QC_1^{\mathrm{T}} + Y^{\mathrm{T}} D_2^{\mathrm{T}} \\ B_1^{\mathrm{T}} & 0 & -\gamma I & D_1^{\mathrm{T}} \\ 0 & C_1 Q + D_2 Y & D_1 & -\gamma I \end{bmatrix} \le 0 \tag{19}$$

with $Q > 0$.

Note that for the closed-loop system given by (14) and (15), (16) implies

$$x^{\mathrm{T}}(k+1)Px(k+1) - x^{\mathrm{T}}(k)Px(k) + \frac{1}{\gamma} z^{\mathrm{T}}(k) z(k) \le \gamma d^{\mathrm{T}}(k) d(k) \tag{20}$$

Now we define the ellipsoid $\varepsilon_\gamma := \{x | x^{\mathrm{T}} \frac{P}{\gamma} x \le 1\}$. Assuming $x(0) = 0$, (20) yields

$$x^{\mathrm{T}}(T)Px(T) \le \gamma \sum_{k=0}^{T-1} d^{\mathrm{T}}(k) d(k) < \gamma \sum_{k=0}^{\infty} d^{\mathrm{T}}(k) d(k) \tag{21}$$

for any $T \in \mathbb{N}$. Assuming[3] $\|d\|_2^2 = 1$, we get

$$x^{\mathrm{T}}(T) \frac{P}{\gamma} x(T) < 1 \tag{22}$$

which shows that $x(T) \in \varepsilon_\gamma$. Since (21) holds for all $T$, $\varepsilon_\gamma$ contains the set of states that are reachable by a unit energy input signal $d$ when the $L_2$ gain of the closed-loop system is bounded by $\gamma$.

[3] It is always possible to scale $d$ such that $\|d\|_2^2 = 1$.

### B. Hard State Constraints

Now we consider the problem of searching for the feedback gain $K$ that minimizes the $L_2$ gain of the closed-loop system subject to polytopic state constraints of the form

$$a_i^{\mathrm{T}} x(k) \leq 1,\ i=1,\ldots,r. \tag{23}$$

To include the state constraints of (23), consider the polytope

$$\mathscr{P}=\{x\in\mathbb{R}^n | a_i^{\mathrm{T}} x \leq 1, i=1,\ldots,r\} \tag{24}$$

associated with (23). We assume that $\mathscr{P}$ has the origin in its interior. To guaranty that (23) holds for all $k>0$ with $x(0)=0$, we must have $\varepsilon_\gamma \subseteq \mathscr{P}$ or equivalently [8]

$$a_i^{\mathrm{T}} \gamma Q a_i \leq 1,\ i=1,\ldots,r \tag{25}$$

Therefore, the following optimization problem needs to be solved:

$$\begin{aligned} &\min_{Q,Y,\gamma}\ \gamma \\ &\text{subject to: } (19),\ (25),\ Q>0 \end{aligned} \tag{26}$$

This problem is not jointly convex in $\gamma$ and $Q$ and $Y$. Moreover, it can be shown in a straightforward manner that the constraints of (18) and (25) do not satisfy the monotonicity property $G(Q,Y,\gamma_1) > G(Q,Y,\gamma_2)$ if $\gamma_1 > \gamma_2$, where $G<0$ represents constraints (19) and (25) combined. Therefore, this problem cannot even be recast as a generalized eigenvalue problem, which is a class of quasiconvex optimization problems [8].

Now we will replace constraint (25) by a more conservative one that is convex in the optimization variables, in the following manner. Note that

$$\gamma Q = \frac{1}{4}(\gamma I+Q)^{\mathrm{T}}(\gamma I+Q) - \frac{1}{4}(\gamma I-Q)^{\mathrm{T}}(\gamma I-Q) \tag{27}$$

Obviously, $\gamma Q < \frac{1}{4}(\gamma I+Q)^{\mathrm{T}}(\gamma I+Q)$. Hence,

$$\frac{1}{4} a_i^{\mathrm{T}}(\gamma I+Q)^{\mathrm{T}}(\gamma I+Q)a_i \leq 1 \implies a_i^{\mathrm{T}}\gamma Q a_i < 1 \tag{28}$$

or equivalently expressed using the Schur complement

$$\begin{bmatrix} I & (\gamma I+Q)a_i \\ a_i^{\mathrm{T}}(\gamma I+Q) & 4 \end{bmatrix} > 0 \tag{29}$$

Clearly, this introduces conservatism as the feasibility set of (29) is a subset of the feasibility set of (25). This conservatism can be reduced if one can find a lower bound for $(\gamma I-Q)^{\mathrm{T}}(\gamma I-Q)\geq 0$ such that $(\gamma I-Q)^{\mathrm{T}}(\gamma I-Q)\geq \alpha^2 I$ or equivalently $\|\gamma I-Q\|\geq\alpha$ (in matrix norm sense) for some $\alpha>0$. Then, instead of (29), one obtains

$$\begin{bmatrix} I & (\gamma I+Q)a_i \\ a_i^{\mathrm{T}}(\gamma I+Q) & 4+a_i^{\mathrm{T}}\alpha^2 a_i \end{bmatrix} > 0 \tag{30}$$

Therefore we consider (26) with (25) replaced by (29) or by (30). This is an eigenvalue problem [8], which is a convex optimization problem that can be solved with currently available LMI optimization toolboxes, e.g., MATLAB LMI toolbox, YALMIP [9], and CVX [10], [11].

### C. Soft State Constraints

In the view of the proposed two-level control scheme, it makes more sense to replace the hard constraints of (25) by soft constraints due to the following observations: i) the constraints are mainly handled at the top level by the MPC controller, ii) if the constraints are too restrictive the conservative version of the original constraints as expressed by (29) may become infeasible, which is not desirable. As an alternative to the approach presented in Section IV-B, one can replace hard constraints by soft ones by considering a multi-objective optimization approach that penalizes the $L_2$ gain of the closed-loop system and, indirectly, the constraint violation at the same time. More precisely, we define the following optimization problem with the objective function that penalizes $\gamma$, and the volume of the ellipsoid $\varepsilon_\gamma$, which is proportional to $\sqrt{\det\ \gamma Q}$ :

$$\begin{aligned} &\min_{Q,Y,\gamma}\ c_\gamma\gamma+\log\big(\det(\gamma Q)\big) \\ &\text{subject to: } Q>0 \text{ and } (19) \end{aligned} \tag{31}$$

where $c_\gamma>0$ is a weight factor. The magnitude of $c_\gamma$ determines the trade-off between the $L_2$ gain and the volume of the ellipsoid that represents the set of reachable states. By minimizing the volume of $\varepsilon_\gamma$, we confine the set of reachable state from the origin. This indirectly minimizes constraint violation since the origin lies in the interior of polytope $\mathscr{P}$. However, this objective function is not convex in the optimization variables $\gamma$ and $Q$. To mitigate this problem, instead of penalizing the volume of $\varepsilon_\gamma$, we penalize an upper bound on the length of semi-major axis of $\varepsilon_\gamma$, which is $\sqrt{\lambda_{\max}(\gamma Q)}$, where $\lambda_{\max}(\gamma Q)$ is the largest eigenvalue of $\gamma Q$. It is clear from (27), that $\lambda_{\max}\big(\frac{1}{4}(\gamma I+Q)^{\mathrm{T}}(\gamma I+Q)\big)$ constitutes an upper bound on $\lambda_{\max}(\gamma Q)$. Then we get

$$\begin{aligned} &\min_{Q,Y,\gamma}\ c_\gamma\gamma+\lambda_{\max}\Big(\frac{1}{4}(\gamma I+Q)^{\mathrm{T}}(\gamma I+Q)\Big) \\ &\text{subject to: } Q>0 \text{ and } (19) \end{aligned} \tag{32}$$

or equivalently

$$\begin{aligned} &\min_{Q,Y,\gamma,\lambda}\ c_\gamma\gamma+\lambda \\ &\text{subject to: } (19),\ Q>0,\ \begin{bmatrix} \lambda I & \gamma I+Q \\ \gamma I+Q & 4I \end{bmatrix} > 0 \end{aligned} \tag{33}$$

This is an eigenvalue problem [8] that can be solved efficiently with currently available LMI solvers such as MATLAB LMI toolbox. Note that, by inspecting (27), the upper bound on $\lambda_{\max}(\gamma Q)$ can be made tighter if one can find an $\alpha>0$ such that $(\gamma I-Q)^{\mathrm{T}}(\gamma I-Q)\geq\alpha^2 I$ or equivalently, in matrix norm[4] sense, $\|\gamma I-Q\|\geq\alpha$. Then the last constraint in (33) will be replaced by

$$\begin{bmatrix} (\lambda+\frac{\alpha^2}{4})I & \gamma I+Q \\ \gamma I+Q & 4I \end{bmatrix} > 0 \tag{34}$$

[4]For matrix norm, we use the definition $\|A\|=\sigma_{\max}(A)$, where $\sigma_{\max}(A)$ is the largest singular value of matrix A.

As an example, consider a discrete-time linear system given by

$$A=\begin{bmatrix}0.1514 & 0.4377 & 0.7293 & 0.1839\\ 0.3958 & 0.3999 & 0.7521 & 0.9368\\ 0.9720 & 0.7636 & 0.8323 & 0.6137\\ 0.7718 & 0.8639 & 0.4821 & 0.6050\end{bmatrix},\ C_1=I$$

$$B_1=\begin{bmatrix}0\\0\\-0.5881\\0.2487\end{bmatrix},B_2=\begin{bmatrix}0.6954\\-0.2837\\-0.9723\\0.6086\end{bmatrix},\ D_1=0,\ D_2=0$$

For $c_\gamma$ taking values in the interval $[0.1,100]$, Fig. 4 illustrates the trade-off between minimizing the $L_2$ gain and the length of the semi-major axis of $\varepsilon_\gamma$.

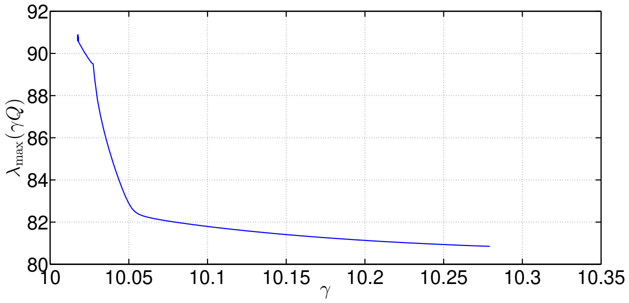


Fig. 4. Trade-off curve between the optimal $\gamma$ and the length of the semi-major axis of $\varepsilon_\gamma$.

## V. Integration of MPC and Feedback Controllers

In this section, we briefly explain how the two control schemes presented in Sections III and IV can be combined. The baggage demand at each origin node is composed of a base demand $d^*$, which is assumed to be predictable over the prediction horizon $N_\mathrm{p}$, and a small additive perturbation $\tilde{d}$ around the base demand that cannot be predicted. Based on a future prediction of $d^*$, the MPC controller computes the optimal DCV flows $u^*$ and system trajectories $z^*$ subject to flow and queue length constraints such that the DCVs arrive at their destinations with minimal energy consumption and with minimal deviation from the time windows. To minimize the adverse effect of $\tilde{d}$ on optimal system trajectories computed by the MPC controller, a feedback gain $K$ minimizing $\frac{\|\tilde{z}\|_2}{\|\tilde{d}\|_2}=\frac{\|z-z^*\|_2}{\|d-d^*\|_2}$ based on the measurement $y(k)-y^*(k)$ is implemented along the MPC controller in the configuration depicted in Fig. 1. Therefore, the control law applied to the system at time step $k$ is $u(k)=u^*(k)+K\big(y(k)-y^*(k)\big)=u^*(k)+K\tilde{y}(k)$.

When we impose constraints on the controlled output $z(k)=z^*(k)+\tilde{z}(k)$, the constraints on $\tilde{z}(k)$ depend on value of $z^*(k)$. As a result, one needs to update the feedback gain $K$ whenever the value of $z^*(k)$ changes. This can be avoided if soft constraints as in Section IV-C are used. Moreover, the MPC control law $u^*$ does not have to be updated at every time instant $kT_\mathrm{s}$. Particularly, if the base demand $d^*$ is varying slowly with time, one can use a controller sampling time $mT_\mathrm{s}$, with $m>1$ being an integer number.

## VI. Case Study

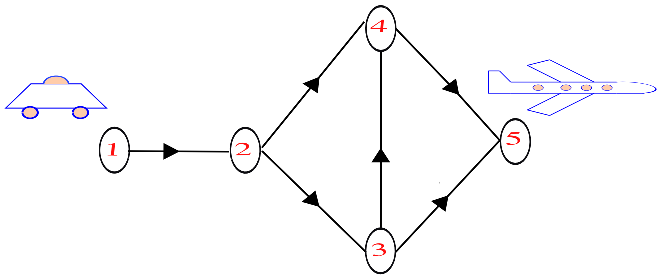


Fig. 5. A layout of baggage handling system with one loading station and one unloading station. The length of each link in the network is 40 m.

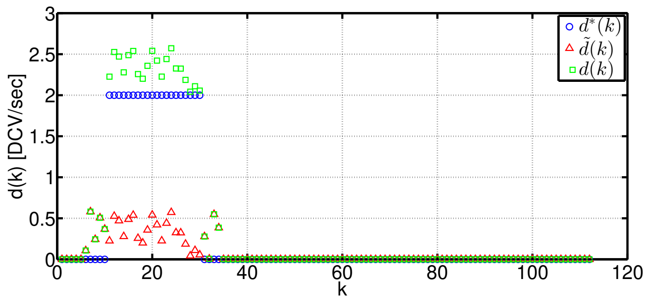


Fig. 6. Base baggage demand,the perturbations on the base demand, and the actual demand at the loading station.

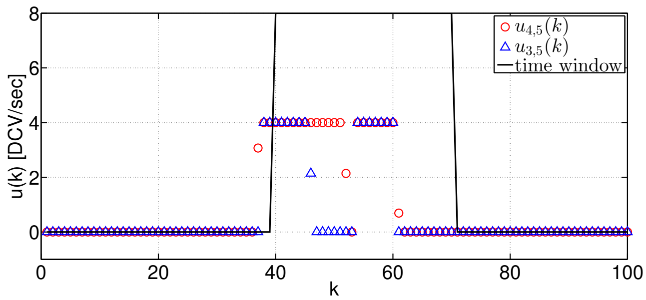


Fig. 7. Optimal flows of DCVs at the unloading station. One can observe that most of the DCVs arrive at the unloading station within the specified time window.

In this section we present a case study to illustrate the performance of our proposed control approach for the baggage handling system. For the sake of simplicity, we consider a simple baggage handling system, the layout of which is depicted in Fig. 5. Here, the focus is to illustrate the effect of the feedback controller on suppressing the adverse effects of an unpredicted baggage demand on the behavior of the system. First, assuming that the demand is fully known, the optimal flows and optimal system trajectories are computed. Next, we consider some unpredictable random perturbations on the base demand and evaluate how closely our proposed two-level control approach can follow the optimal trajectory.

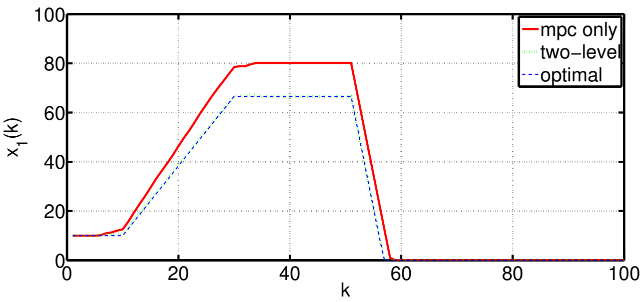


Fig. 8. Queue lengths at node 1. One can observe that for the two-level control approach, the queue length at node 1 is only slightly affected by the disturbance.

TABLE I

CONTROLLER DESIGN AND SIMULATION PARAMETERS

| MPC Parameters | | | | |
|---|---|---|---|---|
| $N_p$ | time window | $u_{max}$[DCV/s] | $(\alpha_1, \alpha_2, \alpha_3)$ | $(c_1^{flow}, c_2^{flow}, c^{tw})$ |
| 12 | $[40, 70]$ | 4 | $(1, 125, 1)$ | $(100, 0.2, 1)$ |
| **Feedback Controller Parameters** | | | | |
| $c_\gamma$ | $\gamma^*_{min}$ | $\gamma_{min}$ | $\lambda^*_{max}$ | $\lambda_{max}$ |
| 40 | 1.4420 | 1.5222 | 2.0001 | 3.8091 |
| **Closed-loop Simulation Parameters** | | | | |
| $T_s$[s] | $N_{sim}$ | $x_0$(initial condition) | demand perturbation | $v_{DCV}$[m/s] |
| 1.41 | 100 | 10 | $\tilde{d} \in \mathscr{U}(0, 1), \|d\|_2^2 = 4.02$ | 1.41 |

For the two-level control approach, we have computed the feedback gain $K$ based the approach of Section IV-C using the MATLAB LMI toolbox. Table I lists the parameters used for the controller design and the closed-loop simulation. In Table I, $\lambda^*_{max}$ and $\gamma^*_{min}$ denote, respectively, the *actual* values of $\lambda_{max}(\gamma Q)$ and $\gamma$ achieved by the closed-loop system whereas $\lambda^*_{max}$ and $\gamma^*_{min}$ denote those values obtained by solving (33).

For the base demand $d^*(k)$ depicted in Fig. 6, the optimal flows to the destination (node 5) are illustrated in Fig. 7 and the resulting optimal queue length at the origin node (node 1) is depicted in Fig. 8. It is clear from Fig. 7 that the optimal flows arrive at the destination within the desired time window. The perturbation on the base demand $\tilde{d}(k) \in \mathscr{U}(0, 1)$ is depicted in Fig. 6. It is obvious from Fig. 8 that in the presence of unpredictable demand perturbations, the two-level controller follows the optimal trajectory very closely whereas the MPC based approach deviates from the optimal trajectory.

## VII. CONCLUSIONS AND FUTURE WORK

The routing problem in baggage handling systems was revisited. A new flow-based model was derived for our control purposes, which are delivering the pieces of baggage at the unloading stations within a pre-specified time window, and minimizing the energy consumption. We proposed a multi-level control approach with an MPC controller at the top level and a constrained feedback controller at the bottom level that minimizes the $L_2$ gain of the closed-loop system. The idea was that based on some prior knowledge on the baggage demand, the MPC controller computes the optimal control inputs and system trajectories such that the pieces of baggage arrive at their destination within a desired time window and with minimal energy consumption. The feedback controller then would guarantee minimal deviation from this optimal trajectory in face of unknown perturbations on the baggage demand.

We showed that the MPC problem can be formulated as a linear programming problem. We proposed two methods to include state constraints in design procedure of the feedback controller that can be recast as LMI constraints. Using a simple case study, we showed the effectiveness of the proposed two-level control approach.

This approach should be extendable to large-scale system. However, for large-scale systems, the conservatism introduced by (28) may render the LMIs in (33) infeasible. Hence, one may need to find a tighter lower bound $\alpha$ in (34).

For future work, the scalability of the proposed two-level approach to large network layouts will be investigated. In addition, we will compare the performance of the two-level control approach with the MPC-based approach for larger network layouts and more elaborate scenarios. As a second extension to the current work, we will include non-polytopic state constraints as well as control signal constraints in the design procedure of the feedback controller.

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