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Origin-Destination Dependent Train Scheduling Problem with Stop-Skipping for Urban Rail Transit Systems

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Abstract.

The train scheduling problem with the origin-destination (O-D) dependent passenger demands is considered for urban rail transit systems. In this paper, trains are allowed to skip any intermediate stations (except the origin station and the final station) to reduce the passenger travel time and to save energy consumption. A model of train movements with stop-skipping and the O-D dependent passenger demands is formulated. A bi-level optimization approach is proposed to solve the train scheduling problem with stop-skipping, which is essentially a mixed integer nonlinear programming problem. The performance of the proposed approach is illustrated via a case study using data of the Beijing Yizhuang subway line.

1 INTRODUCTION

Urban rail transit systems are important for the stability and sustainability for the public transportation, especially in large cities like Beijing, Shanghai, Tokyo, New York, and Paris. They provide safe and effective passenger services. The passenger demand for urban rail transit systems is increasing dramatically; often trains arrives at a station every 2 to 5 minutes to transmit the passengers. Therefore, the scheduling of trains becomes more and more important for reducing the operation costs and for guaranteeing passenger satisfaction. Passenger satisfaction can be characterized by waiting times, on-board times, and the number of transfers.

Many researchers have explored the train scheduling problem for urban rail transit systems using various approaches. In 1980, Cury et al. (*1*) presented a methodology to generate optimal schedules for metro lines based on a model of the train movements and passenger behavior. The performance index includes passenger delay, passenger comfort, and the efficiency of the operation of trains. The resulting nonlinear scheduling problem is recast into many sub-problems by Lagrangian relaxation and then solved in a hierarchical manner (*1*). Since the convergence rate of the hierarchical decomposition algorithm can be quite poor in some cases, Assis and Milani (*2*) proposed a model predictive control algorithm based on linear programming to optimize the train schedule. The algorithm proposed in (*2*) can effectively generate train schedules for the whole day. In addition, Kwan and Chang (*3*) applied a heuristic-based evolutionary algorithm to solve the train scheduling problem, where the operation costs and the passenger dissatisfaction are included in the performance index. Furthermore, a demand-oriented timetable design is proposed in (*4*), where the optimal train frequency and the capacity of trains are first determined and then the schedule of trains are optimized.

In practice, trains do not run exactly according to the predefined schedule. Therefore, several rescheduling approaches have been proposed for urban rail transit systems (*5*): holding, zone scheduling, short turning, deadheading, and dynamic stop-skipping. Wong and Ho (*6*) proposed dwell time and running time control for the real-time rescheduling problem of urban rail transit systems. They applied a dynamic programming approach to the event-based rescheduling model to devise an optimal set of dwell times and running times (*6*). In addition, Goodman and Murata (*7*) formulated the train rescheduling problem from the perspective of passengers, where a gradient calculation method is developed to solve the rescheduling problem in real time. Furthermore, Norio et al. (*8*) proposed to use passenger dissatisfaction as a criterion for the rescheduling and applied a meta-heuristics algorithm to solve the rescheduling problem.

We have proposed a real-time scheduling approach for trains based on the passenger demand in (*9*), where the capacity of the trains, the capacity of the stations, and the safety constraints caused by urban rail transit systems are included. The objective of the real-time scheduling problem was to minimize the total travel time of passengers. Furthermore, the problem was solved by a sequential quadratic programming (SQP) approach in (*9*). However, the train scheduling problem is essentially a multi-objective optimization problem because it should consider both the benefits of the rail transport operators and the passengers (*10, 11*). The rail transport operators prefer to minimize the operation costs (e.g., energy consumption). This conflicts with the benefit of the passengers (e.g., travel time) since a lower operation cost usually results in a longer travel time. In (*12*), we have solved the multi-objective train scheduling problem (i.e., minimizing the energy consumption and the passenger travel time) using a pattern search method, a mixed integer nonlinear programming approach, and a mixed integer linear programming approach.

However, the origin and destination of each passenger are not considered in (*1*)-(*12*). Therefore, this paper addresses the origin-destination (O-D) dependent train scheduling problem. Since the origin and destination of the passengers are known, the train scheduling problem with stop-skipping can then be formulated. In that case, trains can skip any intermediate station (except the origin station and the final station).

This paper is structured as follows. Section 2 formulates the dynamics of the operation of trains, the passenger demand characteristics, and the passenger/vehicle interaction. Section 3 describes the multi-objective cost function and the constraints of the train scheduling problem with stop-skipping. Section 4 proposes a bi-level optimization approach for the train scheduling problem with stop-skipping. Section 5 illustrates the performance of the proposed solution approach with a case study. Finally, Section 6 concludes the paper.

2 MODEL FORMULATION

This paper considers one direction of an urban rail transit line with J stations, where trains depart from station 1 and run to station J. We assume that the intermediate station j for $j \in \{2,3,\ldots,J-1\}$ can only accommodate one train at a time and no overtaking can occur at any point in the line. Furthermore, we select the order in which the trains run such that vehicle $i-1$ always precedes train i for $i \in \{1,2,3,\ldots,I\}$ with I the total number of trains. In addition, we assume that all the trains considered in this paper have the same physical characteristics.

2.1 The operation of a train

The aim of this section is to derive the model equation for the operation of a train with stop-skipping. The operation of trains usually goes as follows: first the train arrives at a station, next it dwells at the station for allowing passengers to alight from the train and to board it, and then the train departs for the next station. However, if a train skips a station, then the train passes that station without stopping and the dwell time is then equal to zero; so passengers are not allowed to get off or on the train. A binary variable $y_{i,j}$ is introduced to indicate whether train i stops at station j or not:

$$y_{i,j} = \begin{cases} 1 & \text{if train } i \text{ stops at station } j, \\ 0 & \text{if train } i \text{ skips station } j. \end{cases} \tag{1}$$

The departure time $d_{i,j}$ of train i at station j can then be written as

$$d_{i,j} = a_{i,j} + \tau_{i,j}, \tag{2}$$

where $a_{i,j}$ is the arrival time of train i at station j and $\tau_{i,j}$ is the dwell time of train i at station j. If train i passes station j without stop, i.e., $y_{i,j}=0$, then the dwell time $\tau_{i,j}$ for train i at station j is equal to zero. However, if train i stops at station j, i.e., $y_{i,j}=1$, the dwell time $\tau_{i,j}$ is influenced by the number of passengers boarding and alighting from the train, as well as the number of waiting passengers at platform (see Section 2.3 for more details).

The arrival time $a_{i,j+1}$ of train i at station $j+1$ equals the sum of the departure time $d_{i,j}$ at station j and the running time $r_{i,j}$on segment j (which is defined as the track between station j and station $j+1$) for train i:

$$a_{i,j+1} = d_{i,j} + r_{i,j}. \tag{3}$$

In this paper, we assume that the operation of trains between two stations (at which it stops) only consists of three phases: the acceleration phase, the speed holding phase, and the deceleration phase. If a train skips station j for $j \in \{2,3,\ldots,J-1\}$, then the deceleration phase for station $j-1$ and the acceleration phase for station j have length zero. In addition, we assume the acceleration $a_{i,j}^{\text{acc}}$ and the deceleration $a_{i,j}^{\text{dec}}$ are known constants. The speed in the speed holding phase is denoted as $v_{i,j}$ and it should satisfy

$$v_{i,j} \in [v_{i,j,\min}, v_{i,j,\max}], \tag{4}$$

where $v_{i,j,\min}$ and $v_{i,j,\max}$ are the minimal and maximal running speed for the speed holding phase of train i at segment j, respectively. The maximum running speed is determined by the train characteristics and the line conditions. If trains run too slow, the passengers may complain. Therefore, a minimum running speed is introduced to avoid passenger dissatisfaction. The running distance of the three phases can then be calculated as

$$s_{i,j}^{\mathrm{acc}} = y_{i,j} \frac{v_{i,j}^2}{2a_{i,j}^{\mathrm{acc}}},$$
$$s_{i,j}^{\mathrm{dec}} = y_{i,j+1} \frac{v_{i,j}^2}{2a_{i,j}^{\mathrm{dec}}},$$
$$s_{i,j}^{\mathrm{hold}} = s_j - s_{i,j}^{\mathrm{acc}} - s_{i,j}^{\mathrm{dec}}.$$

If train i skips station j, the holding speed will not be changed:

$$v_{i,j} = v_{i,j-1} \quad \text{if} \quad y_{i,j} = 0,$$

or equivalently

$$(1 - y_{i,j})(v_{i,j} - v_{i,j-1}) = 0. \tag{5}$$

The running time for train i on segment j is equal to the running times for the three different phases, i.e.,

$$\begin{aligned} r_{i,j} = t_{i,j}^{\mathrm{acc}} + t_{i,j}^{\mathrm{hold}} + t_{i,j}^{\mathrm{dec}} &= \frac{s_{i,j}^{\mathrm{acc}}}{v_{i,j}} + \frac{s_{i,j}^{\mathrm{hold}}}{v_{i,j}} + \frac{s_{i,j}^{\mathrm{dec}}}{v_{i,j}} \\ &= \frac{s_j}{v_{i,j}} + y_{i,j} \frac{v_{i,j}}{2a_{i,j}^{\mathrm{acc}}} + y_{i,j+1} \frac{v_{i,j}}{2a_{i,j}^{\mathrm{dec}}}. \end{aligned} \tag{6}$$

The minimum headway h_0 is defined as the minimum time interval between two successive trains so that they can enter and depart from a station safely (*13*). This leads to the following constraint:

$$a_{i,j} - d_{i-1,j} \geq h_0. \tag{7}$$

2.2 Passenger characteristics

We assume that each passenger only takes one train to arrive at his/her destination, i.e., the transfer between different trains along the line is not allowed. The number of passengers with destination station m remaining at station j immediately after the departure of train $i-1$ is denoted by $w_{i-1,j,m}$. When train i arrives at station j, the number of passengers $w_{i,j,m}^{\mathrm{waiting}}$ waiting at the platform with destination station m is equal to the sum of the number of passengers $w_{i-1,j,m}$ left by train $i-1$ and the number of newly arrived passengers that have station m as their destination and arrive at station j in between the departures of train i and train $i-1$:

$$w_{i,j,m}^{\mathrm{waiting}} = w_{i-1,j,m} + \lambda_{j,m}(d_{i,j} - d_{i-1,j}), \tag{8}$$

where $\lambda_{j,m}$ is the passenger arrival rate at station j for passengers whose destination is station m. The passenger arrival rates in the O-D matrix can be estimated using the real-time data and historical data collected by the advanced automated fare collection system. Note that the passenger arrival rate at the final station J is zero, i.e., no passengers arrival at station J, since we only consider one direction of the line. The total number of passengers $w_{i,j}^{\mathrm{waiting}}$ at station j waiting for train i is then

$$w_{i,j}^{\mathrm{waiting}} = \sum_{m=j+1}^{J} w_{i,j,m}^{\mathrm{waiting}}. \tag{9}$$

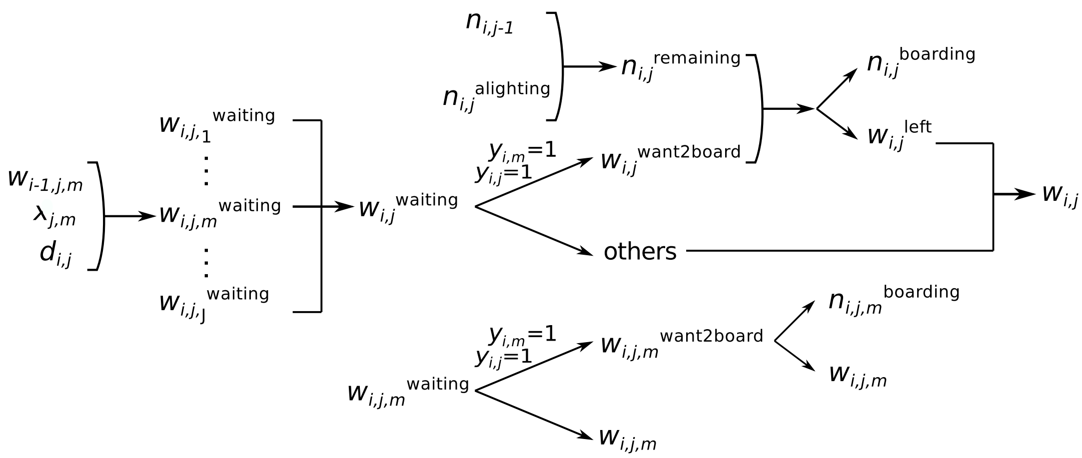

FIGURE 1 The variables for the passenger characteristics

The number of passengers $w_{i,j,m}^{\text{want2board}}$ who want to board train i at station j and have station m as their destination depends on whether train i stops at station j and whether train i stops at station m for $m \in \{j+1, j+2, \ldots, J\}$, i.e.,

$$w_{i,j,m}^{\text{want2board}} = y_{i,j} y_{i,m} w_{i,j,m}^{\text{waiting}}. \tag{10}$$

Note that all the trains stop at the final station J, so $y_{i,J}$ is equal to 1 for $i \in \{1, 2, \ldots, I\}$. The number of passengers $w_{i,j}^{\text{want2board}}$ who want to board train i at station j can be written as

$$w_{i,j}^{\text{want2board}} = y_{i,j} \left(w_{i,j,J}^{\text{waiting}} + \sum_{m=j+1}^{J} y_{i,m} w_{i,j,m}^{\text{waiting}} \right). \tag{11}$$

So if train i skips station j, i.e., $y_{i,j} = 0$, then no passengers want to board train i, i.e., $w_{i,j}^{\text{want2board}} = 0$. If train i stops at station j, i.e., $y_{i,j} = 1$, then the number of passengers who want to board is decided by whether train i stops at their destination m, i.e., $y_{i,m} w_{i,j,m}^{\text{waiting}}$.

We define $n_{i,\ell,j}^{\text{boarding}}$ as the number of passengers that have station j as their destination and that boarded train i at station ℓ (the update equation for $n_{i,\ell,j}^{\text{boarding}}$ is given in (17) below). The number of passengers alighting from train i at station j can be written as

$$n_{i,j}^{\text{alighting}} = \sum_{\ell=1}^{j-1} n_{i,\ell,j}^{\text{boarding}}. \tag{12}$$

In addition, we define the number of passengers on train i immediately after its departure at station j as $n_{i,j}$ (the update equation for $n_{i,j}$ is given in (18) below). The remaining capacity of train i at station j immediately after the alighting process is

$$n_{i,j}^{\text{remaining}} = C_{\max} - n_{i,j-1} - n_{i,j}^{\text{alighting}}. \tag{13}$$

The number of boarding passengers $n_{i,j}^{\text{boarding}}$ equals the minimum of the number of passengers that want to board train i and the remaining capacity of the train, which can be formulated as

$$n_{i,j}^{\text{boarding}} = \min(n_{i,j}^{\text{remaining}}, w_{i,j}^{\text{want2board}}). \tag{14}$$

In addition, the number of passengers $n_{i,j}^{\text{boarding}}$ boarding train i at station j is also equal to

$$n_{i,j}^{\text{boarding}} = \sum_{m=j+1}^{J} n_{i,j,m}^{\text{boarding}}.$$

The number of passengers left by train i depends on whether train i stops at station j or not. We have the following two cases:

- Train i skips station j, i.e., $y_{i,j} = 0$

  If train i skips station j, then the number of boarding passengers $n_{i,j}^{\text{boarding}}$ is equal to zero. All the passengers waiting at station j will then be left by train i.

- Train i stops at station j, i.e., $y_{i,j} = 1$

  Now consider the case that train i stops at station j. If $w_{i,j}^{\text{want2board}} < n_{i,j}^{\text{remaining}}$, then all the passengers that want to board can get on train i. However, there will be passengers left by train i if $w_{i,j}^{\text{want2board}} > n_{i,j}^{\text{remaining}}$. The number of passengers who want to board but cannot get on train i at station j immediately after the departure of train i is

  $$w_{i,j}^{\text{left}} = w_{i,j}^{\text{want2board}} - \min(n_{i,j}^{\text{remaining}}, w_{i,j}^{\text{want2board}}) \quad \text{if } y_{i,j} = 1. \tag{15}$$

  In this case, if train i stops station m for $m \in \{j+1, j+2, \ldots, J-1\}$, i.e., $y_{i,m} = 1$, we assume that the number of passengers that have station m as destination and are left by train i is proportional to the number of passengers who want to board. The number of passengers who have destination m and are left by train i can be formulated as

  $$w_{i,j,m} = w_{i,j}^{\text{left}} \frac{w_{i,j,m}^{\text{want2board}}}{w_{i,j}^{\text{want2board}}} \quad \text{if } y_{i,j} = 1 \text{ and } y_{i,m} = 1.$$

  However, if train i skips station m for $m \in \{j+1, j+2, \ldots, J-1\}$, i.e., $y_{i,m} = 0$, then the number of passengers that have station m as destination will not board. So we have

  $$w_{i,j,m} = w_{i,j,m}^{\text{waiting}}, \quad \text{if } y_{i,j} = 1 \text{ and } y_{i,m} = 0.$$

Hence, the number of passengers who are left by train i and with destination m can be calculated as

$$w_{i,j,m} = y_{i,j}\left(y_{i,m} w_{i,j}^{\text{left}} \frac{w_{i,j,m}^{\text{want2board}}}{w_{i,j}^{\text{want2board}}} + (1 - y_{i,m}) w_{i,j,m}^{\text{waiting}} \right) + (1 - y_{i,j}) w_{i,j,m}^{\text{waiting}}. \tag{16}$$

For the passengers with destination m, the number of passengers boarding on train i is equal to the difference between the number of waiting passengers $w_{i,j,m}^{\text{waiting}}$ and the number of passengers remaining $w_{i,j,m}$ at station j immediately after the departure of train i, i.e.,

$$n_{i,j,m}^{\text{boarding}} = w_{i,j,m}^{\text{waiting}} - w_{i,j,m}. \tag{17}$$

The number of passengers on train i immediately after its departure at station j, denoted as $n_{i,j}$, can be computed using

$$n_{i,j} = n_{i,j-1} - n_{i,j}^{\text{alighting}} + n_{i,j}^{\text{boarding}}. \tag{18}$$

Furthermore, the total number of waiting passengers at station j immediately after the departure of train i is

$$w_{i,j} = \sum_{m=j+1}^{J} w_{i,j,m}. \tag{19}$$

2.3 Passenger/vehicle interaction

The minimum dwell time is influenced by the number of passengers boarding and alighting from a train. In addition, the minimum dwell time is also affected by the number of waiting passengers at station: if there are many passengers waiting at the platform, then the boarding process will be slower. In (*14*), a nonlinear function is given to compute the minimum dwell time:

$$\tau_{i,j,\min} = \alpha_{1,\mathrm{d}} + \alpha_{2,\mathrm{d}} n_{i,j}^{\mathrm{alighting}} + \alpha_{3,\mathrm{d}} n_{i,j}^{\mathrm{boarding}} + \alpha_{4,\mathrm{d}} \left(\frac{w_{i,j}^{\mathrm{waiting}}}{n^{\mathrm{door}}} \right)^3 n_{i,j}^{\mathrm{boarding}}, \tag{20}$$

where $\alpha_{1,\mathrm{d}}$, $\alpha_{2,\mathrm{d}}$, $\alpha_{3,\mathrm{d}}$, and $\alpha_{4,\mathrm{d}}$ are coefficients that can be estimated based on historical data, n^{door} is the number of doors of the train, and $w_{i,j}^{\mathrm{waiting}}/n^{\mathrm{door}}$ is the number of passengers waiting at each door. Note that here we assume that the passengers are equally distributed over all doors of the train. The dwell time $\tau_{i,j}$ should satisfy

$$y_{i,j}\tau_{i,j,\min} \leq \tau_{i,j} \leq y_{i,j}\tau_{i,j,\max}. \tag{21}$$

Note that we impose a maximum dwell time $\tau_{i,j,\max}$ to ensure passenger satisfaction.

3 TRAIN SCHEDULING PROBLEM

Based on the models given in Section 2, we formulate the objective function of the train scheduling problem here. The energy consumption for the acceleration phase of train i on segment j is

$$E_{i,j}^{\mathrm{acc}} = \int_0^{t_{i,j}^{\mathrm{acc}}} \Big((m_\mathrm{e} + n_{i,j} m_\mathrm{p})(a_{i,j}^{\mathrm{acc}} + k_1 + k_2 v(t) + g\sin(\theta_j)) + k_3 v^2(t) \Big) v(t)\mathrm{d}t, \tag{22}$$

where m_e is the mass of the train itself, m_p is the mass of one passenger, k_1, k_2, and k_3 are the resistance coefficients of the train, $v(t)$ is equal to $a_{i,j}^{\mathrm{acc}} t$, g is the gravitational acceleration, and θ_j is the gradient of segment j. The energy consumption for the speed holding phase of train i on segment j is

$$E_{i,j}^{\mathrm{hold}} = \int_{t_{i,j}^{\mathrm{acc}}}^{t_{i,j}^{\mathrm{acc}} + t_{i,j}^{\mathrm{hold}}} \Big((m_\mathrm{e} + n_{i,j} m_\mathrm{p})(k_1 + k_2 v_{i,j} + g\sin(\theta_j)) + k_3 v_{i,j}^2 \Big) v_{i,j}\mathrm{d}t. \tag{23}$$

Note that the deceleration of trains does not consume energy. Since we do not consider the regenerative braking scheme, so the deceleration phase also does not generate energy. The total energy consumption for all I trains running with J stations can then be formulated as

$$E_{\mathrm{total}} = \sum_{i=1}^{I} \sum_{j=1}^{J-1} (E_{i,j}^{\mathrm{acc}} + E_{i,j}^{\mathrm{hold}}). \tag{24}$$

The total travel time is the sum of the passenger waiting time and the passenger in-vehicle time. The passenger waiting time $t_{\mathrm{wait},i,j}$ at station j for train i includes the waiting time of both passengers left by the previous train $i-1$ and the newly arrived passengers, and it can be calculated by

$$t_{\mathrm{wait},i,j} = w_{i-1,j}(d_{i,j} - d_{i-1,j}) + \frac{1}{2} \sum_{m=j+1}^{J} \lambda_{j,m} (d_{i,j} - d_{i-1,j})^2, \tag{25}$$

where the first term represents the waiting time of the passengers left by train $i-1$ at station j, and the second term represents the waiting time of uniformly arriving passengers between the departures of train $i-1$ and train i. The passenger in-vehicle time for train i running from station j to $j+1$ includes the running time for all passengers on train i after its departure form station j and the waiting time of the passengers who do not get off the train at station $j+1$, which can be formulated as

$$t_{\text{in-vehicle},i,j} = n_{i,j} r_{i,j} + (n_{i,j} - n_{i,j}^{\text{alighting}}) \tau_{i,j+1}. \tag{26}$$

The total passenger travel time for all I trains can then be formulated as

$$t_{\text{total}} = \sum_{i=1}^{I} \sum_{j=1}^{J-1} (t_{\text{wait},i,j} + t_{\text{in-vehicle},i,j}). \tag{27}$$

In addition, we also consider the total number of passengers w_I^{left} left by train I (i.e., the last train of the scheduling period) at stations as a penalty term in the objective function. The number of passengers left by train I can be calculated as

$$w_I^{\text{left}} = \sum_{j=1}^{J-1} w_{i,j}^{\text{left}}. \tag{28}$$

We apply the weighted-sum strategy to solve the multi-objective optimization of the O-D dependent train scheduling problem (*10, 11, 15*), i.e. we write the total objective function as a weighted sum of the objectives:

$$f_{\text{opt}} = \gamma_1 \frac{E_{\text{total}}}{E_{\text{total,nom}}} + \gamma_2 \frac{t_{\text{total}}}{t_{\text{total,nom}}} + \gamma_3 \frac{w_I^{\text{left}}}{w_{I,\text{nom}}^{\text{left}}}, \tag{29}$$

where γ_1, γ_2, and γ_3 are non-negative weights, and the normalization factors $E_{\text{total,nom}}$, $t_{\text{total,nom}}$, and $w_{I,\text{nom}}^{\text{left}}$ are "nominal" values of the total energy consumption, the total travel time of passengers, and the total number of passengers left by train I, respectively. These nominal values can e.g. be determined by running trains using a feasible initial schedule.

4 SOLUTION APPROACH

The train scheduling problem (29) with constraints (2)-(21) is a mixed integer nonlinear programming (MINLP) problem, where the objective function (29) is nonlinear and non-convex. In addition, we have the non-smooth min function in constraints (14), (15), and (18). This MINLP problem can be solved using the direct MINLP approach, implemented solver such as MINLP BB (*16*) and SCIP (*17*). Moreover, we could also approximate the nonlinear terms in the MINLP problem using piecewise affine functions and transform the MINLP problem into a mixed integer linear programming problem, which can be solved by existing solvers, e.g., CPLEX (*18*). However, the direct MINLP approach can only solve the small-sized scheduling problem. Furthermore, in the MILP approach we need to approximate many nonlinear terms of the scheduling problem; so the performance of the MILP approach is not optimal due to the big approximation error. In this paper, we propose a bi-level optimization approach to solve the train scheduling problem with stop-skipping.

The variables of the train scheduling problem with stop-skipping include the departure times $d_{i,j}$, the running times $r_{i,j}$, the dwell times $d_{i,j}$, and the binary variables $y_{i,j}$ (which are used to indicate the stop status of train i at station j). The other variables, such as the number of waiting passengers $w_{i,j}^{\text{waiting}}$ at the platform and the number of passengers on-board $n_{i,j}$ immediately after the departure of train i, can be eliminated.

The proposed bi-level optimization method consists of two levels of optimization. The high level optimizes the binary variables using integer programming approaches, such as genetic algorithms or brute force algorithms. For the low-level optimization, a nonlinear non-convex problem is solved since the binary variables are fixed. Multi-start sequential quadratic programming algorithms (*19*) and pattern search (*20*)

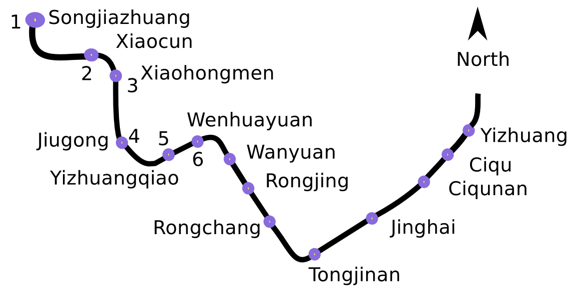

FIGURE 2 The layout of the Yizhuang subway line

can be applied to the nonlinear non-convex problem with real-valued variables in the low level. In this paper, we apply a genetic algorithm for the integer optimization at the high level and the sequential quadratic programming algorithms for the low-level optimization. The procedure of the bi-level optimization method is given in Algorithm 1. The feasibility of the low-level optimization problem depends on the value of the binary variables. If the low-level optimization problem is infeasible, we introduce a relative large penalty value F for the fitness function as shown in Algorithm 1. Furthermore, in order to steer unfeasible binary variables towards feasible ones, we also add the norm of $\delta - \delta_{\mathrm{f}}$ in the fitness function where δ_{f} is a feasible value of δ.

Algorithm 1 The procedure of the bi-level optimization approach

Input : maximum number of generations G, population size s_{p}, initial population P_0 of the binary variables, number of initial points $k_{\max}$ used in the low-level optimization, a relatively large value F for the fitness function, a feasible binary variable δ_{f}, a weight λ_{f};
for $g = 0, 1, \ldots, G-1$ **do**
  for $\ell = 1, 2, \ldots, s_{\mathrm{p}}$ **do**
    binary variables $\delta \leftarrow i$-th parent $P_g(\ell)$ from the g-th generation;
    for $k = 1, 2, \ldots, k_{\max}$ **do**
      generate an initial random feasible solution $d_{i,j}(\ell,0)$, $r_{i,j}(\ell,0)$, $\tau_{i,j}(\ell,0)$, $w_{i,j}(\ell,0)$, and $n_{i,j}(\ell,0)$ for $i = 1, \ldots, I$ and $j = 1, \ldots, J$;
      if low-level optimization problem turns out to be feasible based on current values of δ and initial solutions **then**
        $d_{i,j}(\ell,k)$, $r_{i,j}(\ell,k)$, $\tau_{i,j}(\ell,k)$, $w_{i,j}(\ell,k)$, $n_{i,j}(\ell,k)$, and $f_{\mathrm{opt}}(\ell,k) \leftarrow$ solution of the low-level optimization problem;
      else
        $f_{\mathrm{opt}}(\ell,k) \leftarrow F + \lambda_{\mathrm{f}} \|\delta - \delta_{\mathrm{f}}\|_2$;
      end if
    end for
    value of fitness function $f^*_{\mathrm{opt}}(i)$ for the i-th parent $\leftarrow \min_{k=1,\ldots,k_{\max}} f_{\mathrm{opt}}(\ell,k)$;
  end for
  select new parents from the current population based on the fitness function f_{opt};
  generate a new generation population of binary variables through crossover and mutation;
end for
Output : choose the best offspring solution at generation G and calculate $d_{i,j}$, $r_{i,j}$, $\tau_{i,j}$, $w_{i,j}$, and $n_{i,j}$.

TABLE 1 Information of the Yizhuang subway line

Station number	1	2	3	4	5	6
Distance to next station [m]	1332	1286	2086	2265	2331	–
Minimum running time [s]	87.700	85.628	121.664	129.927	132.700	–

5 CASE STUDY

In order to demonstrate the performance of the bi-level optimization approach proposed in Section 4 for the O-D dependent train scheduling problem with stop-skipping, the train characteristics and a part of the line data of the Yizhuang subway line in Beijing are used as a test case study. There are 14 stations in the Yizhuang line as shown in Figure 2, but we only consider the first 6 stations for the sake of compactness. The number of trains taken into account is 5 for this case study, i.e., train $i \in \{1,2,\ldots,5\}$. Moreover, we allow all trains to skip stations 2, 3, 4, and 5. In addition, it is assumed that train 0 precedes train 1. The schedule of train 0 is given and fixed, which is shown as the red line in Figure 3. Train 0 arrives station 1 at 0 s, runs with the minimum running time, and stops at each station for 120 s. Furthermore, we assume that there are no passengers left by train 0.

The speed limit for the line is 80 km/h (i.e., 22.2 m/s). Detailed information about the 6 stations we consider is listed in Table 1. The minimum running time in Table 1 is calculated by taking a fixed acceleration of 0.8 m/s^2 and a fixed deceleration of -0.8 m/s^2; furthermore, the trains are assumed to run at the maximum speed 22.2 m/s during the holding phase. The maximum running time is assumed as $r_{i,j,\max} = \zeta r_{i,j,\min}$, where ζ is larger than 1. We have chosen ζ as 1.2 to ensure that the passengers do not complain that the train is too slow. The train mass is $1.99 \cdot 10^5$ kg and the mass of one passenger is 60 kg. The passenger arrival rates at stations are shown in as follows

$$\lambda = \begin{bmatrix} 0 & 1.5 & 0.1 & 2.1 & 0.1 & 1.1 \\ 0 & 0 & 0.2 & 2.8 & 0.1 & 1.6 \\ 0 & 0 & 0 & 0.2 & 0.1 & 0.1 \\ 0 & 0 & 0 & 0 & 0.2 & 2.4 \\ 0 & 0 & 0 & 0 & 0 & 0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{30}$$

where e.g., the arrival rate at station 1 of the passengers with destination station 2 is 1.5 passenger/s. The maximal dwell time is chosen as 150 s and the minimal dwell time is decided according to (20). Based on the dwell time research about Beijing subway stations (*14*), the values of the minimum dwell time coefficients $\alpha_{1,\mathrm{d}}$, $\alpha_{2,\mathrm{d}}$, $\alpha_{3,\mathrm{d}}$, and $\alpha_{4,\mathrm{d}}$ are chosen as 4.074, 0.048, 0.051, and $1 \cdot 10^{-5}$, respectively. For the calculation of energy consumption, the coefficients of the resistance k_{1i}, k_{2i}, and k_{3i} are chosen as $1.210 \cdot 10^{-2}$, $5.049 \cdot 10^{-4}$, and 8.521. The capacity of each train is 1468 passengers according to the train characteristics of the Yizhuang line, and the minimum headway h_0 between two successive trains is 90 s.

This paper proposes the bi-level optimization to solve the O-D dependent train scheduling problem with stop-skipping. The high level of the bi-level optimization uses a genetic algorithm for the binary optimization, where the `ga` function in the global optimization toolbox of Matlab is applied. The resulting low-level optimization problem is nonlinear and non-convex; to solve this problem the sequential quadratic programming algorithm of the `fmincon` function in the Matlab Tomlab toolbox is used.

The obtained train schedule with stop-skipping is shown in Figure 3; this schedule is obtained by solving the O-D dependent train scheduling problem with stop-skipping for 5 trains (i.e., train $i \in \{1,2,\ldots,5\}$) and 6 stations (i.e., station $j \in \{1,2,\ldots,6\}$). As we can observe from Figure 3, station 3 is skipped by train 1 and train 4. Furthermore, station 5 is skipped by train 1, train 2, train 3, and train

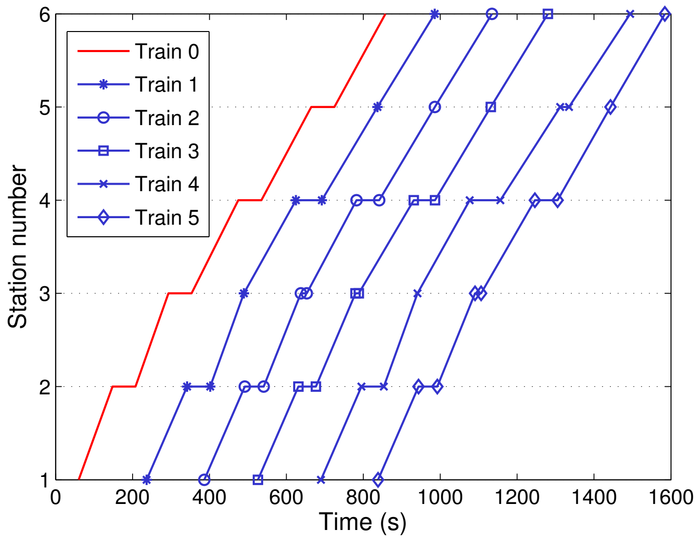

FIGURE 3 The train schedule for 5 trains and 6 stations obtained by the bi-level optimization approach

5. Based on the passenger arrival information given in (30), station 3 and station 5 have lower passenger demands when compared with other stations. Therefore, trains skip station 3 and station 5 as the skipping will reduce the energy consumption and the travel time of most of the passengers. The dwell times and running times of the trains are shown in Figure 4a. The dwell times of train 1 and train 4 at station 3 are equal to zero since they skip these two stations. This also holds for the trains skipping station 5. Furthermore, the nonzero dwell times for the trains stopping at station 3 and 5 are smaller than those at station 1, 2, and 4. This is because the value of the minimum dwell time is influenced by the number of passengers boarding and alighting from trains and there are less passengers getting on and getting off trains at station 3 and 5. In addition, most of the running times of trains obtained by the bi-level approach are larger than the minimum running time as shown in Figure 4b. Therefore, the energy consumption is reduced due to the longer running time. Moreover, since the running times are less than the maximum running times (i.e., $r_{i,j,\max} = 1.2 \cdot r_{i,j,\min}$), the passenger satisfaction constraints are satisfied.

The number of passengers $w_{i,j}$ left at stations and the number of passengers $n_{i,j}$ on-board the trains immediately after departure are shown in Figure 4c and Figure 4d. As we can observe from Figure 4c and Figure 4d, the number of passengers waiting at stations and the number of passengers on-board trains immediately after the departure of trains are affected by the headways between trains and the stop-skipping schedule of trains. If the headways between trains are larger, then there will be more passengers left by trains and on-board trains. For example, the headways between train 0 and train 1 are larger than those between train 1 and train 2; so in general there are more passengers on train 1 than on train 2. Moreover, the number of waiting passengers at station 5 is increasing after the departures of the first three trains because trains 1, 2, and 3 skip station 5. However, the number of waiting passengers reduces to zero after the departure of train 4 because it stops at station 5 and all the waiting passengers get on train 4. In addition, when a train skips one station, the number of the passengers on-board will not change at the skipping station, e.g., the number of passenger on train 1 at station 3 is equal to that at station 2.

Based on the simulation results, the proposed bi-level optimization approach can obtain an acceptable control performance for the O-D dependent train scheduling problem with stop-skipping. The computation time of the bi-level optimization for 5 trains and 6 stations is $2.147 \cdot 10^3$ s on a 1.8 GHz Intel Core2

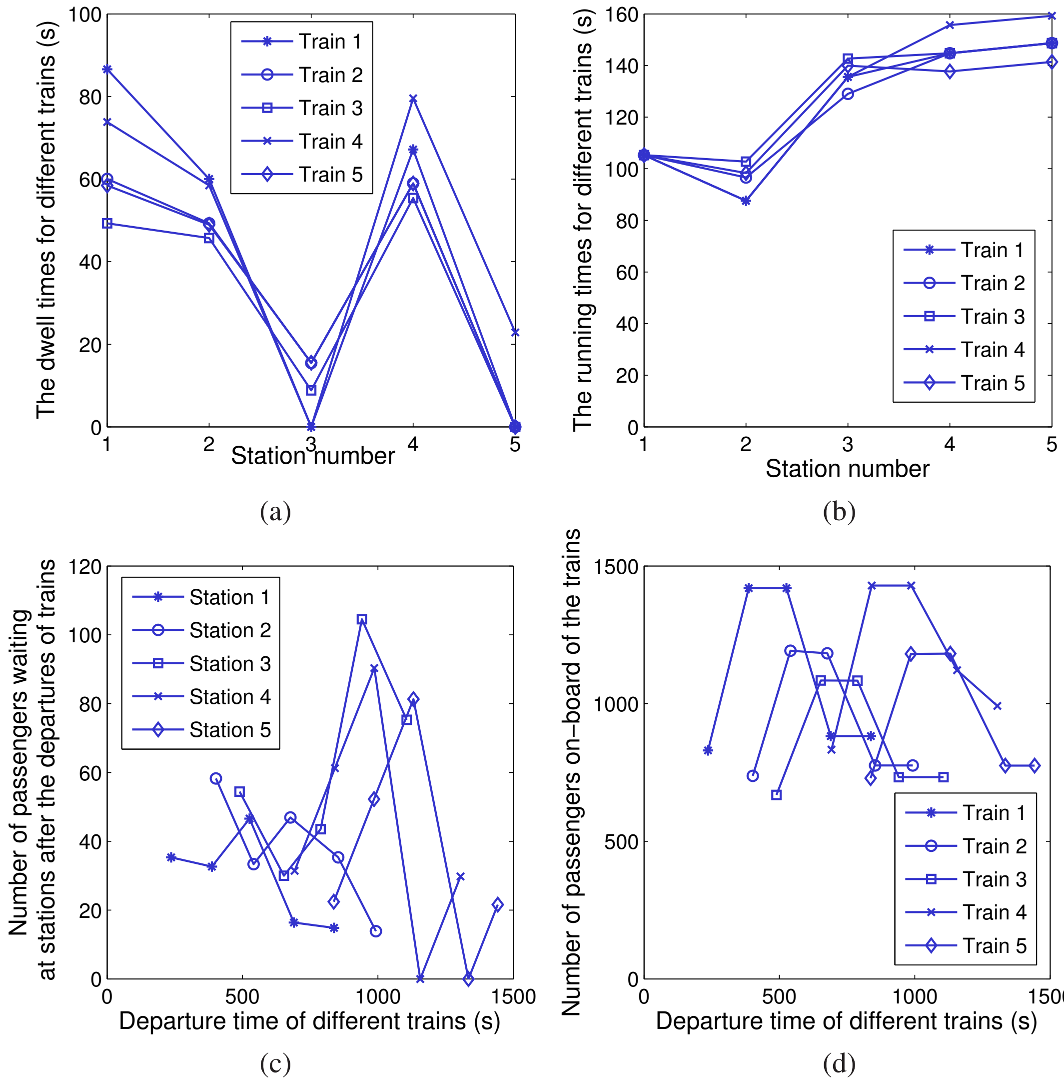

FIGURE 4 The results of trains and passengers obtained by the bi-level optimization approach; (a) running time of trains, (b) dwell time of trains, (c) the number of waiting passengers after the departure of trains, (d) the number of on-board passengers after the departure of trains

Duo CPU running on a 64-bit Linux operating system, which is still tractable for real-time application if using parallel processing. Since steps 4-11 in Algorithm 1 can be performed in parallel. However, this approach is slow for large-scale real-time applications.

6 CONCLUSIONS AND FUTURE WORK

We have considered the origin-destination (O-D) dependent train scheduling problem with stop-skipping. First, the train scheduling model with O-D passenger demands has been formulated. Thereby, we considered that the dwell time at stations is not only dependent on the number of alighting and boarding passengers, but that it is also influenced by the number of passengers waiting at the platform. The train scheduling problem is essentially a mixed-integer nonlinear programming problem. A bi-level optimization approach has been proposed to solve the O-D dependent train scheduling problem with stop-skipping, where the high level employs a genetic algorithm for integer programming and the low-level optimization applies a sequential quadratic programming approach. The simulation results show that the bi-level optimization approach can be applied to small-sized train scheduling problems. However, for large-scale real-time train scheduling problems with stop-skipping, new solution approaches need to be investigated. Furthermore, a comparison between the stop-skipping approach and the conventional all-stop approach will be done in future.

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