Technical report 14-031

Modeling of the dynamics and the energy consumption of a fleet of cybercars*

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Abstract— Automated driving technologies have already been developed for individual vehicles. However, the lack of efficient control strategies for the cooperation of a fleet of vehicles is one of the biggest challenges that cybercars (i.e., fully automatic road vehicles providing on-demand and door-to-door transportation service) are facing. Before an efficient fleet control method can be developed, a reasonably accurate and sufficiently fast model of the dynamics of a fleet of cybercars that is suited for control design is needed. In this paper, we discuss the modeling of the dynamics of a fleet of cybercars in a dedicated road network using a discrete-time modeling description. We consider the total time spent and total energy consumption by all the cybercars in the network and derive how these can be affected by the route choices.

I. INTRODUCTION

In many metropolises, the ever-increasing use of private cars together with the highly disorganized behaviors of human drivers is causing severe problems (e.g., a large amount of injuries and fatalities, frequent congestion, soaring energy consumption and pollution, increased noise levels) that degrade the quality of life and the environment. Being considered suitable solutions to these problems, public transportation systems (e.g., buses, trams, subways, etc) have been widely used and continuously improved. However, in public transportation systems, passengers have to accept pre-defined schedules and routes, and hence have to spend extra time waiting and transferring, and they also have to travel longer distances because of indirect routes. Since they are outperforming public transportation systems on the personal mobility level, private cars still form a large part of the current transport system and the problems caused by the increasing use of private cars are still largely unsolved.

A new and promising option to deal with this situation is to use a cybernetic transportation system i.e., an intelligent transportation system formed by a fleet of cybercars that drive automatically and provide on-demand and door-to-door service [1]–[4]. Cybercars are often small-sized and based on electric power, which is more efficient and less polluting than fossil fuels. They have high flexibility and reactivity (i.e., they can provide on-demand transportation service for any location at any time) and hence offer better urban mobility than conventional public transportation systems [5]. Besides, in terms of energy consumption, they are even competitive on a per passenger-km basis compared with

public transportation [6]. The European project CyberCars [7] is one of the first projects dedicated to developing such a cybernetic transportation system.

In fact, automated driving technologies have been well developed for individual vehicles [8] and some of the technologies (e.g., adaptive cruise control [9], automated lane change [10], etc.) are operating in real-life. But the lack of efficient strategies for the cooperation of a fleet of cybercars is still one of the biggest obstacles that hinder the large-scale application of cybercars.

The cooperation of a fleet of cybercars is necessary for the optimal performance of a cybernetic transportation system. There are several kinds of cooperation among cybercars, such as collision avoidance, platoon merge and split, dynamic routing, etc. When it comes to cooperation, cybercars can be characterized as moving decision-making agents with extensive on-board processing and communication capabilities as well as abundant information of the environment.

Actually, the fleet control problem of cybercars has already been considered in the literature. More specifically, in [6], the problem was studied from a conceptual point of view and a centralized fleet management system was presented. In [11], a new concept of control named open-control was proposed to merge centralized and decentralized control approaches. However, that paper just simply demonstrated how the open-control approach can help to deal with a perturbed environment, but it did not introduce a specific control algorithm. In contrast, in our research, our aim is to explore a specific instance of the fleet control problem, i.e., the dynamic routing of a fleet of cybercars and to develop a distributed control algorithm to solve this problem. But before working on the control solution, as a first step, in this paper we model the problem in such a way that model-based and optimization-based control methods (e.g., model predictive control) can be used.

This paper is organized as follows. In Section II, we present the general description of the dynamic routing problem of a fleet of cybercars and the assumptions we made for modeling the problem. In Section III, considering that the route of each cybercar in a fleet can be decided by itself and also can be assigned by a centralized routing controller, we model the dynamics and the energy consumption of each cybercar in a dedicated road network using a discrete-time modeling description. In Section IV, we consider the total time spent and total energy consumption by all the cybercars and formulate the dynamic routing problem of a fleet of cybercars in a model predictive control framework. In Section V, we present a simple case study of the modeling

and the model predictive route choice control of cybercars. Finally, in Section VI, we summarize the results of this paper and present some ideas for future work.

II. PROBLEM DESCRIPTION

We consider a cybernetic transportation network consisting of a set of dedicated (i.e., only open to cybercars) roads where vehicles are not allowed to turn around, and a set of dedicated intersections. Each road starts and ends at an intersection. Each cybercar can make its desired route decisions or receive route instructions from a centralized routing controller at every simulation time step, but only the route decision or instruction right before the cybercar crosses an intersection will be put into use. In this paper, we simply refer to a road as a 'link', an intersection as a 'node', and a 'cybercar run' (i.e., a cybercar running from its departure point to its destination) as a car. Each link is divided into a number of segments with length typically in the range of 50 to 100 m. In each link, the segments are labeled by consecutive integers starting from 1. At any time, the traffic density (i.e., the number of vehicles per kilometer) in a segment is assumed to determine the speeds of all the cybercars running in that segment. Moreover, each segment has its maximum capacity (i.e., the maximal allowed number of cars at the same time). More specifically, if the number of vehicles running in a segment reaches or exceeds the maximum capacity, that segment will be blocked and no vehicle is allowed to enter that segment. The energy consumption of a car is a function of its velocity and also the variation of its velocity (i.e., acceleration or deceleration). We assume that the departure point and destination of a car are always at a node, and all cybercars have the same length. We also assume that within a simulation time interval, no car can cover a distance longer than the length of the segment it is running in. Namely, we assume that $v_{\text{free},m,j} \cdot T < L_{m,j}$ holds for all m and j, where $v_{\text{free},m,j}$ denotes the free speed of segment m of link j, T denotes the simulation time interval (typically 1 second), and $L_{m,j}$ denotes the length of segment m of link j.

III. DISCRETE-TIME MODELING

A. Definitions

Let k be the discrete-time counter and i be the car index. Let $T_{\text{start},i}$ denote the time instant when car i is due to run from its departure point and $T_{\text{stop},i}$ denote the time instant when car i arrives at its destination. Let $l_i(k)$ and $s_i(k)$ be the link and the segment in which car i is running at time kT, respectively. Let $x_i(k)$ be the position (measured along the longitudinal axis of link) of car i in the link $l_i(k)$ at time kT, and $l_{\text{final},i}$ be the final link of car i, i.e., the end of $l_{\text{final},i}$ is the destination of car i. Let $r_i(k)$ be the route selected by car i at time kT, and $u_i(k)$ be the next link of car i after $l_i(k)$ following the current route $r_i(k)$.

Let $p_{m,j}^{\text{start}}$ and $p_{m,j}^{\text{end}}$ denote the positions of the starting point and the ending point of segment m of link j, respectively. Defining $N_{m,j}(k)$ as the number of vehicles running in segment m of link j at time kT and $L_{m,j}$ as the length of
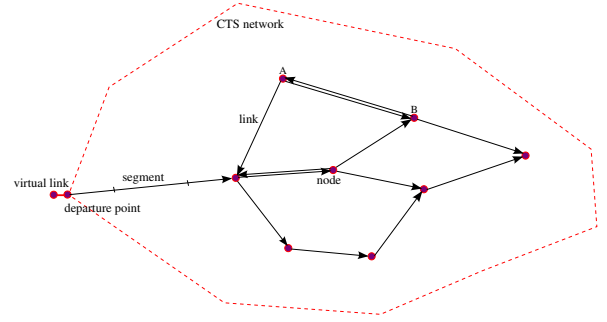

Fig. 1. Cybernetic transportation network

that segment, then the traffic density in segment m of link j at time kT is

$$\rho_{m,j}(k) = \frac{N_{m,j}(k)}{L_{m,j}} \quad (1)$$

Besides, let $C_{m,j}(k)$ denote the maximum capacity of segment m of link j. Let $b_{m,j}(k)$ define the blocking signal of segment m in link j at time kT. More specifically, $b_{m,j}(k) = 1$ means that the number of vehicles running in segment m of link j at time kT has reached or exceeded the maximum capacity and hence the segment is blocked, while $b_{m,j}(k) = 0$ means that segment is not blocked at time kT. Finally, let $\Delta_{m,j}$ denote the temporary variable that collects the change of the number of vehicles in segment m of link j during one simulation time interval.

Note that at the start of the simulation, for each car i, $T_{\text{stop},i}$ is initialized with a sufficient large number, which is used for (14) before $T_{\text{stop},i}$ can be determined. In addition, at the start of every simulation time interval, $\Delta_{m,j}$ for all m and j are set to be 0.

B. Network Set-Up

Considering the cybercars that are not able to enter the network due to a blocked departure link, we introduce a virtual link with zero length and infinite capacity to each departure point. Then the layout of the network can be represented by the following figure.

In the network, as shown in Figure 1, a link is represented by a directed line with the arrow indicating the heading direction, and a node is represented by a small solid circle.

C. Dynamics of a Single Vehicle

When $T_{\text{start},i}$ comes, car i will enter the network. At each simulation time step kT, with $x_i(k)$, $l_i(k)$, $s_i(k)$, $r_i(k)$ of car i and the information of the network $N_{m,j}(k)$, $\rho_{m,j}(k)$, $b_{m,j}(k)$ for all j and m given, the variables $x_i(k+1)$, $l_i(k+1)$, and $s_i(k+1)$ of car i need to be updated. As car i may go from one segment (or link) to a different segment (or link) during $[kT,(k+1)T]$, we also need to capture the change of the number of vehicles in the segments related to car i.

The update process consists of five main cases, which are described as

- "**same segment, same link**": car i runs in the same link and the same segment between time $t = kT$ and $t = (k+1)T$.

- "**different segments, same link**": car i runs in the same link but goes from the current segment $s_i(k)$ to the next one between $t = kT$ and $t = (k+1)T$.
- "**desired link blocked**": car i arrives at the end of its current link, but the first segment of its desired next link is blocked between $t = kT$ and $t = (k+1)T$.
- "**different links**": car i arrives at the end of its current link and the first segment of its desired next link is not blocked between $t = kT$ and $t = (k+1)T$. The car goes from its current link to its desired link.
- "**arrival**": car i arrives at its destination between $t = kT$ and $t = (k+1)T$.

For the sake of simplicity of notation, we assume $l_i(k) = j$ and $s_i(k) = m$, then present the conditions of all the five cases and describe how the dynamics of car i are updated in each of the cases.

For the case of **same segment, same link**, the following conditions must be satisfied:

$$T_{\text{start},i} < (k+1)T$$
$$x_i(k) + f_{m,j}\Big(\rho_{m,j}(k)\Big)T > p_{m,j}^{\text{start}}$$
$$x_i(k) + f_{m,j}\Big(\rho_{m,j}(k)\Big)T \leq p_{m,j}^{\text{end}}$$

where the function $f_{m,j}(\cdot)$ describes how the speed of cars in a segment depends on the traffic density in that segment. Since cybercars are automated vehicles, according to the fundamental flow-density curve for automated traffic presented in [12], one possible way to define $f_{m,j}(\cdot)$ based on the constant time headway policy is given by

$$f_{m,j}\Big(\rho_{m,j}(k)\Big) = \begin{cases} v_{\text{free},m,j}, & \text{if } \rho_{m,j}(k) \leq \rho_{\text{crit},m,j} \\ \frac{1}{h_{\text{con}}}\left(\frac{1}{\rho_{m,j}(k)} - L_{\text{veh}}\right), & \text{if } \rho_{m,j}(k) > \rho_{\text{crit},m,j} \end{cases} \tag{2}$$

where [12]

$$\rho_{\text{crit},m,j} = \frac{1}{h_{\text{con}}v_{\text{free},m,j} + L_{\text{veh}}} \tag{3}$$

is the critical traffic density of segment m of link j at which the maximal flow is obtained, h_{con} is the constant time headway of automated vehicles, and L_{veh} is the length of the vehicle. In this case, the dynamics of car i are updated by

$$x_i(k+1) \leftarrow x_i(k) + f_{m,j}\Big(\rho_{m,j}(k)\Big)T$$
$$l_i(k+1) \leftarrow l_i(k)$$
$$s_i(k+1) \leftarrow s_i(k)$$

Actually, in this case, only the position of car i is changed.

For the case of **different segments, same link**, the conditions are:

$$T_{\text{start},i} < (k+1)T$$
$$x_i(k) + f_{m,j}\Big(\rho_{m,j}(k)\Big)T > p_{m,j}^{\text{end}}$$
$$x_i(k) + f_{m,j}\Big(\rho_{m,j}(k)\Big)T \leq p_{m+1,j}^{\text{end}}$$
$$b_{m+1,j}(k) = 0$$

In this case, during $[kT, (k+1)T]$ car i first keeps speed $f_{m,j}\Big(\rho_{m,j}(k)\Big)$ until it arrives at the end of the current segment m. After that, it runs at $f_{m,j}\Big(\rho_{m+1,j}(k)\Big)$ in the next segment $m+1$ for the rest of the time. Then the dynamics of car i are updated by

$$x_i(k+1) \leftarrow p_{m,j}^{\text{end}} + f_{m,j}\Big(\rho_{m+1,j}(k)\Big)\left[T - \frac{p_{m,j}^{\text{end}} - x_i(k)}{f_{m,j}\Big(\rho_{m,j}(k)\Big)}\right]$$
$$l_i(k+1) \leftarrow l_i(k)$$
$$s_i(k+1) \leftarrow s_i(k) + 1$$

and the changes of the number of vehicles in segment m and segment $m+1$ caused by car i are captured by

$$\Delta_{m,j} \leftarrow \Delta_{m,j} - 1$$
$$\Delta_{m+1,j} \leftarrow \Delta_{m+1,j} + 1$$

Next, assuming the desired next link of car i at kT is $u_i(k) = j^*$, the conditions of **desired link blocked** are:

$$T_{\text{start},i} < (k+1)T$$
$$x_i(k) + f_{m,j}\Big(\rho_{m,j}(k)\Big)T > p_{m,j}^{\text{end}}$$
$$b_{1,j^*}(k) = 1$$

Since the desired next link j^* is blocked during $[kT, (k+1)T]$, after car i arrives at the end of the current link, it has to wait there. Then the dynamics at $(k+1)T$ are updated by

$$x_i(k+1) \leftarrow p_{m,j}^{\text{end}}$$
$$l_i(k+1) \leftarrow l_i(k)$$
$$s_i(k+1) \leftarrow s_i(k)$$

Then for the case of **different links**, also assuming $u_i(k) = j^*$, the conditions are:

$$T_{\text{start},i} < (k+1)T$$
$$x_i(k) + f_{m,j}\Big(\rho_{m,j}(k)\Big)T > p_{m,j}^{\text{end}}$$
$$b_{1,j^*}(k) = 0$$

In this case, car i is allowed to enter its desired next link, and its dynamics are updated by

$$x_i(k+1) \leftarrow f_{m,j}\Big(\rho_{1,j^*}(k)\Big)\left[T - \frac{p_{m,j}^{\text{end}} - x_i(k)}{f_{m,j}\Big(\rho_{m,j}(k)\Big)}\right]$$
$$l_i(k+1) \leftarrow u_i(k)$$
$$s_i(k+1) \leftarrow 1$$

Further, the changes of the number of vehicles in segment m of link j and segment 1 of link j^* caused by car i are captured by

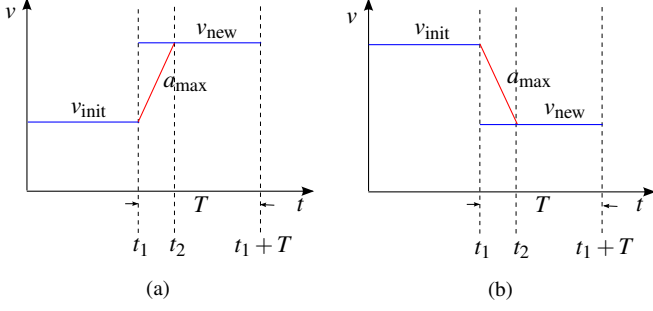$$\Delta_{m,j} \leftarrow \Delta_{m,j} - 1$$
$$\Delta_{1,j^*} \leftarrow \Delta_{1,j^*} + 1$$

Fig. 2. Two possible cases of the speed change of a car

Finally, for the case of **arrival**, the conditions are:

$$T_{\text{start},i} < (k+1)T$$

$$x_i(k) + f_{m,j}\big(\rho_{m,j}(k)\big)T > p_{m,j}^{\text{end}}$$

$$l_{\text{final},i} = j$$

In this case, car i stops running after arriving at its destination. Then $T_{\text{stop},i}$ is obtained by

$$T_{\text{stop},i} = kT + \frac{p_{m,j}^{\text{end}} - x_i(k)}{f_{m,j}\big(\rho_{m,j}(k)\big)} \qquad (4)$$

After that, car i leaves the network and $\Delta_{m,j}$ is updated by

$$\Delta_{m,j} \leftarrow \Delta_{m,j} - 1$$

D. Network Dynamics

After every simulation time step, with the updated information of all cars, the update of the states of the whole network, for all m and j, is given by

$$N_{m,j}(k+1) = N_{m,j}(k) + \Delta_{m,j}$$

$$\rho_{m,j}(k+1) = \frac{N_{m,j}(k+1)}{L_{m,j}}$$

$$b_{m,j}(k+1) = \mathbf{1}_{x \geq 0}\big(N_{m,j}(k+1) - C_{m,j}\big)$$

where $\mathbf{1}_{x \geq 0}(\cdot)$ is an indicator function defined by

$$\mathbf{1}_{x \geq 0}(a) = \begin{cases} 1, & \text{if } a \geq 0 \\ 0, & \text{if } a < 0 \end{cases} \qquad (5)$$

E. Energy Consumption of a Single Vehicle

The energy consumption of a electric vehicle consists of four consuming factors [13], which are:

- speeding up
- air drag
- rolling resistance
- energy losses in the energy-conversion chain

Starting from the speeds of the vehicle at respectively time $t = (k-1)T$ and $t = kT$, we consider all of the four factors described above and make the graph of Figure 2 to describe how the energy consumption of a car is calculated, where t_1 and t_2 are absolute time instants while T is the simulation time interval. We assume that $t_2 - t_1 < T$ always holds and the car has the same acceleration and deceleration rate. It

should be noted that different from Section III.*C* where the dynamics of all cars and the network are updated using constant speeds for all cars in every simulation interval, in this subsection, the acceleration and deceleration processes are approximated and then taken into account in the energy consumption calculation since energy consumption formulas for cars are acceleration dependent. In fact, we could also consider acceleration and deceleration in the update of the position of the cybercar (cf. Section III.*C*). But the relative effect of that will be smaller than for the energy consumption formulas and it also create a lot of extra complexity in the position update equations.

According to Figure 2, the car is accelerating or decelerating between t_1 and t_2. Let us call $[t_1, t_2]$ the acceleration-deceleration period. Then define $E_{\text{var\_kin}}$ as the kinetic energy change of the car due to the speed change from v_{init} to v_{new}, $E_{\text{var\_air}}$ as the energy consumption of the car needed to overcome the air drag during the acceleration-deceleration period, and also $E_{\text{var\_rol}}$ as the energy consumption of the car needed to overcome the rolling resistance during this period. Based on [13], $E_{\text{var\_kin}}$, $E_{\text{var\_air}}$ and $E_{\text{var\_rol}}$ are given by

$$E_{\text{var\_kin}} = \int_{t_1}^{t_2} M \frac{dv}{dt} v \, dt = M \int_{v_{\text{init}}}^{v_{\text{new}}} v \, dv$$
$$= \frac{1}{2} M(v_{\text{new}}^2 - v_{\text{init}}^2) \qquad (6)$$

$$E_{\text{var\_air}} = \int_{t_1}^{t_2} \frac{1}{2} \rho_{\text{air}} A_{\text{front}} v^3 \, dt$$
$$= \frac{1}{2} \rho_{\text{air}} A_{\text{front}} \int_{t_1}^{t_2} [v_{\text{init}} \pm a_{\text{max}}(t - t_1)]^3 \, dt$$
$$= \frac{\rho_{\text{air}} A_{\text{front}} \left| v_{\text{new}}^4 - v_{\text{init}}^4 \right|}{8 a_{\text{max}}} \qquad (7)$$

$$E_{\text{var\_rol}} = \int_{t_1}^{t_2} c_{\text{r}} M g v \, dt = c_{\text{r}} M g \int_{t_1}^{t_2} [v_{\text{init}} \pm a_{\text{max}}(t - t_1)] \, dt$$
$$= \frac{c_{\text{r}} M g \left| v_{\text{new}}^2 - v_{\text{init}}^2 \right|}{2 a_{\text{max}}} \qquad (8)$$

where M is the mass of the vehicle, a_{max} is the maximal acceleration and deceleration rate, ρ_{air} is the air density, A_{front} is the effective frontal area of the vehicle, g is the gravitational acceleration, and c_{r} is the rolling resistance coefficient. As a_{max} is assumed to be positive and fixed, for compactness, we use the combination expression $v_{\text{init}} \pm a_{\text{max}}(t - t_1)$ for the speed of a vehicle during acceleration-deceleration period in (7) and (8). For clarity, $v_{\text{init}} + a_{\text{max}}(t - t_1)$ denotes the speed of the vehicle when it is accelerating while $v_{\text{init}} - a_{\text{max}}(t - t_1)$ denotes the speed of the vehicle when it is decelerating.

After the acceleration-deceleration period, the car keeps the constant speed v_{new} for time length $T - (t_2 - t_1)$, with $t_2 - t_1 = \frac{|v_{\text{new}} - v_{\text{init}}|}{a_{\text{max}}}$. Let us call this the constant-speed period. Since the car keeps a constant speed during this period, its kinetic energy does not change. Now by defining $E_{\text{ct\_air}}$ and $E_{\text{ct\_rol}}$ as the energy consumption of the car needed to

overcome the air drag and the rolling resistance during the constant-speed period, respectively, we find

$$
\begin{aligned}
E_{\text{ct\_air}} &= \int_{t_2}^{T+t_1} \frac{1}{2}\rho_{\text{air}}A_{\text{front}}v_{\text{new}}^3 dt \\
&= \frac{1}{2}\rho_{\text{air}}A_{\text{front}}v_{\text{new}}^3(T+t_1-t_2) \\
&= \frac{1}{2}\rho_{\text{air}}A_{\text{front}}v_{\text{new}}^3\left(T - \frac{|v_{\text{new}}-v_{\text{init}}|}{a_{\max}}\right)
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
E_{\text{ct\_rol}} &= \int_{t_2}^{T+t_1} \frac{1}{2}c_{\text{r}}Mgv_{\text{new}} dt \\
&= \frac{1}{2}c_{\text{r}}Mgv_{\text{new}}\left(T - \frac{|v_{\text{new}}-v_{\text{init}}|}{a_{\max}}\right)
\end{aligned}
\tag{10}
$$

Besides, we define η_{motor} as the efficiency of electric motors[1]. Then, the actual energy consumption of the vehicle during the simulation time interval $[t_1, t_1+T]$ is

$$
E(v_{\text{init}}, v_{\text{new}}, a_{\max}, T) =
$$
$$
\begin{cases}
\dfrac{1}{\eta_{\text{motor}}}(E_{\text{var\_kin}} + E_{\text{var\_air}} + E_{\text{var\_rol}} + E_{\text{ct\_air}} + E_{\text{ct\_rol}}), \\
\qquad\qquad\qquad\qquad\qquad \text{if } v_{\text{new}} \geq v_{\text{init}} \\
\dfrac{1}{\eta_{\text{motor}}}(E_{\text{ct\_air}} + E_{\text{ct\_rol}}), \qquad \text{if } v_{\text{new}} < v_{\text{init}}
\end{cases}
\tag{11}
$$

Note that since a vehicle does not consume energy during the acceleration-deceleration period when it is decelerating, only the energy consumption of the vehicle during the constant-speed period is calculated in the second part of (11).

However, if a vehicle uses regenerative braking (i.e., using the vehicle's momentum to recharge the on-board batteries), it could save part of the kinetic energy lost in braking. We define γ_{recover} as the round-trip energy recovery coefficient[2] of the regenerative braking system. Then when regenerative braking is used, the second part of (11) becomes

$$
\begin{aligned}
E(v_{\text{init}}, v_{\text{new}}, a_{\max}, T) &= \frac{\gamma_{\text{recover}}}{\eta_{\text{motor}}}(E_{\text{var\_kin}} + E_{\text{var\_air}} + E_{\text{var\_rol}}) \\
&+ \frac{1}{\eta_{\text{motor}}}(E_{\text{ct\_air}} + E_{\text{ct\_rol}}), \text{ if } v_{\text{new}} < v_{\text{init}}
\end{aligned}
\tag{12}
$$

where $E_{\text{var\_kin}} + E_{\text{var\_air}} + E_{\text{var\_rol}}$ is negative and represents the part of the car's kinetic energy that can be used to recharge the on-board batteries.

Finally, by defining $E_i(k)$ as the energy car i consumes during $[kT, (k+1)T]$, we have

$$
E_i(k) = E\left(v_i(k-1), v_i(k), a_{\max}, T\right)
\tag{13}
$$

---

[1]Energy dissipates in the energy chain and the maximal efficiency of electric motors is about 85% to 90% [13]. That means for the best case, only 90% percent of the electricity used to charge the onboard batteries is available to power the vehicle.

[2]According to [14], the round-trip energy recovery coefficient (i.e., the ratio between the electric energy recovered from braking and the electric energy spent on accelerating) is around 38%.

IV. ROUTE CHOICE CONTROL PROBLEM FORMULATION

In our research, our aim is to design a control strategy that enables all cybercars to receive dynamic routes so that the total time spent (TTS) and total energy consumption (TEC) by all the cars are minimized.

Model predictive control (MPC) is widely recognized as a high-performance control approach that can be used to determine optimal control actions for complex and constrained systems [15], [16]. MPC determines these control actions by solving a constrained finite-horizon optimal control problem in a receding horizon fashion. As presented in the previous section, the dynamics of all cars and of the network are highly complex and subject to many constraints. Therefore, we adopt an MPC scheme to formulate the dynamic routing problem of a fleet of cybercars based on the discrete-time model given in the previous section.

Based on the model of Section III, the total time spent and total energy consumption of all cars in the network during the prediction period $[kT, (k+N_{\text{p}})T]$ are given by

$$
\begin{aligned}
J_{\text{TTS}}(k) = \sum_{i\in I(k,N_{\text{p}})} \min\Big(& (k+N_{\text{p}})T - T_{\text{start},i}, \; T_{\text{stop},i} - kT, \\
& T_{\text{stop},i} - T_{\text{start},i}, \; N_{\text{p}}T\Big) + J_{\text{TTS}}^{\text{end}}(k)
\end{aligned}
\tag{14}
$$

$$
J_{\text{TEC}}(k) = \sum_{n=0}^{N_{\text{p}}-1} \sum_{i\in I(k,N_{\text{p}})} E_i(k+n) + J_{\text{TEC}}^{\text{end}}(k)
\tag{15}
$$

with N_{p} the prediction horizon and $I(k, N_{\text{p}})$ the set of all cars that are in the network during $[kT, (k+N_{\text{p}})T]$, $J_{\text{TTS}}^{\text{end}}(k)$ and $J_{\text{TEC}}^{\text{end}}(k)$ are measures of the expected remaining total time spent and expected remaining total energy consumption from their positions at that time to their destinations for all the cars that are still in the network at $t = (k+N_{\text{p}})T$. One way to compute $J_{\text{TTS}}^{\text{end}}(k)$ and $J_{\text{TEC}}^{\text{end}}(k)$ is using the speeds of the cars at $t = (k+N_{\text{p}})T$ and shortest time routes computed by *Dijkstra's algorithm* [17] based on those speeds.

Considering $J_{\text{TTS}}(k)$ and $J_{\text{TEC}}(k)$ have different orders of magnitude and different units, in order to properly balance them in optimization, we design the following objective function

$$
J(k) = w_1 \frac{J_{\text{TTS}}(k)}{J_{\text{TTS,typical}}} + w_2 \frac{J_{\text{TEC}}(k)}{J_{\text{TEC,typical}}}
\tag{16}
$$

where $J_{\text{TTS,typical}}$ and $J_{\text{TEC,typical}}$ are the "typical" values[3] of the total time spent and total energy consumption of all cars in one prediction period $N_{\text{p}}T$, and $w_1, w_2 > 0$ are weights.

At MPC step k, the decision variables for all car i with $i \in I(k, N_{\text{p}})$ are routes to be taken leading to the final destinations (i.e., at step k, each car i has to select one route from a finite discrete set of possible routes $R_i(k)$). Note that once a route $r_i(k) \in R_i(k)$ has been selected by car i, the link choice

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[3]These values are e.g., the values of the total time spent and total energy consumption of all cars in one prediction period in a numerical simulation where the routes of all cars are fixed or a simple route control strategy (e.g., fastest route) is used.
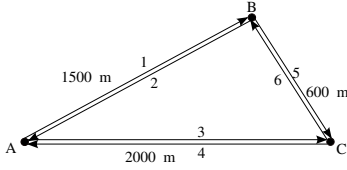
Fig. 3.   Road network

TABLE I

TRANSPORTATION DEMAND

Origin	Destination	Departure times
A	B	0.2s, 1.4s, 2.6s, 3.8s, 5.0s, 6.2s, 7.4s, 8.2s, 8.6s, 9.0s, 9.4s
A	C	0.4s, 0.6s, 1.0s, 1.2s, 1.6s, 1.8s, 2.2s, 2.4s, 2.8s, 3.0s, 3.4s, 3.6s, 4.0s, 4.2s, 4.6s, 4.8s, 5.2s, 5.4s, 5.8s, 6.0s, 6.4s, 6.6s, 7.0s, 7.2s, 7.6s, 7.8s
B	C	0.8s, 2.0s, 3.2s, 4.4s, 5.6s, 6.8s, 8.0s, 8.4s, 8.8s, 9.2s, 9.6s

sequence $u_i(k)$ can be determined and used as input for the model of Section III.

Finally, together with the nonlinearity of the dynamics of all cars and the objective function, the discrete nature of the decision variables of all cars leads to a *mixed integer nonlinear programming* (MINLP) problem, for which several algorithms are available, such as genetic algorithm, branch and bound, simulated annealing, etc [18]. In general however this problem is computationally very hard to solve, in particular if the number of cybercars is large. Hence, a major challenge is to find efficient approximated solution methods, possibly based on a distributed control approach. This will be a topic for future research.

V. CASE STUDY

The modeling and the dynamic routing formulation of a fleet of cybercars developed in the previous sections are applied to the case study network depicted in Figure 3. The network contains three origins and destinations connected by three bidirectional links. The link lengths are indicated in the figure. All segments in all links have a length of 50 m. Table I lists the demands.

Taking $T = 1$ s, $v_{\text{free},m,j} = 60$ km/h, $h_{\text{con}} = 0.3$ s, $L_{\text{veh}} = 3.2$ m, $a_{\max} = 2.45$ m/s^2, $M = 1000$ kg, $\rho_{\text{air}} = 1.3$ kg/m^3, $A_{\text{front}} = 0.8$ m^2, $g = 9.8$ m/s^2, $c_r = 0.01$, $\eta_{\text{motor}} = 0.85$, $\gamma_{\text{recover}} = 0.38$, $N_p = 15$, $w_1 = 0.6$ and $w_2 = 0.4$, we compare the performance of MPC route choice control and the shortest routes strategy. The result are listed in Table II.

Note that with optimized routes assigned by MPC controller, the fleet of cybercars successfully avoids congestion

TABLE II

COMPARISON: SHORTEST ROUTE AND MPC ROUTE CHOICE CONTROL

Method	TTS	TEC
Shortest Route	6258.0 s	15.20 kWh
MPC Route Choice Control	4517.6 s	7.64 kWh

in the network, so that they save time (i.e., travel at higher speeds) and save energy (i.e., less accelerating and braking).

VI. CONCLUSION

Considering cybercars as moving decision-making agents that can decide their desired route choices or receive route instructions from a higher-level controller, we have presented a discrete-time model of the dynamics and energy consumption of a fleet of cybercars running in a dedicated road network. We have considered the total time spent and total energy consumption by all the cybercars in the network and have derived how these performance measures can be affected by the route choices. Finally, with the proposed model, we have formulated the dynamic routing control problem according to a model predictive control scheme.

In our future work, we will seek to work out efficient distributed model predictive control solutions for the dynamic routing problem of a fleet of cybercars.

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