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Customer-Oriented Optimal Vehicle Assignment in Mobility-on-Demand Systems

Francesco Acquaviva, Alfredo Núñez, Donato Di Paola, Alessandro Rizzo and Bart De Schutter

Abstract— **In this paper, we introduce a novel optimization framework for a station-to-door mobility-on-demand system that aims at ensuring an efficient transportation service for the daily mobility of passengers in densely populated urban areas. We propose a mixed integer linear programming approach that maximizes both the customers' satisfaction and the provider's revenues, keeping at the same time the number of vehicles in each station within given bounds towards the improvement of system balancing. The proposed customer-oriented approach aims at meeting as many customer requests as possible, while maximizing the provider revenues and reducing the customers' impatience, thus increasing their satisfaction. This implies, in turn, a better reputation for the service provider. The performance of the proposed approach is assessed through an extensive Monte Carlo simulation campaign. In particular, through the analysis of different performance indices, we compare the optimal solution of the proposed approach with the optimal solution achieved by a previously presented approach based on the Profitable Tour Problem.**

I. Introduction

Nowadays, mobility in urban areas is a topic of great importance for any modern city. The capacity of transportation infrastructures is very often insufficient; thus, the growing use of privately-owned cars heavily contributes to increase traffic congestion levels. Consequences are not only economic, yet they also have an impact on the environment, since congestion leads to higher levels of pollution, and on the society, in terms of lower quality of life. Beyond the classic public transportation services, an opportunity to facilitate people's mobility comes from more flexible solutions like car-sharing [1], [2], shared mobility systems [3], [4], and Mobility-On-Demand (MOD) systems [5], [6]. All these solutions, also known as flexible transportation systems, are dedicated to users that are willing to pay more than the cost of public services, but expect to have a service closer to the one obtained when using a private vehicle or a taxi. Flexible transportation systems can work in synergy with the public transportation service to reduce, for example, the pressure over a highly occupied bus line or congested subway line, enriching the user experience by providing a nearly door-to-door service. In this context, MOD systems aim at providing citizens with an affordable service offering the same benefits of flexible transportation systems through the use of a fleet of shared vehicles. Crucial aspects in the management of MOD systems are optimal vehicle assignment [1], fleet balancing [6], [7], and maintenance of low operational costs and good level of service [8]. Fleet balancing, vehicle assignment, and user behavior have been usually tackled separately in recent research efforts. The objective of balancing [5] is to achieve a uniform distribution of vehicles throughout the stations, to avoid the presence of empty pick-up stations and/or full drop-off ones. Balancing can be performed manually by operators [6], or by self-driving vehicles [9]. Another balancing technique consists of adopting pricing strategies [5], [7], promoting drop-offs to stations with high demand that are still reasonably close to the customer's destination. Beyond balancing, pricing strategies can be designed to reduce traffic congestion by encouraging shared mobility with other users [10]. Strategies for optimization of the manual relocation of vehicles have been designed with the objective of optimizing several performance indices [1], [11], [12].

The design of novel mobility solutions based on the optimization of both the provider's profits and the user's satisfaction is a hot research topic at present [8], [13], [14]. On the one hand, the system should generate income that allows its sustainability, covering all costs and generating revenues. On the other hand, a high-quality service should be assured, to avoid users to abandon the shared service in favor of the private one [14], [15]. In the literature, most of the studies on shared mobility services focus on the optimization of operational costs only, without including the customer satisfaction in the decision making process [13], [14]. Nowadays, the extremely fast pace of modern cities makes quality of service and customer satisfaction crucial issues in the design of urban transportation systems. Flexible approaches should aim at satisfying as many requests as possible, while reducing customers' waiting time through a balanced distribution of vehicles over the stations, and generating revenue for the provider.

In this paper, an optimal vehicle assignment strategy for a station-to-door MOD system is proposed. The approach is inspired to [8], where the system has been modeled leveraging the Profitable Tour Problem [16] to design an optimization strategy for a station-to-station MOD system. The aim of the strategy defined in [8] is to optimize costs and revenues for both customers and the provider, while trying to keep the system balanced. Here, we extend the

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approach in [8] by modeling a station-to-door MOD system and defining an objective function that explicitly accounts for the customer's impatience, which is posited to be related to the waiting time at the departure station. This characteristic makes the proposed model representative of the real way customers experience urban traveling [17], [18], [19]. Similarly to [8], the proposed approach aims at optimizing the system performance through trading-off between the satisfaction of the customer requests with a good quality of service, the optimization of the provider's revenues, and the vehicle balancing throughout the stations of the system. In the following, we will refer to the approach presented in this paper as Customer-oriented Vehicle Assignment (CVA), whereas the approach presented in [8] will be referred to as Profitable Vehicle Assignment (PVA).

The paper is organized as follows. In Section II, the description of the system is given. Section III presents the mathematical formulation of the optimization problem. In Section IV, numerical results are discussed and compared with those obtained by PVA. Finally, our conclusions and an outline of possible future work are proposed.

II. System Description

In this section, we describe the main concepts underlying the model of the proposed MOD system. We consider a given urban area where S stations, with index set $\mathcal{S} = \{1, \ldots, S\}$, are located, and V vehicles, with index set $\mathcal{V} = \{1, \ldots, V\}$, are present. Each station $i \in \mathcal{S}$ is equipped with a maximum number of parking lots $V_i^{\max} \in \mathbb{N}$. At each discrete-time step $k \in \mathbb{N}$, a number $V_i(k)$ of vehicles is parked at each station i. Finally, we consider a number of C waiting customers, with index set $\mathcal{C} = \{1, \ldots, C\}$, with whom C destinations and C pick-up stations are univocally associated. We denote with d_c the destination of each customer. Since we are modeling a station-to-door problem, the position of d_c does not generally coincide with the location of a drop-off station. Furthermore, we define $i_c \in \mathcal{S}$ as the pick-up station where customer $c \in \mathcal{C}$ is waiting for the service, and $i_v \in \mathcal{S}$ as the station where vehicle v is parked.

We define $\mathcal{V}_{i_c} \subseteq \mathcal{V}$ as the set of vehicles parked at station i_c where customer c is waiting for service, while $\mathcal{C}_{i_v}^{\text{veh}} \subseteq \mathcal{C}$ represents the set of customers waiting at station i_v where vehicle v is parked. We also define $\mathcal{C}_i^{\text{sta}} \subseteq \mathcal{C}$ as the set of C_i customers that are waiting at station $i \in \mathcal{S}$. Given the aforementioned definitions, we define the customer's request assignment problem as follows.

***Problem** 1 (Customer-to-Vehicle Assignment):* Each customer $c \in \mathcal{C}$, traveling from pick-up station i_c where he/she is located, who has to reach his/her destination d_c through a commuting station $j \in \mathcal{S}$ where he/she would drop-off the vehicle, has to be univocally assigned to one and only one vehicle $v \in \mathcal{V}_{i_c}$.

We assume that each customer $c \in \mathcal{C}$ is characterized by a different attitude to accept the assignment proposed by the service provider, waiting up to a certain limit time to obtain service, before giving up and switching to another transportation mode. To capture this aspect, we define the customer impatience function I_{cj} as the amount of money that a customer, who is not happy for the provided service, expects to receive as compensation for the unsatisfactory service quality. In other words, the customer impatience function models the difference between the actual rental price and the rental price that the customer wishes to pay due to a detriment in the service quality. The customer impatience is modeled as a piecewise-linear function of the service time t_{cj}^{ser}, which is the time that elapses between the arrival of customer c at his/her pick-up station i_c and the instant at which he/she arrives at his/her desired destination d_c after having dropped the vehicle off at the commuting station j. In particular, we define $t_{cj}^{\text{ser}} = t_{cj}^{\text{wait}} + t_{cj}^{\text{travel}} + t_{cj}^{\text{walk}}$, where t_{cj}^{wait} is the waiting time for customer c to obtain the service and start his/her ride, t_{cj}^{travel} is the time needed for customer c to travel from his/her pick-up station i_c to the commuting station j, and t_{cj}^{walk} is the time spent by customer c to walk from the commuting station j to which he/she is routed to his/her desired destination d_c. An example of such a function is illustrated in Fig. 1.

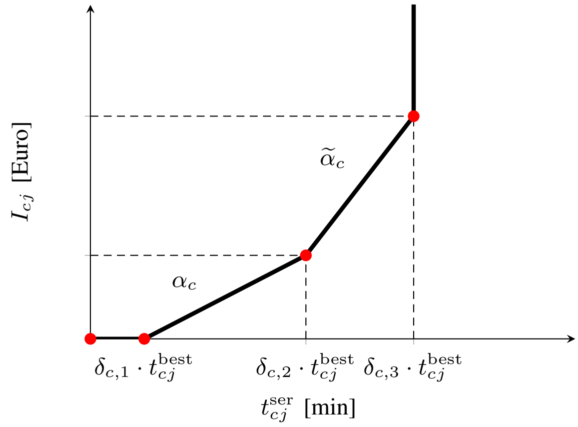


Fig. 1: Customer's Impatience Function

The piecewise-linear function modeling the customer impatience is described by:

$$I_{cj} = \begin{cases} 0 & t_{cj}^{\text{ser}} < \delta_{c,1} \cdot t_{cj}^{\text{best}} \\ \alpha_c \cdot (t_{cj}^{\text{ser}} - \delta_{c,1} \cdot t_{cj}^{\text{best}}) & \delta_{c,1} \cdot t_{cj}^{\text{best}} \leq t_{cj}^{\text{ser}} < \delta_{c,2} \cdot t_{cj}^{\text{best}} \\ \widetilde{\alpha}_c \cdot (t_{cj}^{\text{ser}} - \delta_{c,2} \cdot t_{cj}^{\text{best}}) + \Gamma_{cj} & \delta_{c,2} \cdot t_{cj}^{\text{best}} \leq t_{cj}^{\text{ser}} < \delta_{c,3} \cdot t_{cj}^{\text{best}} \\ \infty & t_{cj}^{\text{ser}} \geq \delta_{c,3} \cdot t_{cj}^{\text{best}} \end{cases}$$

where $\Gamma_{cj} = \alpha_c \cdot (t_{cj}^{\text{ser}} - \delta_{c,1} \cdot t_{cj}^{\text{best}})$. Here and henceforth, the dependence of $I_{cj}(t_{cj}^{\text{ser}})$ on the service time t_{cj}^{ser} has been dropped to enhance readability.

The parameters of the piecewise-linear function model the subjective attitude of the customer to assess the value of time spent while waiting, according to his/her schedule and needs. In particular, the customer impatience rates α_c and $\widetilde{\alpha}_c$ reflect how different customers evaluate the quality of service and are modeled as the slopes of the piecewise-linear function, while the customer impatience turning points $\delta_{c,\bullet} \cdot t_{cj}^{\text{best}}$ are the time instants at which a customer modifies his/her behavior, modeled as changes in the slope of the impatience function, as depicted in Fig. 1. Customer impatience turning points are defined as fractions of the estimated shortest travel time t_{cj}^{best} for customer c to travel from his/her pick-up station i_c to his/her destination d_c. These fractions are expressed by a set of customer impatience breakpoints $\delta_{c,\bullet}$, such that the turning points are modeled as $\delta_{c,\bullet} \cdot t_{cj}^{\text{best}}$. In the proposed approach, two different customer impatience rates, $\alpha_c, \widetilde{\alpha}_c$, and three customer impatience breakpoints $\delta_{c,1}, \delta_{c,2}, \delta_{c,3}$, are considered. We observe that, if necessary, a different number of breakpoints and slopes can be used to shape the impatience function towards a more detailed characterization of customers' behavior. Given the definition of customer impatience and considering Problem 1, the following assumption holds.

***Assumption** 1 (Customer's Request):* We assume that each customer $c \in \mathcal{C}$, starting from the pick-up station i_c, is associated with a unique *request*, which is defined by the tuple $(i_c, d_c, \delta_{c,1}, \delta_{c,2}, \delta_{c,3}, \alpha_c, \widetilde{\alpha}_c)$, where $\delta_{c,1}, \delta_{c,2}, \delta_{c,3}, \alpha_c, \widetilde{\alpha}_c$ model the customer impatience, while the other parameters are related to the customer request.

Finally, we define the cost $J_{cvj} = \beta_{\text{veh},v} \cdot t_{cj}^{\text{travel}}$ as the price that customer c has to pay to rent vehicle v to drive from his/her pick-up station i_c to the assigned commuting station j. The cost J_{cvj} is a function of the travel time t_{cj}^{travel} (the time customer c spent to drive from station i_c to the station j) and the factor $\beta_{\text{veh},v} \in \mathbb{N}$, that is the rental rate for the vehicle v.

III. Mathematical Formulation

In this section, we introduce the framework adopted to simultaneously optimize the provider's cost and the customers' satisfaction, henceforth called Customer-oriented Vehicle Assignment (CVA). The devised mathematical formulation accounts both for the customer impatience I_{cj} and the customer trip cost J_{cvj}, and tends to maintain the number of vehicles parked in stations within given bounds to fulfill forthcoming requests. The main optimization problem studied in this paper is now defined.

***Problem** 2 (Vehicle Assignment with Balancing):* Given a set of C customers directed towards their desired destinations, determine the best assignment of V vehicles, parked in $\mathcal{S}$, maximizing both the provider's income and the customers' satisfaction in terms of minimization of the impatience function, serving as many requests as possible in a given time window of T discrete-time steps.

To formulate Problem 2, we define the binary decision variables x_{cvj}, which take the value 1 if vehicle v is assigned to customer c to drive from pick-up station i_c to commuting station j, with the objective of reaching the desired destination d_c, and 0 otherwise. Hence, Problem 2 is formulated as the following mixed integer linear programming problem:

$$\max_{x_{cvj}} \sum_{c\in\mathcal{C}} \sum_{v\in\mathcal{V}_{i_c}} \sum_{j\in\mathcal{S}} (J_{cvj} - I_{cj}) \cdot x_{cvj} \tag{1}$$

subject to:

$$\sum_{j\in\mathcal{S}} x_{cvj} \leq 1 \quad \forall c \in \mathcal{C}, \forall v \in \mathcal{V}_{i_c}; \tag{2}$$

$$\sum_{c\in\mathcal{C}_{i_v}^{\text{veh}}} \sum_{j\in\mathcal{S}} x_{cvj} \leq 1 \quad \forall v \in \mathcal{V}; \tag{3}$$

$$\sum_{v\in\mathcal{V}_{i_c}} \sum_{j\in\mathcal{S}} x_{cvj} \leq 1 \quad \forall c \in \mathcal{C}; \tag{4}$$

$$\sum_{c\in\mathcal{C}_i^{\text{sta}}} \sum_{v\in\mathcal{V}_{i_c}} \sum_{j\in\mathcal{S}} x_{cvj} \leq \min\{C_i, V_i\} \quad \forall i \in \mathcal{S} : C_i > 0; \tag{5}$$

$$\sum_{c\in\mathcal{C}_i^{\text{sta}}} \sum_{v\in\mathcal{V}_{i_c}} \sum_{j\in\mathcal{S}} x_{cvj} - \sum_{c\in\mathcal{C}} \sum_{v\in\mathcal{V}_{i_c}} x_{cvi} \leq V_i - V_i^{\min} \quad \forall i \in \mathcal{S}; \tag{6}$$

$$\sum_{c\in\mathcal{C}} \sum_{v\in\mathcal{V}_{i_c}} x_{cvi} - \sum_{c\in\mathcal{C}_i^{\text{sta}}} \sum_{v\in\mathcal{V}_{i_c}} \sum_{j\in\mathcal{S}} x_{cvj} \leq V_i^{\max} - V_i \quad \forall i \in \mathcal{S}; \tag{7}$$

$$x_{cvj} \in \{0, 1\} \quad \forall (c, v, j) \in \mathcal{C} \times \mathcal{V}_{i_c} \times \mathcal{S}.$$

Customer-to-vehicle and vehicle-to-route assignment constraints are expressed in Eqs. (2)-(5). In constraint (2), we consider all the possible pairings of each customer c with vehicles in $\mathcal{V}_{i_c}$ parked in the pick-up station $i_c \in \mathcal{S}$ where he/she is located. Thus, constraint (2) states that for each possible customer-vehicle pairing, at most one route starting from the selected pick-up station i_c and directed towards one of the possible commuting stations j can be assigned. Constraint (3) states that each vehicle v can be driven at most by one of the customers who are waiting for service at station i_v, where vehicle v is parked. Constraint (4) states that each customer c can drive at most one of the vehicles that are parked in station i_c, where customer c is located. Constraint (5) considers all the possible routes that start from each pick-up station i, where at least one customer request has been made, and states that the sum of the possible routes starting from station i should be at most equal to the number of the possible matches between the C_i waiting customers and the V_i vehicles parked at station i, at the time instant at which the assignment is performed. Constraints (6)-(7) represent station capacity restrictions, where $V_i^{\max}$ and $V_i^{\min}$ are, respectively, the maximum number of vehicles allowed and the minimum number of vehicles needed in station i at each discrete-time step k.

When no requests are made at a given station $i \in \mathcal{S}$, $C_i = 0$ is set. As a consequence, both sets $\mathcal{C}_i^{\text{sta}}$ and $\mathcal{C}_{i_v}^{\text{veh}}$ are empty. Thus, for all vehicles in $\mathcal{V}_i$, all the decision variables x_{cvj} are set to 0 for all customers c and commuting stations j. Furthermore, all the decision variables x_{cvj} related to vehicles $v \notin \mathcal{V}_{i_c}$, $\forall c \in \mathcal{C}$, that are vehicles parked in stations different from the one where customer c is located, are set to 0. Moreover, we observe that when the number of parked vehicles V_i is larger than the number of waiting customers C_i, the number of corresponding decision variables x_{cvj} can be reduced without loss of optimality. To this aim, we assign the waiting customers to the first C_i most expensive vehicles, based on their vehicle rental rate factor $\beta_{\text{veh},v}$. Thus, the decision variables x_{cvj}, corresponding to the remaining $V_i - C_i$ vehicles, are set to 0.

The proposed problem is based on the Profitable Vehicles Assignment (PVA) [8], which is a generalization of the Traveling Salesman Problem. The latter is a well-known NP-hard problem. Indeed, the proposed CVA problem considers additional aspects of the customer model, increasing the complexity of the formulation, also leading to an NP-hard problem.

IV. Numerical Results

A. Setup of the Case Study

In this section, the performance of the CVA approach is assessed through the analysis of an extensive Monte Carlo simulation campaign. We compare the optimal solution of CVA with the one obtained solving the Profitable Vehicles Assignment (PVA) problem. The analysis is performed using a simulation framework implemented in MATLAB, where the solution of the mixed integer linear programming problem (1)-(7) is performed using the `ANSI-C` MATLAB MEX Interface for the `GLPK` library. Simulations have been run on a workstation with a 2.5 GHz quad-core processor with 16 GB of RAM. In this simulation campaign, the following assumptions have been made. We consider two scenarios with 4- and 6-stations, respectively. Each station is randomly located in a simulated square environment of 3×3 km.

For each scenario analyzed, we vary the minimum and maximum number of both customers and vehicles, as well as their distribution over the stations. These features have been selected according to the outcomes of a statistical analysis conducted on the NYC CitiBike service data [20]. The analysis on the NYC CitiBike data has been performed taking into account the data of the most used stations in the period from July 2013 to December 2014. The following data have been analyzed: trip duration, station ID and position, number of available parking lots and vehicles parked, customer type and station position. From the available data, we have determined for each station: the number of customers that are waiting for service at each time step, their type, the number of vehicles parked and the average time taken to travel between two stations placed at a given distance. Furthermore, based on the data gathered, each customer c is associated with a commuting station $j_c^\star$, that is the closest to the customer destination d_c, and a best travel time t_{cj}^{best}, that is, the shortest time spent to travel from his/her pick-up station to his/her actual drop-off station. In particular, a number of customers that range from 17 to 52, and of vehicles that range from 4 to 15 are considered in each station for the 4-stations scenario, while in the 6-stations scenario 15 to 80 customers are considered to wait at each station, which contains a number of vehicles ranging from 6 to 23. In each trial, the numbers of customers and vehicles are increased in steps of 5 and 1 units, respectively. The simulator initially places the vehicles over the stations in order to meet the balancing constraints (6)-(7). In this simulation campaign, we consider a single class of vehicles and two classes of customers, namely, subscribers and non-subscribers. We define a realistic pricing scheme, with a different rental rate factor $\beta_{\text{veh},v}$ for subscribers and non-subscribers. In particular, we set the rental rate factor for non-subscribers to the Car2go price of 0.29 Euro/min [21]. Inspired by the Citibike [20] policy that reduces the cost for subscribers of about 50%, we set the subscriber rental rate factor to 0.15 Euro/min.

The parameters of the impatience function have been modeled as follows. Customers' impatience rates α_c have been uniformly set to 1 for all customers, while the rates $\widetilde{\alpha}_c$ have been set as independent and identically distributed realizations of a random variable with uniform distribution $\mathcal{U}(0.01, 1.0)$. Furthermore, the customers' impatience breakpoints, $\delta_{c,1}, \delta_{c,2}, \delta_{c,3}$ have been drawn as independent and identically distributed realizations of random variables with uniform distributions $\mathcal{U}(1, 20)$, $\mathcal{U}(\delta_{c,1}, \delta_{c,1}+50)$, $\mathcal{U}(\delta_{c,2}, \delta_{c,2}+10)$, respectively. Note that the values of $\alpha_c, \widetilde{\alpha}_c$ are expressed in Euro/min, while the customers' impatience breakpoints $\delta_{c,\bullet}$, are expressed in minutes.

The results achieved show that a feasible assignment can be obtained within a reasonable computation time for small instances of the problem. In particular, we obtained that the CVA approach is able to compute the solution of a single instance of Problem 2 in an average computation time of 0.05 seconds with a maximum of 0.07 seconds rather than the 27.63 and 33.2 seconds of PVA, when tested on the worst-case scenario, *i.e.*, a 6-stations scenario with 23 vehicles parked and 80 waiting customers.

B. Assessment Criteria

We define a set of metrics to analyze and compare the performance of the proposed approach. The metrics defined do not depend on the particular approach, nor on a specific score function. Thus, they can be used to assess global performance in MOD-related problems. Inspired by the analysis in [7], we define the *mean balancing error* (MBE) as follows:

$$\text{MBE} = \frac{1}{T}\frac{1}{S}\sum_{k=1}^{T}\sum_{i\in\mathcal{S}}\left|\text{V}_i(k) - \frac{V}{S}\right|.$$

The MBE can be considered as a metric of the balancing status of the network: if vehicles are uniformly distributed over all stations, MBE is equal to 0. On the other hand, the greater MBE is, the more unbalanced the system is. We consider the *revenue of the provider* (REV), which quantifies the total incomes earned by the MOD service provider as

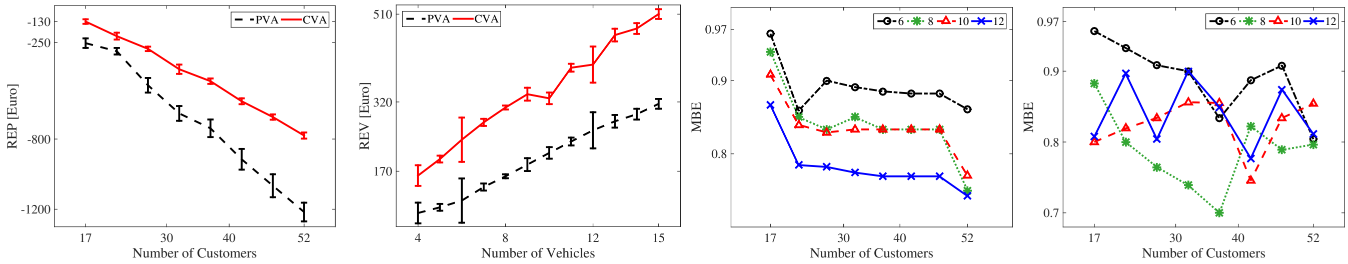
$$\text{REV} = \sum_{k=1}^{T}\sum_{c\in\mathcal{C}}\sum_{v\in\mathcal{V}_{i_c}} R_c(k)\cdot J_{cvj}(k),$$

where $R_c(k)$ is 1 if the request made by customer c is fulfilled, and 0 otherwise. We recall that J_{cvj} represents the cost, in terms of money spent by customer c to reach his/her destination d_c through the commuting station j. Note that, by assumption, a constant number of customers $c \in \mathcal{C}(k)$ are waiting for service. Thus, at each time step k, a number of waiting customers equal to the number of customers served at the time step $k-1$, is added to the system. Here and henceforth, to enhance readability, we will refer to the set of customers $\mathcal{C}(k)$ as $\mathcal{C}$, and to the set $\mathcal{V}_{i_c}(k)$ of the vehicles parked at station i_c as $\mathcal{V}_{i_c}$.

We define the *reputation of the service* (REP), as the total revenue missed by the provider because of unserved customers. This is computed considering, for each missed trip, the revenue that the provider would have obtained by directing the unserved customer to the commuting station $j_c^\star$ closest to his/her destination,

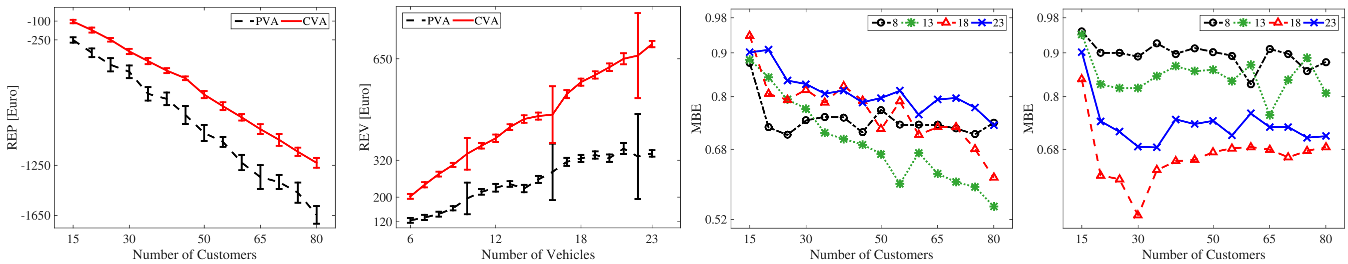
$$\text{REP} = \sum_{k=1}^{T}\sum_{c\in\mathcal{C}}(R_c(k)-1)\cdot J_{cvj_c^\star}(k).$$

We remark that in the REP definition, the commuting station $j_c^\star$ is the one closest to the customer destination, while in



(a) (b) (c) (d)

Fig. 2: Simulation results for the 4-stations scenario: (a) REP versus Number of Customers with 11 vehicles; (b) REV versus Number of Vehicles with 42 customers; MBE metric for different sets of Vehicles versus Number of Customers for the PVA approach (c), and for the CVA approach (d).



(a) (b) (c) (d)

Fig. 3: Simulation results for the 6-stations scenario: (a) REP versus Number of Customers with 11 vehicles; (b) REV versus Number of Vehicles with 40 customers; MBE metric for different sets of Vehicles versus Number of Customers for the PVA approach (c), and for the CVA approach (d).

the REV formula it is the one to which customer c has been routed by the provider, *i.e.*, station j.

Remark 1: Note that the proposed metrics may be combined as extra performance indices to be optimized, in combination with the CVA score function, along the lines of the approach described in [22].

C. Performance assessment and comparison

In this section, the outcomes of the Monte Carlo simulation campaign are presented and discussed, and the performance of CVA is compared with that of PVA. For each simulation instance, 10 independent trials are realized, randomizing customers' impatience and requests, and varying the distribution over the network of both customers and vehicles. Averaged results are shown in Figs. (2)-(3), where error bars indicate the 95% confidence interval. From the analysis of CitiBike data [20], the sampling time has been set to the maximum trip time between any two stations, that is, 10 min. Under the simplifying assumption of synchronous operations, this choice guarantees that at each time step all the vehicles are parked in the stations before being reassigned to new customers. We observe that in a more realistic framework, where travel times have a greater variability, the choice of the sampling time should be refined, operations should be considered as asynchronous, and a technique to account for vehicles that are traveling while others are being assigned should be devised. Results have been obtained over an observation time window of two hours, thus, $T = 12$ in our simulation campaign. It is worth noting that, in a real urban environment, different traffic conditions may lead to customer travel times greater than the ones mentioned before. Figures 2 and 3 offer a comparison between the performance obtained using the CVA- and PVA-based approaches. Figure 2 deals with the 4-stations scenario, while Fig. 3 illustrates the 6-stations case. In particular, Figs. 2(a) and 3(a) display the values of REP, as a function of the number of customers with 11 vehicles available. In Figs. 2(b) and 3(b), the values of REV as a function of the number of vehicles are illustrated. From the analysis of CitiBike data, a number of 40 and 42 customers is considered for these simulations in the 4- and 6-stations scenarios, respectively. As shown in Figs. 2(a)-3(a), CVA offers a better performance than PVA in satisfying customers' requests, in terms of a higher REP. Moreover, the gap in performance between the two strategies increases when the number of customers in the system increases. However, PVA achieves better performance than CVA in terms of fulfilled requests, as summarized in Table I.

The performance is also assessed in terms of the provider's revenue, REV, whose trend is shown in Figs. 2(b)-3(b) as a function of the number of vehicles. We observe that the revenue achieved with CVA is always higher than that achieved with PVA, and that the gap increases when the number of vehicles in the system increases. Our results confirm that our approach, while aiming at optimizing customers' satisfaction, also provides higher revenues to the provider.

Finally, we analyze the CVA performance in terms of system balancing. We recall that constraints (6)-(7) have been designed to maintain the system balanced keeping the number of parked vehicles in each station within a certain bound, that is, between $V_i^{\min} = 1$ and $V_i^{\max}$. Figs. 2(c)-(d) and Figs. 3(c)-(d) display the trend of the MBE metric for the 4- and 6-stations scenarios, respectively. In particular, the values of MBE versus the number of customers, for different numbers of vehicles, are displayed in Figs. 2(c) and 3(c) for PVA while corresponding results obtained using CVA are illustrated in Figs. 2(d) and 3(d). We observe that CVA tries to favor the fulfillment of customers' requests, whereas PVA tends towards a more balanced satisfaction of both the provider's and the customers' needs. Due to their

	One Vehicle per Station				Max Num Vehicles			
	4 stations		6 stations		4 stations		6 stations	
	CVA	PVA	CVA	PVA	CVA	PVA	CVA	PVA
Min N_C	84.6%	81.3%	87%	90.6%	96.7%	99%	99.9%	99.2%
Max N_C	42.2%	57%	44.9%	45%	80%	80%	80%	80.4%

TABLE I: Percentage of fulfilled requests considering the minimum and the maximum number of customers waiting for service in the 4- and 6-stations scenarios, with different set of vehicles.

different prioritization schemes, CVA tends to route vehicles towards commuting stations selected by customers, whereas PVA rather proposes commuting stations that are close to the selected ones.

We observe from Figs. 2(c)-(d) and Figs. 3(c)-(d) that, with a small number of vehicles (i.e., black-circled lines), the MBE is high both for CVA and PVA, and the performance gap is small, whereas it tends to increase for a greater number of vehicles (i.e., blue-crossed lines). This is due to the fact that PVA tends to favor system balance rather than revenue and reputation. Conversely, CVA aims to concurrently increase the providers' revenue and the customers' satisfaction, rather than keeping the system balanced to fulfill forthcoming requests.

V. Conclusions

In this paper, a customer-oriented optimization framework for MOD systems, called Customer-oriented Vehicle Assignment (CVA), has been proposed. The presented formulation aims at offering an efficient station-to-door transportation system that is able to fulfill the customer requests at its best, maximize the provider's revenues, and tends to keep the system balanced within prescribed limits. The problem has been tackled by defining a constrained mixed integer linear programming problem, which takes into account the customers' impatience and the trip costs. The customers' impatience has been modeled through a piecewise-linear function of parameters that have been suitably identified through the analysis of different aspects that dictate the customers' behavior, such as the affordability of the service, its quality, and the overall time used to reach his/her final destination. The performance of the proposed approach has been assessed through an extensive Monte Carlo simulation campaign in several operational conditions. The simultaneous satisfaction of often contrasting needs is sometimes achieved at the cost of a reduction in performance in the system balancing. In particular, this occurs when the number of vehicles in the system is large.

Future work will seek to further enrich our model by considering door-to-door services, and by developing a more accurate model of the customers' impatience and a more detailed cost function definitions in which the vehicles' fixed costs and other features will be accounted for. The objective function can also be refined by including metrics inspired to the performance parameter defined in this work. Furthermore, since the solution of the optimization problem for large instances of the system involves a high computational burden, we aim to devise sub-optimal optimization strategies, which can solve the problem for systems of large size in a reasonable time. Finally, models with different classes of vehicles will be considered.

References

[1] K. Uesugi, N. Mukai, and T. Watanabe, "Optimization of vehicle assignment for carsharing system," *Knowledge-Based Intelligent Information and Engineering System*, vol. 4693, pp. 1105–1111, 2007.

[2] D. Jorge and G. Correia, "Carsharing systems demand estimation and defined operations: A literature review," *The European Journal of Transport and Infrastructure Research*, vol. 13, no. 3, pp. 201–220, 2013.

[3] C.E. Cortés, A. Núñez, and D. Sáez, "Hybrid adaptive predictive control for a dynamic pickup and delivery problem including traffic congestion," *International Journal of Adaptive Control and Signal Processing*, vol. 22, pp. 103–123, 2008.

[4] J. Pfrommer, J. Warrington, G. Schildbach, and M. Morari, "Dynamic vehicle redistribution and online price incentives in shared mobility systems," *IEEE Transactions on Intelligent Transportation Systems*, vol. 15, no. 4, pp. 1567–1578, 2014.

[5] W. Mitchell, "Mobility on-demand: Future of transportation in cities," MIT Media Laboratory, Tech. Rep., 2008.

[6] S.L. Smith, M. Pavone, M. Schwager, E. Frazzoli, and D. Rus, "Rebalancing the rebalancers: Optimally routing vehicles and drivers in mobility-on-demand systems," in *Proc. of the American Control Conference*, Washington DC, USA, June 17-19 2013, pp. 2362–2367.

[7] A. Bianchessi, S. Formentin, and S. M. Savaresi, "Active fleet balancing in vehicle sharing systems via feedback dynamic pricing," in *Proc. of the 16th International IEEE Conference on Intelligent Transportation Systems*, The Hague, The Netherlands, Oct. 6-9 2013, pp. 1619–1624.

[8] F. Acquaviva, D. Di Paola, and A. Rizzo, "A novel formulation for the distributed solution of load balancing problems in mobility on-demand systems," in *Proc. of the 3rd IEEE International Conference on Connected Vehicles and Expo*, Vienna, Austria, Nov. 3-7 2014, pp. 902–907.

[9] M. Pavone, S.L. Smith, E. Frazzoli, and D. Rus, "Robotic load-balancing for mobility-on-demand systems," *International Journal of Robotics Research*, vol. 31, no. 7, pp. 839–854, 2011.

[10] M.J. Barth, M. Todd, and L. Xue, "User-based relocation techniques for multiple-station shared-use vehicle systems," in *Proc. of the Transportation Research Board 83th Annual Meeting*, Washington DC, USA, Jan. 11-15 2004.

[11] A. Kek, R. Cheu, Q. Meng, and C. Fung, "A decision support system for vehicle relocation operations in carsharing systems," *Transportation Research Part E: Logistics and Transportation Review*, vol. 45, no. 1, pp. 149–158, 2009.

[12] L. Li, "Design and analysis of a car sharing systems offering one-way journeys," University of Wisconsin - Milwaukee, Tech. Rep., 2011.

[13] J. Paquette, J. Cordeau, and G. Laporte, "Quality of service in dial-a-ride operations," *Computers & Industrial Engineering*, vol. 56, no. 4, pp. 1721–1734, 2009.

[14] A. Núñez, C.E. Cortés , D. Sáez, B. De Schutter, and M. Gendreau, "Multiobjective model predictive control for dynamic pickup and delivery problems," *Control Engineering in Practice*, vol. 32, pp. 73–86, 2014.

[15] N. Paulley, R. Balcombe, R. Mackett, H. Titheridge, J. Preston, M. Wardman, J. Shires, and P. White, "The demand for public transport: The effects of fares, quality of service, income and car ownership," *Transport Policy*, vol. 13, no. 4, pp. 295–306, 2006.

[16] D. Feillet, P. Dejax, and M. Gendreau, "Travelling salesman problems with profits: An overview," *Transportation Science*, vol. 39, no. 2, pp. 188–205, 2001.

[17] M. Pavone, N. Bisnik, and E. Frazzoli, "A stochastic and dynamic vehicle routing problem with time windows and customer impatience," *Mobile Networks and Applications*, vol. 14, no. 3, pp. 350–364, 2008.

[18] I.P. Levin, M.K. Mosell, C.M. Lamka, B.E. Savage, and M.J. Gray, "Measurement of psychological factors and their role in travel behavior," *Transportation Research Record*, no. 649, pp. 1–7, 1977.

[19] A. Di Febbraro, E. Gattorna, and N. Sacco., "Optimizing dynamic ride-sharing systems," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 2359, pp. 44–50, 2013.

[20] New York City Citibike Service, 2015. [Online]. Available: https://www.citibikenyc.com

[21] Car2go N.A., LLC, 2015. [Online]. Available: https://www.car2go.com/en/newyorkcity/

[22] A. Bemporad and M. Morari, "Control of systems integrating logic, dynamics, and constraints," *Automatica*, vol. 35, no. 3, pp. 407–427, 1999.