Delft University of Technology

Delft Center for Systems and Control

Technical report 17-012

Power scheduling in islanded-mode microgrids using fuel cell vehicles*

F. Alavi, N. van de Wouw, and B. De Schutter

Delft Center for Systems and Control
Delft University of Technology
Mekelweg 2, 2628 CD Delft
The Netherlands
phone: +31-15-278.24.73 (secretary)
URL: https://www.dcsc.tudelft.nl

* This report can also be downloaded via https://pub.bartdeschutter.org/abs/17_012.html

Power Scheduling in Islanded-Mode Microgrids using Fuel Cell Vehicles

Farid Alavi, Nathan van de Wouw, and Bart De Schutter

Abstract— We consider power scheduling in a microgrid operated in the islanded mode. It is assumed that at any time all the renewable energy sources are generating the maximum achievable electrical power based on the weather conditions and the power balance of the microgrid is exclusively done by a fleet of fuel cell cars. As a result, the uncertainty in the prediction of the load will also make the future power generation of the fuel cell cars uncertain and, hence, a robust control method should be used to operate the fuel cell cars. We develop a min-max model predictive control approach to schedule the power generation profile of the fuel cell cars. Furthermore, we develop an alternative approach, a min-max disturbance feedback approach, in order to reduce the conservatism of the min-max approach. Finally, an illustrative case study shows the performance of the proposed approaches.

I. INTRODUCTION

A microgrid that includes some loads and renewable energy sources and that is operated in the islanded mode is a promising structure for future power grids as the distributed power generation units can be included in this structure [1]. If the wind and solar energy are the only available sources of renewable energy, the power balance of a microgrid cannot be guaranteed solely based on the renewable energy sources (RES) because the variation in the generated power is an inherent feature in any wind turbine or solar photo-voltaic system. Such an islanded-mode microgrid needs a mechanism to store energy whenever the power generation of the RESs is higher than the load. In addition, another mechanism to regenerate electricity from the stored energy is necessary when the generation of RESs is less than the load. The ability to store energy and to recover electrical power from the stored energy can provide the necessary flexibility to the microgrid.

Fuel cell cars are a new type of vehicles that use hydrogen as the main source of energy. This type of cars are equipped with a fuel cell stack, in which the chemical energy of hydrogen is converted into electricity. The generated electricity is used to drive an electrical motor; as such, the fuel cell is being used for transportation. It is also possible to connect the fuel cell to the power grid and, consequently, to use the fuel cell car as an electrical power generation unit. This is the car as power plant concept [2]. Therefore, a fleet of fuel cell cars can create part of the required flexibility in the microgrid. Another part of the required flexibility, i.e.,

The authors are with the Delft Center for Systems and Control, Delft University of Technology, The Netherlands. E-mail: `f.alavi`, `n.vandewouw`, `b.deschutter@tudelft.nl`

Nathan van de Wouw is also with the Department of Mechanical Engineering, Eindhoven University of Technology, The Netherlands, and the Department of Civil, Environmental & Geo-Engineering, University of Minnesota, Minneapolis, U.S.A.

a mechanism to store energy, can be realized by using a water electrolysis system. The generated hydrogen in an electrolyzer can then be used by the cars as a fuel for transportation or for the generation of electricity at a later time.

In [3], the problem of power scheduling is formulated as a mixed integer linear programming problem. Herein, the uncertainty in the prediction of the load is not considered. To deal with the uncertainty in the microgrid, robust and stochastic methods have been developed in [4] and [5]. [6] developed a min-max MPC strategy that is able to deal with the uncertainty in the prediction of the load. However, the approach developed in [6] assumes curtailment in the generation of renewable energy sources. In addition, the concept of using fuel cell cars and water electrolysis system is not considered there.

In this paper, we consider a microgrid in the islanded mode that includes a fleet of fuel cell cars, a water electrolysis system, and some RES units. This scenario is different from [7] and [8] because we consider a microgrid in the islanded mode. As will be shown later, to satisfy the energy balance of the islanded mode microgrid, the power generation of fuel cell cars cannot be determined accurately in advance. Therefore, the future power generation profile of the fuel cell cars and also the future evolution of the system states are uncertain. A min-max robust model predictive control method is developed to tackle this problem. Toward this goal, a model of the microgrid is developed that considers different operation modes of the devices. We show that the proposed control method guarantees the satisfaction of the system constraints and how it can be used in the energy management system of the microgrid. Another development in the control method of this paper compared to that in [7] is the reduction of the conservatism by getting closer to the optimal operational cost compared to the min-max approach, where we propose an alternative method, called the min-max disturbance feedback approach. Finally, a numerical simulation is done to indicate the system performance and to explicate the enhancement that is achieved by the developed min-max disturbance feedback approach.

II. PROBLEM STATEMENT

A microgrid is considered that contains uncontrollable loads, renewable energy sources, a water electrolysis system, and fuel cell cars. It is assumed that the fuel cell cars are connected to the electrical network of the microgrid and that they can be used for both power generation and transportation; a specific car might be used in one of these two tasks, but not both tasks at the same time. A storage

tank of hydrogen is considered inside the microgrid and it is assumed that the stored hydrogen in this tank is used to refill the fuel cell cars. The hydrogen produced by the water electrolysis system will be added to the storage tank.

Even though the power generation of RES units and some part of the microgrid's load can be controlled, in this paper we assume that only the fleet of fuel cell cars is under control and that at any time, the RES units generate the maximum electrical power possible given the weather conditions. It is assumed that a prediction of the future generation of RES and the electricity demand of the load is available, as it is a standard assumption in the literature on MPC for such applications. Therefore, an estimate of the profile of the residual load, i.e., the difference between the household electrical demand and the RES generation, is assumed to be available. Due to the inaccuracy that exists in such prediction, the profile of the residual load is considered to be uncertain.

The energy management system is responsible for maintaining the power balance of the microgrid. The assumption of an uncontrollable load and the maximum generation of the RES means that the residual load of the microgrid should be compensated by the fuel cell cars and the water electrolysis system. In order to guarantee the power balance condition of the microgrid, a lower-level controller is embedded in each fuel cell car. These lower-level controllers aim to maintain the voltage and the frequency of the grid at the desired set-points. Note that the design of lower-level controllers in an islanded mode microgrid is an open issue and the interested reader is referred to [9] and the references therein for more information about the challenges in this area. However, the focus of this paper is on the higher-level control system and we assume that each fuel cell car is equipped with a lower-level control system that can guarantee the stability of the microgrid in short time scales, from milliseconds to seconds. The presence of lower-level controllers will result in a deviation between the scheduled and the actual power generation profile for each fuel cell car. This deviation is due to the uncertainty that exists in predicting the load and RES generation inside the microgrid.

Even though the lower-level controllers of the cars are able to guarantee the stability of the system, the physical constraints of the fuel cells and the desire to operate the system with the minimum cost necessitate the use of higher-level control. This level of control considers the scheduling of the power generation or demand of every controllable device inside the microgrid and the outputs of this control level are the set-points of the lower-level controllers. The power scheduling should be done in such a way that the operational cost of the microgrid is minimized, while the physical constraints of the system are satisfied.

III. MODELING

A. Model of the fuel cell cars

Because the aim of modeling is to describe the system behavior from the energy point of view, we only consider the elements of a fuel cell car that influence the power generation and storage of energy. The level of fuel in the storage tank of the car number i, $x_{\mathrm{f},i}$, is considered as a system state. The fuel cell stack consumes the stored hydrogen in order to generate electricity. When the fuel cell of car number i is operated in order to generate electricity, we have [10]:

$$x_{\mathrm{f},i}(k+1) = x_{\mathrm{f},i}(k) - (\alpha_{\mathrm{f},i} u_{\mathrm{f},i}^*(k) + \beta_{\mathrm{f},i})T_{\mathrm{s}}, \quad (1)$$

where $\alpha_{\mathrm{f},i}$ and $\beta_{\mathrm{f},i}$ are model parameters. The time step interval is represented by T_{s} and the actual power generation of the fuel cell stack at time step k is assumed to be $u_{\mathrm{f},i}^*(k)$.

The lower-level controllers will cause a difference, $w_{\mathrm{f},i}(k)$, between the actual power generation, $u_{\mathrm{f},i}^*(k)$, and the scheduled power generation, $u_{\mathrm{f},i}(k)$;

$$u_{\mathrm{f},i}^*(k) = u_{\mathrm{f},i}(k) + w_{\mathrm{f},i}(k). \quad (2)$$

The dynamics of the stored hydrogen of car number i during generation of electricity can be written as:

$$x_{\mathrm{f},i}(k+1) = x_{\mathrm{f},i}(k) - \big(\alpha_{\mathrm{f},i}(u_{\mathrm{f},i}(k) + w_{\mathrm{f},i}(k)) + \beta_{\mathrm{f},i}\big)T_{\mathrm{s}}. \quad (3)$$

The mismatch between the prediction and the actual residual load of the microgrid, $w(k)$, is compensated with the total unscheduled power generation of fuel cell cars, i.e.,

$$w(k) = \sum_{i=1}^{N_{\mathrm{veh}}} w_{\mathrm{f},i}(k).$$

A binary control input, $s_{\mathrm{f},i}(k)$, indicates the on/off operating mode of the fuel cell i. We assume that whenever $s_{\mathrm{f},i}(k)$ is equal to 0 or 1, the fuel cell i is in the off or on mode, respectively. Another binary control input, $s_{\mathrm{r},i}(k)$, indicates the refilling process of fuel cell car i. Whenever $s_{\mathrm{r},i}(k) = 1$, the fuel cell is in the refilling mode; during this mode, the fuel cell car is disconnected from the microgrid and the fuel tank of the car is filled with an amount $R_{\mathrm{f},i}$ at each time step. Therefore, during refilling the dynamics of the fuel level of the car can be represented by $x_{\mathrm{f},i}(k+1) = x_{\mathrm{f},i}(k) + R_{\mathrm{f},i}$.

A sequence of binary numbers, $\lambda_{\mathrm{f},i}(k), \lambda_{\mathrm{f},i}(k+1), \ldots,$ is used to indicate the transportation mode of the car. We assume that $\lambda_{\mathrm{f},i}(k+j) = 1$ indicates that at time step $k+j$, fuel cell car i is in the transportation mode. Otherwise, $\lambda_{\mathrm{f},i}(k+j) = 0$, and the car is ready to be connected to the microgrid. Considering historical data and the behavior of each driver, it is possible to predict a time interval that a car is used for transportation purposes. As a result, it is possible to determine the sequence of $\lambda_{\mathrm{f},i}$ for the future. The fuel consumed by car i during a trip that starts from time step k and ends at time step $k+j$, is assumed to be $h_{\mathrm{f},i}(k+j)$.

Summarizing, the piecewise affine model of the fuel cell car i can be written in the following form:

$$x_{\mathrm{f},i}(k+1) = \begin{cases} x_{\mathrm{f},i}(k) + R_{\mathrm{f},i} & \text{refilling} \\ x_{\mathrm{f},i}(k) & \text{no generation} \\ x_{\mathrm{f},i}(k) - (\alpha_{\mathrm{f},i} u_{\mathrm{f},i}^*(k) + \beta_{\mathrm{f},i})T_{\mathrm{s}} & \text{generation} \\ x_{\mathrm{f},i}(k) & \text{transportation} \\ x_{\mathrm{f},i}(k) - h_{\mathrm{f},i}(k) & \text{arrival.} \end{cases}$$

The interested reader is referred to [7] for more details and the motivation of this model.

There are several constraints in the operation of a fuel cell car that are related to either the operation mode or

the physical constraints of the system. The first constraint is related to the transportation mode of the car: if a car is used for transportation at time step k, then it cannot be in the refilling process nor in the generation mode:

$$\text{if } \lambda_{\mathrm{f},i}(k) = 1 \text{ then } s_{\mathrm{r},i}(k) = 0$$
$$\text{if } \lambda_{\mathrm{f},i}(k) = 1 \text{ then } s_{\mathrm{f},i}(k) = 0.$$

In addition, we assume that a car is disconnected from the grid during the refilling process, i.e.

$$\text{if } s_{\mathrm{r},i}(k) = 1 \text{ then } s_{\mathrm{f},i}(k) = 0.$$

The physical constraints consist of the maximum power generation of a fuel cell, $\bar{u}_{\mathrm{f},i}$, and the minimum and maximum level of the fuel tank, $\underline{x}_{\mathrm{f},i}$ and $\bar{x}_{\mathrm{f},i}$. We assume that whenever the fuel level of a car is under the minimum level, the fuel cell stack is turned off, i.e. if $x_{\mathrm{f},i}(k) \leq \underline{x}_{\mathrm{f},i}$ then $s_{\mathrm{f},i}(k) = 0$. These constraints can be expressed in the following form:

$$0 \leq u_{\mathrm{f},i}(k) \leq \bar{u}_{\mathrm{f},i}$$
$$\underline{x}_{\mathrm{f},i} s_{\mathrm{f},i}(k) \leq x_{\mathrm{f},i}(k) \leq \bar{x}_{\mathrm{f},i}.$$

B. Model of the water electrolysis system

The water electrolysis system is used in order to produce hydrogen and to store it in the hydrogen storage system. It is assumed that the hydrogen production is exclusively done via water electrolysis. Therefore, one model is derived for both the hydrogen storage system and the water electrolysis system. The level of stored hydrogen, x_{el}, is considered as the system state and the power demand of the electrolyzer at time step k, $u_{\mathrm{el}}(k)$, is the control input. Following an approach similar to [7], the piecewise affine model of the electrolyzer and the hydrogen storage system is:

$$x_{\mathrm{el}}(k+1) = \begin{cases} x_{\mathrm{el}}(k) - \sum_{i=1}^{N_{\mathrm{veh}}} s_{\mathrm{r},i}(k) R_{\mathrm{f},i} & \text{OFF} \\ x_{\mathrm{el}}(k) - \sum_{i=1}^{N_{\mathrm{veh}}} s_{\mathrm{r},i}(k) R_{\mathrm{f},i} \\ \qquad + T_{\mathrm{s}} \alpha_{\mathrm{el}} u_{\mathrm{el}}(k) & \text{ON}, \end{cases}$$

where N_{veh} is the total number of fuel cell cars and α_{el} is a model parameter. The operating modes of the electrolyzer, OFF and ON, are determined at each time step k by a binary control input, $s_{\mathrm{el}}(k)$. Whenever $s_{\mathrm{el}}(k) = 1$, the water electrolysis system is turned on. The hydrogen consumption of the cars is present in both modes and expressed by the term $\sum_{i=1}^{N_{\mathrm{veh}}} s_{\mathrm{r},i}(k) R_{\mathrm{f},i}$.

The physical constraints of the hydrogen storage system impose a minimum and a maximum limit on the level of stored hydrogen, $\underline{x}_{\mathrm{el}}$ and $\bar{x}_{\mathrm{el}}$, respectively. In addition, there is a maximum power demand for the water electrolysis system. These constraints can be written as:

$$0 \leq u_{\mathrm{el}}(k) \leq \bar{u}_{\mathrm{el}} \tag{4}$$
$$\underline{x}_{\mathrm{el}} \leq x_{\mathrm{el}}(k) \leq \bar{x}_{\mathrm{el}}. \tag{5}$$

The definition of the binary control input $s_{\mathrm{el}}(k)$ implies that whenever $s_{\mathrm{el}}(k) = 0$, the power demand of the electrolyzer should be equal to zero. Using (4), this can be expressed as:

$$\text{if } s_{\mathrm{el}}(k) = 0 \text{ then } u_{\mathrm{el}}(k) \leq 0. \tag{6}$$

C. Total model of the system

The piecewise affine models developed in Sections III-A and III-B can be converted into a mixed logical dynamical (MLD) model [11] using standard techniques [12]. Therefore, the overall system model can be written as:

$$x(k+1) = x(k) + B_1(w(k))u(k) + B_3(k)z(k) + B_4(k), \tag{7}$$

where x, u, and z are defined as:

$$x = \begin{bmatrix} x_{\mathrm{f},1} & \dots & x_{\mathrm{f},N_{\mathrm{veh}}} & x_{\mathrm{el}} \end{bmatrix}^T \tag{8}$$
$$u = \begin{bmatrix} u_{\mathrm{f},1} & s_{\mathrm{r},1} & s_{\mathrm{f},1} & \dots & u_{\mathrm{f},N_{\mathrm{veh}}} & s_{\mathrm{r},N_{\mathrm{veh}}} & s_{\mathrm{f},N_{\mathrm{veh}}} & u_{\mathrm{el}} & s_{\mathrm{el}} \end{bmatrix}^T$$
$$z = \begin{bmatrix} z_{\mathrm{f},1} & \dots & z_{\mathrm{f},N_{\mathrm{veh}}} & z_{\mathrm{el}} \end{bmatrix}^T.$$

The variables $z_{\mathrm{f},i} := s_{\mathrm{f},i} u_{\mathrm{f},i}$ and $z_{\mathrm{el}} := s_{\mathrm{el}} u_{\mathrm{el}}$ are continuous auxiliary variables and $B_1(k)$, $B_3(k)$, and $B_4(k)$ are:

$$B_1(k) = \begin{bmatrix} \mathrm{blkdiag}\left(b_1^1(k), \dots, b_1^{N_{\mathrm{veh}}}(k)\right) & \mathbf{0} \\ \begin{bmatrix} b_2^1 & \dots & b_2^{N_{\mathrm{veh}}} \end{bmatrix} & 0 \end{bmatrix}$$

$$B_3(k) = \begin{bmatrix} \mathrm{blkdiag}\left(b_3^1(k), \dots, b_3^{N_{\mathrm{veh}}}(k)\right) & \mathbf{0} \\ \mathbf{0} & b_3^{\mathrm{el}}(k) \end{bmatrix}$$

$$B_4(k) = \begin{bmatrix} -\lambda_{\mathrm{f},1}(k) h_1(k) & \dots & -\lambda_{\mathrm{f},N_{\mathrm{veh}}}(k) h_{N_{\mathrm{veh}}}(k) & 0 \end{bmatrix}^T,$$

where $\mathrm{blkdiag}\{.\}$ indicates a block diagonal matrix with the arguments as diagonal blocks. For all $i \in \{1, \dots, N_{\mathrm{veh}}\}$, $b_1^i(k) = \begin{bmatrix} 0 & R_{\mathrm{f},i} & -\lambda_{\mathrm{f},i}(k) T_{\mathrm{s}}(\beta_{\mathrm{f},i} + \alpha_{\mathrm{f},i} w_{\mathrm{f},i}(k)) \end{bmatrix}$, $b_2^i = \begin{bmatrix} 0 & -R_{\mathrm{f},i} & 0 \end{bmatrix}$, $b_3^i(k) = -\lambda_{\mathrm{f},i}(k) T_{\mathrm{s}} \alpha_{\mathrm{f},i}$, and $b_3^{\mathrm{el}}(k) = \alpha_{\mathrm{el}} T_{\mathrm{s}}$.

All the mentioned constraints of the system at time step k can be written as one inequality of the form:

$$E_1 u(k) + E_4 x(k) + E_5(k) \geq E_3 z(k). \tag{9}$$

IV. CONTROL ALGORITHM

To schedule the power generation of the fuel cell cars and the power demand of the water electrolysis system, an MPC algorithm is designed. Assuming that predictions about the household loads and the renewable energy generation are available, the power scheduling of the other devices is done via minimizing the operational cost of the system, subject to all the operational constraints. To deal with the uncertainty that exists in the cost function and in the constraints, two approaches, namely a min-max approach and a min-max disturbance feedback approach, are developed in this section.

Three important factors define the operational cost [7]: the cost of power generation or power demand of the devices, the switching cost of the devices, and the cost of stabilizing actions done by the low-level controllers of the fuel cell cars.

Switching the operation mode of the fuel cells and the electrolyzer will decrease their lifetime [13] and, hence, it is considered as the first factor influencing the operational cost. The first line of (10) represents this part of the cost function.

The second factor is related to the cost of power generation of fuel cell cars and power demand of water electrolysis system. This part of the cost originates from two sources. The first source is the degradation of the devices during the usage. It is assumed that one part of the degradation of the

device is related to the generated or consumed power. The second source is the price of the hydrogen consumed in the fuel cell cars or produced in the water electrolysis system. Considering the linear relation between the consumed hydrogen and the generated electricity in the fuel cell cars, and between the produced hydrogen and electricity consumed in the electrolyzer, one may express the price of consumed or produced hydrogen in terms of the generated or consumed electricity. The second line of (10) represents this factor.

The third factor is related to the compensation of the effort made by the low-level controllers of the fuel cell cars. The low-level controllers of the fuel cell cars are responsible for stabilizing the voltage and frequency of the microgrid. The effort of fuel cell cars in stabilizing the voltage and frequency of the microgrid is measured by the deviation between the actual and scheduled power generation and it should be rewarded. In the current set-up of the microgrid, the resulting rewards for the fuel cell cars can be considered as an extra part of the operational cost, which is represented by the third line of (10).

The definition of the cost function is then as follows:

$$J(k) = \sum_{j=0}^{N_\mathrm{p}-1} \Big[\sum_{i=1}^{N_\mathrm{veh}} W_\mathrm{sf}|\Delta s_{\mathrm{f},i}(k+j)| + W_\mathrm{sel}|\Delta s_\mathrm{el}(k+j)|$$
$$+ \sum_{i=1}^{N_\mathrm{veh}} W_\mathrm{pf} u_{\mathrm{f},i}(k+j) + W_\mathrm{pel} u_\mathrm{el}(k+j)$$
$$+ C_\mathrm{e}(k+j) w(k+j) \Big], \qquad (10)$$

where N_p and N_veh represent the prediction horizon and the number of vehicles, respectively and where $\Delta s(j) = s(j) - s(j-1)$. The weights W_sf, W_sel, W_pf, and W_pel indicate the influence of the switching and the power generation or power demand cost of the devices in the total operational cost. The total reward that is paid to the fuel cell cars for stabilizing the microgrid is equal to the total unscheduled power generation, $w(k)$, multiplied by an electricity tariff $C_\mathrm{e}(k)$ at time step k. Therefore, the term $C_\mathrm{e}(k)w(k)$ represents a part of the operational cost that is related to the reward for fuel cell cars. Here, the total mismatch between the scheduled power generation and the actual power generation is equal to the total unscheduled power generation of all the fuel cell cars, i.e. $w(k) = \sum_{i=1}^{N_\mathrm{veh}} w_{\mathrm{f},i}(k)$.

The operational cost can be written as:

$$J(k) = W_\mathrm{v}(k)\tilde{V}(k) + W_\mathrm{d}(k)\tilde{w}(k). \qquad (11)$$

The vector $\tilde{V}(k)$ in (11) contains all the optimization variables; i.e. $\tilde{V}(k) = \begin{bmatrix} \tilde{u}(k)^T & \tilde{z}(k)^T \end{bmatrix}^T$, where the tilde notation indicates a vector containing the value of its operand from time step k to $k + N_\mathrm{p} - 1$. Therefore, $\tilde{w}(k) = \begin{bmatrix} w(k) & \dots & w(k+N_\mathrm{p}-1) \end{bmatrix}^T$.

It is easy to verify that the system constraints for all the time steps in the prediction horizon can be written as:

$$F_1(\tilde{w}(k))\tilde{V}(k) \le F_2(k) + F_3(k)x(k). \qquad (12)$$

A. Min-max approach

In the min-max approach, the optimization problem that should be solved at each time step k in the model predictive controller is as follows:

$$\min_{\tilde{V}(k)} \max_{\tilde{w}(k)} W_\mathrm{v}(k)\tilde{V}(k) + W_\mathrm{d}(k)\tilde{w}(k) \qquad (13)$$
$$\text{subject to (12),} \quad \text{for all } \tilde{w}(k).$$

The presence of a maximum over all possible realizations of the uncertain parameter $w(k)$ in the cost function (11) and the necessity to satisfy the constraint (12) for any realization of $\tilde{w}(k)$ make the optimization problem (13) hard to solve. However, using the following assumption and Lemma 1 below will result in a simpler problem formulation.

Assumption 1: The uncertainty in the prediction of the residual load of the microgrid is bounded for all k, i.e., $\underline{w} \le w(k) \le \bar{w}$.

Lemma 1: Defining $\tilde{w}_1 = \begin{bmatrix} \bar{w} & \bar{w} & \dots & \bar{w} \end{bmatrix}^T_{1 \times N_\mathrm{p}}$, $\tilde{w}_2 = \begin{bmatrix} \bar{w} & \bar{w} & \dots & \underline{w} \end{bmatrix}^T_{1 \times N_\mathrm{p}}$, $\dots$, $\tilde{w}_N = \begin{bmatrix} \underline{w} & \underline{w} & \dots & \underline{w} \end{bmatrix}^T_{1 \times N_\mathrm{p}}$, the inequality (12) holds for all possible disturbances $w(k)$ satisfying Assumption 1 if the following $N = 2^{N_\mathrm{p}}$ inequalities hold:

$$F_1(\tilde{w}_1)\tilde{V}(k) \le F_2(k) + F_3(k)x(k) \qquad (14)$$
$$\vdots$$
$$F_1(\tilde{w}_N)\tilde{V}(k) \le F_2(k) + F_3(k)x(k).$$

Proof: Considering the structure of $F_1(\tilde{w}(k))$, $F_2(k)$, and $F_3(k)$, each row of inequality (12) can be written in the form:

$$\gamma_1 w(k) + \gamma_2 w(k+1) + \dots + \gamma_{N_\mathrm{p}} w(k+N_\mathrm{p}-1) \le a, \qquad (15)$$

where $a, \gamma_i \in \mathbb{R}$ for all i. The maximum value of the left-hand side of (15) will be realized at a specific realization of the uncertainty $\tilde{w}^*(k) = \begin{bmatrix} w^*(k) & \dots & w^*(k+N_\mathrm{p}-1) \end{bmatrix}^T$. Considering that the left-hand side of (15) is linear with respect to w and also that $\underline{w} \le w(k+j) \le \bar{w}$ for all j, the value of $\tilde{w}^*(k)$ will be equal to one of the $\tilde{w}_i$ defined in Lemma 1. Based on (14), we know that (15) holds for all $\tilde{w}_i$. Therefore, (15) holds for any realization of $\tilde{w}(k)$ that satisfies Assumption 1.

Equation (12) consists of several rows, all in the form of (15) and we have shown that (14) is a sufficient condition for the validity of (15). Therefore, the satisfaction of (14) implies the satisfaction of (12). $\square$

As a result of Lemma 1 and using a similar reasoning for the maximum in the objective function of (13), the optimization problem (13) can be simplified and expressed in the following form:

$$\min_{\tilde{V}(k)} \max \{ \quad W_\mathrm{v}(k)\tilde{V}(k) + W_\mathrm{d}(k)\tilde{w}_1(k), \dots \\ W_\mathrm{v}(k)\tilde{V}(k) + W_\mathrm{d}(k)\tilde{w}_N(k) \} \qquad (16)$$
$$\text{subject to (14).}$$

Note that if the Assumption 1 holds, the optimization problems (13) and (16) are equivalent and no conservatism is

added with respect to the original problem in (13). Optimization problem (16) consists of N mixed integer linear programming (MILP) problems.

B. Min-max disturbance feedback approach

The min-max optimization problem introduced in Section IV-A is based on a worst-case scenario. However, in reality the worst-case scenario will be realized rarely and, hence, such conservatism is a disadvantage of the min-max approach. In this section, an alternative method, called min-max disturbance feedback, is developed based on [14]. Even though the realized value of the disturbance is unknown to the controller, the presence of a disturbance feedback mechanism prevents the expansion of possible state trajectories in the prediction horizon. As a result, the min-max disturbance feedback controller will act less conservatively compared to the regular min-max controller of Section IV-A.

In the min-max disturbance feedback approach, a control law is considered for the sequence of future control inputs of fuel cell cars. For each time step $k+j$, the control input of fuel cell car i is determined as follows:

$$u_{\mathrm{f},i}(k+j) = v_{\mathrm{f},i}(k+j) + K_{\mathrm{f}}(k+j)w_{\mathrm{f},i}(k+j-1),$$
(17)

where $v_{\mathrm{f},i}$ is a real number determined by MPC controller and represents the scheduled power generation of fuel cell car i. The first part of the control input, $v_{\mathrm{f},i}$, and the feedback gain, K_{f}, are determined via solving an optimization problem. The value of $w_{\mathrm{f},i}(k+j-1)$ is an uncertainty and it can be only determined after time step $k+j$.

Using (17), the actual power generation of fuel cell cars can be still represented by (2). In addition, the model of fuel cell cars and water electrolysis system will remain the same. However, the matrices B_1 in (7) and E_1 in (9) will become a function of $w_{\mathrm{f},i}(k-1)$. The resulting MLD model is:

$$x(k+1) = x(k) + B_1(w(k))u(k) + B_3(w(k-1))z(k) + B_4(k)$$
$$E_1(w(k-1))u(k) + E_4x(k) + E_5(k) \geq E_3z(k),$$
(18)

where the definition of x remains the same as in (8). The new definition of u and z is as follows:

$$u = \begin{bmatrix} u_{\mathrm{f},1} & s_{\mathrm{r},1} & s_{\mathrm{f},1} & \cdots & u_{\mathrm{f},N_{\mathrm{veh}}} & s_{\mathrm{r},N_{\mathrm{veh}}} & s_{\mathrm{f},N_{\mathrm{veh}}} & K_{\mathrm{f}} & u_{\mathrm{el}} & s_{\mathrm{el}} \end{bmatrix}^T$$
$$z = \begin{bmatrix} z_{\mathrm{f},1}z_{\mathrm{kf},1} & \cdots & z_{\mathrm{f},N_{\mathrm{veh}}}z_{\mathrm{kf},N_{\mathrm{veh}}} & z_{\mathrm{el}} \end{bmatrix}^T.$$

The variable K_{f} represents the disturbance feedback gain for all the fuel cell cars and the new auxiliary variables are defined as $z_{\mathrm{kf},i} := K_{\mathrm{f}}s_{\mathrm{f},i}$. By extending the inequality constraints in (18) over the prediction horizon we have:

$$F_1^{\mathrm{df}}(\tilde{w}(k))\tilde{V}(k) \leq F_2^{\mathrm{df}}(k) + F_3^{\mathrm{df}}(k)x(k).$$
(19)

The matrices F_1^{df}, F_2^{df}, and F_3^{df} are different from the min-max approach, but the format given in (15) still applies to it. As a result, Lemma 1 holds also for the current approach. Therefore, the inequality (19) will be satisfied if the following inequalities are satisfied:

$$F_1^{\mathrm{df}}(\tilde{w}_1(k))\tilde{V}(k) \leq F_2^{\mathrm{df}}(k) + F_3^{\mathrm{df}}(k)x(k)$$
(20)
$$\vdots$$
$$F_1^{\mathrm{df}}(\tilde{w}_N(k))\tilde{V}(k) \leq F_2^{\mathrm{df}}(k) + F_3^{\mathrm{df}}(k)x(k).$$

The operational cost of the system is given by (10). Considering that the sequence of control inputs is determined based on (17) and following a similar procedure as in Section IV-A, the cost function can be written in the following form:

$$J(k) = W_{\mathrm{v}}^{\mathrm{df}}(\tilde{w}(k))\tilde{V}(k) + W_{\mathrm{d}}^{\mathrm{df}}(k)\tilde{w}(k),$$
(21)

where $W_{\mathrm{v}}^{\mathrm{df}}$ and $W_{\mathrm{d}}^{\mathrm{df}}(k)$ can be determined based on the system model and the future electricity tariff, respectively.

To minimize the worst-case operational cost of the system, the following optimization problem should be solved:

$$\min_{\tilde{V}(k)} \max_{\tilde{w}(k)} J(k)$$
(22)
$$\text{subject to (19)}, \qquad \text{for all } \tilde{w}(k)$$

The operational cost of the system (21) has an affine relation with the disturbance of the system, $\tilde{w}(k)$. Therefore, with considering Assumption 1 for the disturbance, the worst-case cost of the system will be realized in one of the sequences of $\tilde{w}_i$. Therefore, by using Lemma 1, the optimization problem (22) can be simplified to:

$$\min_{\tilde{V}(k)} \max\{ \quad W_{\mathrm{v}}(k)\tilde{V}(k) + W_{\mathrm{d}}(k)\tilde{w}_1(k), \ \ldots$$
$$W_{\mathrm{v}}(k)\tilde{V}(k) + W_{\mathrm{d}}(k)\tilde{w}_N(k)\}$$
(23)
$$\text{subject to (20)}.$$

This is also a collection of MILP problems.

V. NUMERICAL EXAMPLE

We now present a case study to illustrate the proposed approaches using a microgrid containing 4 fuel cell cars, a water electrolysis system, and a storage tank of hydrogen. For the sake of simplicity, we assume that the fuel cell cars are not used in the transportation mode. The values of $(\alpha_{\mathrm{f},i}, \beta_{\mathrm{f},i})$ for $i = 1,2,3,4$ are $(0.0502, 0.095)$, $(0.0504, 0.10)$, $(0.0506, 0.105)$, and $(0.0508, 0.11)$. For all the cars, the maximum power generation, $\bar{u}_{\mathrm{f},i}$, the minimum, $\underline{x}_{\mathrm{f},i}$, and maximum, $\bar{x}_{\mathrm{f},i}$, limit of fuel level and the refilling rate, $R_{\mathrm{f},i}/T_{\mathrm{s}}$, are 15 kW, 2 kg, 5 kg, and 2 kg per time step, respectively. The interval between any two consecutive time steps, T_{s}, is 15 minutes. The maximum power consumption of the water electrolysis system, $\bar{u}_{\mathrm{el}}$, is 100 kW. The value of α_{el} is 0.02 kg/kWh. A storage tank of hydrogen with the maximum capacity, $\bar{x}_{\mathrm{el}}$, of 200 kg is connected to the electrolyzer to refill the fuel cell cars. The values of W_{sf}, W_{sel}, W_{pf}, and W_{pel} in the cost function (10) are equal to 0.1 €, 0.1 €, 0.6 €/kWh, and 0.1 €/kWh, respectively.

We have used Gurobi to solve the MILP problems in the simulation. The residual load of the microgrid is depicted in Figure 1. In order to maintain the power balance condition, the scheduled power generation of all the fuel cell cars and the electrolyzer should be equal to the residual load of the microgrid. The uncertainty in the prediction of the residual load is less than 4 kWh, i.e. $\bar{w} = |\underline{w}| = 4$ kWh. For the sake of comparing the two control approaches, we have considered the same realizations of the uncertainty in simulating the system behavior for both the min-max approach and the min-max disturbance feedback approach.
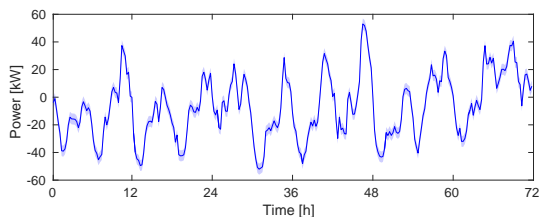
Fig. 1. Residual load of the microgrid; the blue line indicates the prediction of the residual load, while the actual residual load will be realized in the shaded area.
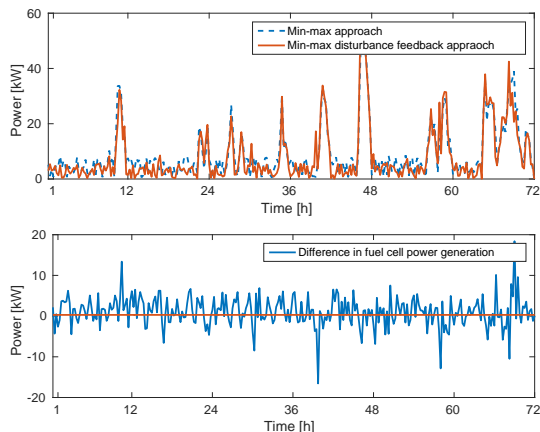

Fig. 2. Operation of the control system. Top: actual power generation of all the fuel cell cars using the min-max approach and the min-max disturbance feedback approach. Bottom: difference between the total power generation using the disturbance feedback approach and the min-max approach.

Based on the problem set-up, there is a lower-level controller inside each fuel cell car that maintains the power balance in the time scales less than T_s. Therefore, the uncertainty in the prediction of the residual load will influence the actual power generation of fuel cell cars. This phenomenon is clearly observable in Figure 2(top). Figure 2(bottom) shows the difference between the total power generation of the fuel cell cars for the two approach. As can be seen, the difference is often positive, which indicates that the fuel cell cars generate more power by using the min-max approach compared to using the disturbance feedback approach. Here, the former method results in generating 2.6 MWh electrical energy of fuel cells during the entire simulation time, while this value is 2.3 MWh for the latter method. Because the power generation of the fuel cell cars determines the majority of the operational costs, less power generation of fuel cell cars is a desirable behavior. Hence, the disturbance feedback approach results in a better performance.

TABLE I

REALIZED OPERATIONAL COST OF THE SYSTEM FOR MIN-MAX (MM) AND MIN-MAX DISTURBANCE FEEDBACK (DF) APPROACHES.

N_p	4		6		8		10	
$\bar{w}$	MM	DF	MM	DF	MM	DF	MM	DF
1	1651	1208	1650	1223	1650	1205	1649	1217
2	1854	1378	1855	1388	1855	1390	1855	1398
3	1850	1380	1849	1384	1850	1367	1850	1340
4	2051	1500	2051	1528	2050	1538	2051	1536

The realized operational cost of the system with respect to different values of $\bar{w} = |\underline{w}|$ and N_p is reported in Table I. It is assumed that the disturbance, w, is realized in the domain $[\underline{w}, \bar{w}]$ with a uniform probability function. It can be seen that the realized operational cost of the system for the min-max disturbance feedback approach is significantly less than for the min-max approach.

VI. CONCLUSIONS

We have developed a min-max model predictive control approach that is able to guarantee the power balance condition in a microgrid with fuel cell cars and a water electrolysis system, while the operational cost of the system is minimized for the worst-case scenario of the disturbance. In addition, we also developed a min-max disturbance feedback approach that reduces the conservativeness of the min-max approach. The case study illustrates the system performance of the min-max approach, in addition to the reduction of conservatism by the proposed min-max disturbance feedback approach.

ACKNOWLEDGMENTS

REFERENCES

[1] J. A. P. Lopes, C. L. Moreira, and A. G. Madureira, "Defining control strategies for microgrids islanded operation," *IEEE Trans. on Power Systems*, vol. 21, pp. 916–924, May 2006.

[2] A. J. M. van Wijk and L. Verhoef, *Our Car as Power Plant*. Delft University Press, 2014. ISBN: 9781614993766.

[3] A. Parisio, E. Rikos, and L. Glielmo, "A model predictive control approach to microgrid operation optimization," *IEEE Trans. on Control Systems Technology*, vol. 22, pp. 1813–1827, Sept. 2014.

[4] D. Bertismas, E. Litvinov, X. A. Sun, J. Zhao, and T. Zheng, "Adaptive robust optimization for the security constrained unit commitment problem," *IEEE Trans. on Power Systems*, vol. 28, pp. 52–63, Feb. 2013.

[5] R. Jiang, J. Wang, M. Zhang, and Y. Guan, "Two-stage minimax regret robust unit commitment," *IEEE Transactions on Power Systems*, vol. 28, pp. 2271–2282, Aug. 2013.

[6] C. A. Hans, V. Nenchev, J. Raisch, and C. Reincke-Collon, "Minimax model predictive operation control of microgrids," in *19th World Congress of the Int. Federation of Automatic Control*, Aug. 2014.

[7] F. Alavi, E. Park Lee, N. van de Wouw, B. De Schutter, and Z. Lukszo, "Fuel cell cars in a microgrid for synergies between hydrogen and electricity networks," *Applied Energy*, vol. 192, pp. 296–304, 2017.

[8] F. Alavi, N. van de Wouw, and B. De Schutter, "Min-max control of fuel-cell-car-based smart energy systems," in *European Control Conference*, pp. 1223–1228, July 2016.

[9] J. Schiffer, R. Ortega, A. Astolfi, J. Raisch, and T. Sezi, "Conditions for stability of droop-controlled inverter-based microgrids," *Automatica*, vol. 50, 2014.

[10] P. Rodatz, G. Paganelli, A. Sciarretta, and L. Guzzella, "Optimal power management of an experimental fuel cell/supercapacitor-powered hybrid vehicle," *Control Eng. Practice*, vol. 13, no. 1, pp. 41–53, 2005.

[11] A. Bemporad and M. Morari, "Control of systems integrating logic, dynamics, and constraints," *Automatica*, vol. 35, no. 3, pp. 407–427, 1999.

[12] W. P. M. H. Heemels, B. De Schutter, and A. Bemporad, "Equivalence of hybrid dynamical models," *Automatica*, vol. 37, pp. 1085–1091, July 2001.

[13] S. Zhang, X. Yuan, H. Wang, W. Mérida, H. Zhu, J. Shen, S. Wu, and J. Zhang, "A review of accelerated stress tests of MEA durability in PEM fuel cells," *Int. Journal of Hydrogen Energy*, vol. 34, no. 1, pp. 388–404, 2009.

[14] D. H. van Hessem and O. H. Bosgra, "A full solution to the constrained stochastic closed-loop MPC problem via state and innovations feedback and its receding horizon implementation," in *42nd IEEE Conference on Decision and Control*, pp. 929–934, IEEE, Dec. 2003.