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Technical report 17-019

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If you want to cite this report, please use the following reference instead:
S. Yuan, B. De Schutter, and S. Baldi, "Robust adaptive tracking control of uncertain slowly switched linear systems," *Nonlinear Analysis: Hybrid Systems*, vol. 27, pp. 1–12, Feb. 2018. doi:10.1016/j.nahs.2017.08.003

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Robust adaptive tracking control of uncertain slowly switched linear systems

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Abstract

In this paper, robust adaptive tracking control schemes for uncertain switched linear systems subject to disturbances are investigated. The robust adaptive control problem requires the design of both adaptive and switching laws. A novel adaptive law is proposed based on an extended leakage approach, which does not require knowledge of the bounds of the uncertainty set. Two switching laws are developed based on extended dwell time (DT) strategies: a) mode-dependent dwell time (MDDT); b) mode-mode-dependent dwell time (MMDDT). MDDT exploits the information of the known reference model for every subsystem, i.e., the dwell time is realized in a subsystem sense. MMDDT is a variant of MDDT that can guarantee stability under faster switching than MDDT, provided that the next subsystem to be switched on is known. The proposed adaptive schemes can achieve global uniform ultimate boundedness for shorter switching intervals than state-of-the-art adaptive approaches based on DT. In addition to global uniform ultimate bounded stability, transient and steady-state performance bounds are derived for the tracking error. The numerical example of a highly maneuverable aircraft technology vehicle is adopted to demonstrate the effectiveness of the proposed adaptive methods.

Keywords:
Robust adaptive tracking control, uncertain switched linear systems, mode-dependent dwell time, mode-mode-dependent dwell time

1. Introduction

Switched systems are an important class of hybrid systems consisting of subsystems with continuous dynamics, called *modes*, and a rule, called *switching law*, to regulate the switching action between the modes. Switched systems appear in a wide range of applications, such as intelligent transportation systems, power electronics, and smart energy systems [1].

To date, productive research has been conducted on switched systems with known parameters, such as stability and stabilization problems [2, 3, 4, 5, 6, 7]. This research direction is mainly based on two families of switching laws: dwell time (DT) and average dwell time (ADT) [8]. In DT switching, the switching interval between two consecutive discontinuities of the switching law should be larger than a sufficiently large constant to guarantee the stability of the switched system. In ADT switching, the switching interval between two consecutive discontinuities of the switching law should be sufficiently large in an average sense: this means that very short switching intervals are allowed provided that they are compensated by long ones. Recently, conservativeness[1] of ADT has been further decreased by a new switching strategy proposed in [9]: mode-dependent average dwell time (MDADT). The peculiarity of this switching strategy consists in exploiting the information of every mode, such as the exponential rate of the Lyapunov function associated to each mode.

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[1] In this work, the term conservativeness is adopted to indicate the lower bound on the length of the switching intervals for which stability can be guaranteed.

On the other hand, research on uncertain switched systems is not equally mature. As a matter of fact, in real-life problems parametric uncertainty is a ubiquitous condition. This creates additional difficulties when designing control and switching laws. In general, there are two main families of techniques dealing with stabilization of uncertain systems: robust control and adaptive control. It is well recognized that a single robust controller may lead to very conservative performance for a large uncertainty set [10, 11]. Therefore, when the uncertainties are polytopic, using a family of robust controllers has been proposed to improve the performance of a single controller [12]. As a complement to robust control, adaptive approaches for non-switched uncertain systems have been investigated to improve the performance of robust approaches over large non-polytopic uncertainties [13, 14, 15, 16]. However, adaptive control of uncertain switched systems is more challenging. This is because not only an adaptive law should be developed to estimate the unknown parameters, but also a switching law should be carefully designed to guarantee the stability of the closed-loop system. Recently, some research has been conducted on adaptive tracking control of uncertain switched systems, i.e., switched nonlinear systems [17, 18] where adaptive fuzzy approaches are adopted, state-dependent switched systems [19, 20, 21, 22] where minimal control synthesis algorithm is used, and time-constraint switched systems [23, 24, 25, 26]. However, the following two gaps can be identified in the state of the art on adaptive tracking control for uncertain slowly switched systems: first, the set where the nominal parameters reside should be known *a priori* [23, 24, 25, 26]; second, not much attention has been paid to switching laws that exploit the information of each subsystem. While the importance of overcoming the knowledge of the uncertainty set is evident, the need to address less conservativeness switching laws stems from the following research problem: ADT and MDADT switching strategies might cause undesired transient performance of the switched system due to overshoot of the Lyapunov function [1, 27]. Therefore, it is relevant to address the following question: can we design an adaptive law and a switching law for uncertain switched linear systems such that the knowledge of the residing space of the parameters is not necessary, and undesired transient behavior of the tracking error due to fast switchings can be avoided?

The main contribution of this paper is twofold. On the one hand, a robust adaptive law with a leakage approach is developed without requiring a priori knowledge of the uncertainty set. On the other hand, two switching laws with shorter switching intervals than dwell time are introduced. In particular, new adaptive tracking control scheme for uncertain switched linear systems is developed based on a mode-dependent dwell time (MDDT) switching law by exploiting the information of every subsystem [28]. Furthermore, to address scenarios for which the next subsystem to be switched on is known, we introduce a new switching scheme: mode-mode-dependent dwell-time (MMDDT). MMDDT is relevant in many applications, such as automobile power train [29], power converters [30], thermostatic control [31], train trajectory planning [32], where the next mode to be switched on is known in advance. Exploiting this information allows even shorter switching intervals than MDDT. Global uniform ultimate stability of the switched system via the proposed robust adaptive tracking control schemes is shown. An upper bound and the ultimate bound characterizing the global uniform ultimate boundedness of the tracking error are also given.

The paper is organized as follows. The problem and some definitions are presented in Section 2. In Section 3, an adaptive law and two switching laws based on mode-dependent dwell time and mode-mode-dependent dwell time are explained. In Section 4, stability results of the closed-loop system are given. In Section 5, a practical example of highly maneuverable aircraft technology is used to illustrate the proposed control schemes. The paper is concluded with Section 6.

Notations: The notations used in this paper are as follows: $\mathbb{R}$ and $\mathbb{N}^+$ represent the set of real numbers and positive natural numbers, respectively. For a symmetric matrix $P > 0$ means P is positive definite. In addition, the superscript T represents the transpose of matrix. The operator $\mathrm{tr}(\cdot)$ represents the trace of a matrix. The notation $\|\cdot\|$ represents the Euclidean norm. The identity matrix with dimension n is denoted with $I_{n\times n}$. The notation $\Omega = \{1,2,\cdots,N\}$ represents the set of subsystems and N is the number of subsystems.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the uncertain switched linear system described by:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + d(t), \quad \sigma(t) \in \Omega \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control input, σ is the switching signal, and $d \in \mathbb{R}^n$ is a bounded disturbance with known upper bound $\overline{d}$. We say that a subsystem $p \in \Omega$ is uncertain when the entries of the matrices $A_p \in \mathbb{R}^{n\times n}$ and $B_p \in \mathbb{R}^{n\times m}$ are unknown.

A group of switched reference models representing the desired behavior of each subsystem is given as follows:

$$\dot{x}_{\mathrm{m}}(t) = A_{\mathrm{m}\sigma(t)} x_{\mathrm{m}}(t) + B_{\mathrm{m}\sigma(t)} r(t), \quad \sigma(t) \in \Omega \tag{2}$$

where $x_{\mathrm{m}} \in \mathbb{R}^n$ is the desired state vector, and $r \in \mathbb{R}^m$ is a bounded reference input. The matrices $A_{\mathrm{m}p} \in \mathbb{R}^{n\times n}$ and $B_{\mathrm{m}p} \in \mathbb{R}^{n\times m}$ are known and $A_{\mathrm{m}p}$, $p \in \Omega$, are Hurwitz matrices. The state-feedback mode-dependent control that makes the switched systems behave like the reference models is $u(t) = K^{*T}_{\sigma(t)}(t)x(t) + L^*_{\sigma(t)}(t)r(t)$, where $K^*_p \in \mathbb{R}^{n\times m}$ and $L^*_p \in \mathbb{R}^{m\times m}$, $p \in \Omega$, are nominal parameters satisfying the following matching conditions:

$$A_p + B_p K^{*T}_p = A_{\mathrm{m}p}, \quad B_p L^*_p = B_{\mathrm{m}p}. \tag{3}$$

Since we cannot obtain K^*_p and L^*_p from (3) with unknown A_p and B_p, the following mode-dependent controller is introduced:

$$u(t) = K^T_{\sigma(t)}(t)x(t) + L_{\sigma(t)}(t)r(t) \tag{4}$$

where K_p and L_p are the estimates of K^*_p and L^*_p, $p \in \Omega$, respectively. The tracking error is defined as $e(t) = x(t) - x_{\mathrm{m}}(t)$. Substituting (4) into (1) and subtracting (2) leads to the following dynamics of the tracking error:

$$\dot{e}(t) = A_{\mathrm{m}\sigma(t)} e(t) + B_{\sigma(t)}(\tilde{K}^T_{\sigma(t)}(t)x(t) + \tilde{L}_{\sigma(t)}(t)r(t)) + d(t) \tag{5}$$

where $\tilde{K}_p = K_p - K^*_p$ and $\tilde{L}_p = L_p - L^*_p$ are the parameter estimation errors.

We use the notation $\{(\sigma(t_0),\, t_0), (\sigma(t_1),\, t_1) \cdots\, (\sigma(t_l),\, t_l),\, \cdots \mid l \in \mathbb{N}^+\}$ to represent the set of mode-switching instant pairs. The sequence of switch-in instants of subsystem p, $p \in \Omega$, is given as: $\{t_{p_1},\, t_{p_2},\, \cdots t_{p_l}, \cdots \mid l \in \mathbb{N}^+\}$, and the sequence of switch-out instants of subsystem p, $p \in \Omega$, is given as: $\{t_{p_1+1},\, t_{p_2+1},\, \cdots t_{p_l+1}, \cdots \mid l \in \mathbb{N}^+\}$. The following preliminary definitions are given. First, we define switching signals based on extended dwell time.

Definition 1 (Mode-dependent dwell time). [27] A switching signal is said to be *admissible with mode-dependent dwell time* if there exists a number $\tau_p > 0$ for $p \in \Omega$ such that the constraint $t_{p_l+1} - t_{p_l} \geq \tau_p$ holds for all $l \in \mathbb{N}^+$. Any positive number τ_p for which this constraint holds is called *mode-dependent dwell time*.

Definition 2 (Mode-mode-dependent dwell time). The switching signal $\sigma(\cdot)$ is said to have *mode-mode-dependent dwell time* (MMDDT) if there exist positive numbers τ_{pq} such that $t_{p_l+1} - t_{p_l} \geq \tau_{pq}$ with $\sigma(t_{p_l}) = p$ and $\sigma(t_{p_l+1}) = q$, $\forall l \in \mathbb{N}^+$. Furthermore, we indicate the fact that the next mode to be switched on after p is q with $\mathcal{N}(p) = q$. The MMDDT switching law is defined for every p, q such that $\mathcal{N}(p) = q$.

Secondly, we characterize the type of stability sought in this work.

Definition 3 (Global uniform ultimate stability). The uncertain switched system (1) under switching signal $\sigma(\cdot)$ is *globally uniformly ultimately bounded* if there exists a convex and compact set $\mathcal{C}$ such that for every initial condition $x(0) = x_0$, there exists a finite $T(x_0)$ such that $x(t) \in \mathcal{C}$ for all $t \geq T(x_0)$.

Definition 4 (Ultimate bound). A signal $\phi(\cdot)$ is said to be globally uniformly ultimately bounded with *ultimate bound b* if there exists a positive constant b, and for any $a \geq 0$, there exists $T = T(a,b)$, where b and T are independent of t_0, such that $\|\phi(t_0)\| \leq a \Rightarrow \|\phi(t)\| \leq b, \quad \forall t \geq t_0 + T$.

Given these definition, the control objective for the uncertain switched linear system (1) can be formulated as:

Problem 1. Develop an adaptive law for the control parameters in (4) and a switching law based on extended dwell time that, without requiring the knowledge of the nominal values of A_p and B_p, $\forall p \in \Omega$, assures the global uniform ultimate stability of all signals in the switched system (1) with controller (4).

3. Methodology

In this section, an adaptive law and two switching laws are proposed to solve *Problem 1*.

3.1. Adaptive control

Before introducing the adaptive law, an assumption is required: there exists a family of matrices $S_p \in \mathbb{R}^{m\times m}$, $p \in \Omega$, such that $M_p := L_p^* S_p = \left(L_p^* S_p\right)^T = S_p^T L_p^{*T} > 0, \forall p \in \Omega$. This assumption, which is adopted for adaptive control of multiple-input non-switched systems [33], is analogous to knowing the sign of L^* in the signal-input case. Let p denote the index of the subsystem active for $t \in [t_l, t_{l+1})$. Suppose that there exists a matrix $P_p > 0$ and a constant $\kappa_p > 0$ such that

$$A_{\mathrm{mp}}^T P_p + P_p A_{\mathrm{mp}} + (1+\kappa_p)P_p \leq 0. \tag{6}$$

Then, the following adaptive law using a leakage approach is introduced, for $t \in [t_l, t_{l+1})$

$$\dot{K}_p^T(t) = -S_p^T B_{\mathrm{mp}}^T P_p e(t) x^T(t) - \delta_p M_p K_p^T(t) \tag{7a}$$
$$\dot{L}_p(t) = -S_p^T B_{\mathrm{mp}}^T P_p e(t) r^T(t) - \delta_p M_p L_p(t) \tag{7b}$$
$$\dot{K}_q^T(t) = -\delta_q M_q K_q^T(t) \tag{7c}$$
$$\dot{L}_q(t) = -\delta_q M_q L_q(t) \tag{7d}$$
$$q = 1, \dots, p-1, p+1, \dots, N$$

where the leakage rate δ_p should satisfy the following condition:

$$\delta_p - \max_{p\in\Omega}\left\{\kappa_p\right\}\lambda_{\max}(M_p^{-1}) \geq 0. \tag{8}$$

Remark 1. The difference between the adaptive law (7) and the laws proposed in literature [23, 24, 25, 26] consists in the following two aspects. Firstly, the knowledge of the uncertainty set where the nominal control parameters reside is not required in (7) thanks to the leakage approach. Secondly, the parameter estimates of each subsystem are continuously updated even when the corresponding mode is inactive: two updating laws are exploited depending on status of the subsystem, i.e., active (update law (7a)–(7b)), or inactive (update law (7c)–(7d)).

3.2. Switching Laws

In this section, two switching laws are proposed based on the MDDT and MMDDT strategies, respectively. We denote with $\overline{\lambda}_p$ and $\underline{\lambda}_p$ the largest and the smallest eigenvalue of P_p, respectively, and we define $\kappa_{\max} = \max_{p\in\Omega}\kappa_p$, $\alpha = \max_{p\in\Omega}\overline{\lambda}_p$ and $\beta = \min_{p\in\Omega}\underline{\lambda}_p$. We propose a switching law based on the following MDDT:

$$\tau_p > \tau_p^* = \frac{1}{\xi\kappa_p}\ln\mu_p, \quad \forall p \in \Omega \tag{9}$$

where $\mu_p = \alpha/\underline{\lambda}_p$ and $\xi \in (0,\ 1)$ is a design positive constant.

For the scenario when the next subsystem q to be switched on after subsystem p is known, a different switching law than (9) is proposed with the following MMDDT:

$$\tau_{pq} > \tau_{pq}^* = \frac{1}{\xi\kappa_p}\ln\mu_{pq}, \quad \forall p \in \Omega,\ q = \mathcal{N}(p) \tag{10}$$

where $\mu_{pq} = \overline{\lambda}_q/\underline{\lambda}_p$ and $\xi \in (0,\ 1)$ is a design positive constant. Note that, when the switching sequence is given, the MMDDT (10) represents a larger class of switching signals than (9), for which GUUB of the closed-loop switched system can be guaranteed.

Remark 2. When a prespecified switching sequence is known in advance, optimal control has been investigated to find the optimal switching instants and the optimal controller [29, 34]. However, the methods in [29, 34] are not applicable when the switched system is uncertain. In light of this, the MMDDT switching law is presented to deal with the scenario when the switching sequence is known. Comparing with (9) and (10), it can be observed that MMDDT can allow faster switching than MDDT by exploiting the information of the next subsystem to be switched on.

4. Main result

In this section, the robust stability results deriving from the adaptive law (7)–(8) and switching laws (9)–(10) are presented.

The following lemma will be exploited to prove the stability results.

Lemma 1. *[35] Let $\phi \in \mathbb{R}^g$, $\varphi \in \mathbb{R}^s$ be vector-valued signals, and let $W \in \mathbb{R}^{g\times g}$, $V \in \mathbb{R}^{g\times s}$ be constant matrices. Then, the following inequality holds: $2\phi^T WV\varphi \leq \phi^T WW^T\phi + \varphi^T V^T V\varphi$.*

Theorem 1. *With the adaptive law* (7)–(8) *and the switching law with MDDT* (9)*, the GUUB stability of the unknown switched system* (1) *with controller* (4) *can be guaranteed. In addition, the tracking error is bounded as:*

$$\|e(t)\|^2 \leq \frac{1}{\beta^2}\max\left\{c_1, \frac{\alpha\left(c_2+\alpha\|\bar{d}\|^2\right)}{(1-\xi)\max_{p\in\Omega}\kappa_p}\right\} \tag{11}$$

where c_1 and c_2 are two positive constants that depend on the initial estimates and on the actual values of the controller parameters. In addition, the tracking error is GUUB with an ultimate bound b in the interval:

$$b \in \left[0,\ \frac{1}{\beta}\sqrt{\frac{\alpha\left(c_2+\alpha\|\bar{d}\|^2\right)}{(1-\xi)\max_{p\in\Omega}\kappa_p}}\right]. \tag{12}$$

Proof. The proof is organized as follows: we adopt a Lyapunov function which is quadratic in the tracking error and estimation errors. The behavior of the Lyapunov function is studied with the proposed adaptive law (7)–(8) and switching law (9). It is shown that the Lyapunov function is decreasing at an exponential rate when the value of Lyapunov function is located outside of a bound. Then, it is proven that there exists a finite bound such that after some time the Lyapunov function will stay inside the bound, which implies that the closed-loop system is GUUB. The following Lyapunov function is adopted:

$$V(t) = e^T(t)P_{\sigma(t)}e(t) + \sum_{p=1}^{N}\mathrm{tr}\left[\tilde{K}_p(t)M_p^{-1}\tilde{K}_p^T(t)\right] + \sum_{p=1}^{N}\mathrm{tr}\left[\tilde{L}_p^T(t)M_p^{-1}\tilde{L}_p(t)\right]. \tag{13}$$

In general, P_p is different for different subsystems, which indicates that $V(\cdot)$ might be continuous w.r.t. time only in the intervals between two consecutive switches. In light of this, to investigate the behavior of $V(\cdot)$, first, we need to establish the characteristics of $V(\cdot)$ at the discontinuous instants. Without loss of generality, we study the Lyapunov function at the switching instant t_{l+1}, $l \in \mathbb{N}^+$. Subsystem $\sigma(t_{l+1}^-)$ is active when $t \in [t_l, t_{l+1})$ and subsystem $\sigma(t_{l+1})$ is active when $t \in [t_{l+1}, t_{l+2})$.

At the switching instant t_{l+1}, we have before switching

$$V(t_{l+1}^-) = e^T(t_{l+1}^-)P_{\sigma(t_{l+1}^-)}e(t_{l+1}^-) + \sum_{p=1}^{N}\mathrm{tr}\left[\tilde{K}_p(t_{l+1}^-)M_p^{-1}\tilde{K}_p^T(t_{l+1}^-)\right] + \sum_{p=1}^{N}\mathrm{tr}\left[\tilde{L}_p^T(t_{l+1}^-)M_p^{-1}\tilde{L}_p(t_{l+1}^-)\right]$$

and after switching

$$V(t_{l+1}) = e^T(t_{l+1})P_{t_{l+1}}e(t_{l+1}) + \sum_{p=1}^{N}\mathrm{tr}\left[\tilde{K}_p(t_{l+1})M_p^{-1}\tilde{K}_p^T(t_{l+1})\right] + \sum_{p=1}^{N}\mathrm{tr}\left[\tilde{L}_p^T(t_{l+1})M_p^{-1}\tilde{L}_p(t_{l+1})\right].$$

According to the continuity of the tracking error $e(\cdot)$ in (5) and the continuity of the parameter estimates updated via (7), we have $e(t_{l+1}) = e(t_{l+1}^-)$, $\tilde{K}_p(t_{l+1}) = \tilde{K}_p(t_{l+1}^-)$, and $\tilde{L}_p(t_{l+1}) = \tilde{L}_p(t_{l+1}^-)$ for any switching law. Due to $e^T(t)P_{\sigma(t)}e(t) \leq \alpha e^T(t)e(t)$ and $e^T(t)P_{\sigma(t)}e(t) \geq \underline{\lambda}_{\sigma(t)}e^T(t)e(t)$, we have $V(t_{l+1}) - V(t_{l+1}^-) = e^T(t_{l+1})(P_{\sigma(t_{l+1})} - P_{\sigma(t_{l+1}^-)})e(t_{l+1}) \leq (\alpha - \underline{\lambda}_{\sigma(t_{l+1}^-)})/\underline{\lambda}_{\sigma(t_{l+1}^-)} \cdot e^T(t_{l+1})P_{\sigma(t_{l+1}^-)}e(t_{l+1}) \leq (\alpha - \underline{\lambda}_{\sigma(t_{l+1}^-)})/\underline{\lambda}_{\sigma(t_{l+1}^-)} \cdot V(t_{l+1}^-)$. Then, the following relationship of $V(t)$ at the switching instant t_{l+1} can be established:

$$V(t_{l+1}) \leq \mu_{\sigma(t_{l+1}^-)}V(t_{l+1}^-) \tag{14}$$

with $\mu_{\sigma(t_{l+1}^-)} = \alpha/\underline{\lambda}_{\sigma(t_{l+1}^-)}$, which is a positive constant no smaller than 1. Moreover, the dynamics of $V(t)$ between two consecutive switching instants is studied. When $t \in [t_l,\, t_{l+1})$, the derivative of $V(t)$ w.r.t. time according to (5) and (7) is:

$$
\begin{aligned}
\dot{V}(t) = & e^T(t)(A^T_{\mathrm{m}\sigma(t_{l+1}^-)} P_{\sigma(t_{l+1}^-)} + P_{\sigma(t_{l+1}^-)} A_{\mathrm{m}\sigma(t_{l+1}^-)})e(t) - 2\sum_{p=1}^{N} \mathrm{tr}\left[\tilde{K}_p(t)\delta_p K_p^T(t)\right] - 2\sum_{p=1}^{N} \mathrm{tr}\left[\tilde{L}_p^T(t)\delta_p L_p(t)\right] \\
& + d^T(t)P_{\sigma(t_{l+1}^-)} e(t) + e^T(t) P_{\sigma(t_{l+1}^-)} d(t)
\end{aligned} \tag{15}
$$

Since $P_{\sigma(t_{l+1}^-)}$ is a positive definite matrix, there exists a nonsingular matrix $H_{\sigma(t_{l+1}^-)}$ such that $P_{\sigma(t_{l+1}^-)} = H_{\sigma(t_{l+1}^-)} H^T_{\sigma(t_{l+1}^-)}$. Then, (15) can be recast into

$$
\begin{aligned}
\dot{V}(t) = & e^T(t)(A^T_{\mathrm{m}\sigma(t_{l+1}^-)} P_{\sigma(t_{l+1}^-)} + P_{\sigma(t_{l+1}^-)} A_{\mathrm{m}\sigma(t_{l+1}^-)})e(t) - 2\sum_{p=1}^{N} \mathrm{tr}\left[\tilde{K}_p(t)\delta_p K_p^T(t)\right] - 2\sum_{p=1}^{N} \mathrm{tr}\left[\tilde{L}_p^T(t)\delta_p L_p(t)\right] \\
& + d^T(t)H_{\sigma(t_{l+1}^-)} H^T_{\sigma(t_{l+1}^-)} e(t) + e^T(t) H_{\sigma(t_{l+1}^-)} H^T_{\sigma(t_{l+1}^-)} d(t) \\
\leq & e^T(t)(A^T_{\mathrm{m}\sigma(t_{l+1}^-)} P_{\sigma(t_{l+1}^-)} + P_{\sigma(t_{l+1}^-)} A_{\mathrm{m}\sigma(t_{l+1}^-)})e(t) - 2\sum_{p=1}^{N} \mathrm{tr}\left[\tilde{K}_p(t)\delta_p K_p^T(t)\right] - 2\sum_{p=1}^{N} \mathrm{tr}\left[\tilde{L}_p^T(t)\delta_p L_p(t)\right] \\
& + e^T(t)P_{\sigma(t_{l+1}^-)} e(t) + d^T(t) P_{\sigma(t_{l+1}^-)} d(t) \\
\leq & -\kappa_{\sigma(t_{l+1}^-)} e^T(t) P_{\sigma(t_{l+1}^-)} e(t) - 2\sum_{p=1}^{N} \mathrm{tr}\left[\tilde{K}_p(t)\delta_p(\tilde{K}_p^T(t) + K_p^{*T})\right] - 2\sum_{p=1}^{N} \mathrm{tr}\left[\tilde{L}_p^T(t)\delta_p(\tilde{L}_p^T(t) + L_p^{*T})\right] \\
& + d^T(t)P_{\sigma(t_{l+1}^-)} d(t)
\end{aligned} \tag{16}
$$

where the first inequality holds according to Lemma 1, and the last inequality holds due to (6). Since $\mathrm{tr}\left[\tilde{K}_p K_p^{*T}\right] = \mathrm{tr}\left[K_p^{*T}\tilde{K}_p\right]$, we have $-2\,\mathrm{tr}\left[\tilde{K}_p\tilde{K}_p^T\right] - 2\,\mathrm{tr}\left[\tilde{K}_p K_p^{*T}\right] \leq -\mathrm{tr}\left[\tilde{K}_p\tilde{K}_p^T\right] + \mathrm{tr}\left[K_p^* K_p^{*T}\right]$. Hence, it follows from (16) that

$$
\begin{aligned}
\dot{V}(t) \leq & -\kappa_{\sigma(t_{l+1}^-)} e^T(t) P_{\sigma(t_{l+1}^-)} e(t) - \sum_{p=1}^{N} \mathrm{tr}\left[\delta_p \tilde{K}_p(t)\tilde{K}_p^T(t)\right] + \sum_{p=1}^{N} \mathrm{tr}\left[\delta_p K_p^* K_p^{*T}\right] - \sum_{p=1}^{N} \mathrm{tr}\left[\delta_p \tilde{L}_p^T(t)\tilde{L}_p^T(t)\right] \\
& + \sum_{p=1}^{N} \mathrm{tr}\left[\delta_p L_p^* L_p^{*T}\right] + d^T(t) P_{\sigma(t_{l+1}^-)} d(t) \\
\leq & -\kappa_{\sigma(t_{l+1}^-)} V(t) + \sum_{p=1}^{N} \mathrm{tr}\left[\tilde{K}_p(t)\tilde{K}_p^T(t)(\kappa_{\sigma(t_{l+1}^-)}\lambda_{\max}(M_p^{-1}) - \delta_p)\right] + \sum_{p=1}^{N} \mathrm{tr}\left[\delta_p K_p^* K_p^{*T}\right] \\
& + \sum_{p=1}^{N} \mathrm{tr}\left[\tilde{L}_p^T(t)\tilde{L}_p(t)(\kappa_{\sigma(t_{l+1}^-)}\lambda_{\max}(M_p^{-1}) - \delta_p)\right] + \sum_{p=1}^{N} \mathrm{tr}\left[\delta_p L_p^* L_p^{*T}\right] + d^T(t) P_{\sigma(t_{l+1}^-)} d(t).
\end{aligned} \tag{17}
$$

Using (8), we have for $t \in [t_l,\, t_{l+1})$, $\forall \xi \in (0,\ 1)$:

$$
\dot{V}(t) \leq -\xi\kappa_{\sigma(t_{l+1}^-)} V(t) - (1-\xi)\kappa_{\sigma(t_{l+1}^-)} V(t) + \sum_{p=1}^{N} \mathrm{tr}\left[\delta_p K_p^* K_p^{*T}\right] + \sum_{p=1}^{N} \mathrm{tr}\left[\delta_p L_p^* L_p^{*T}\right] + d^T(t) P_{\sigma(t_{l+1}^-)} d(t). \tag{18}
$$

We define a positive number $\mathcal{B}$ as

$$
\mathcal{B} = \frac{\alpha\|\bar{d}\|^2 + c_2}{(1-\xi)\max_{p\in\Omega}\kappa_p} \tag{19}
$$

where $c_2 = \sum_{p=1}^{N} \mathrm{tr}\left[\delta_p K_p^* K_p^{*T} + \delta_p L_p^* L_p^{*T}\right]$. To analyze the behavior of the Lyapunov function during two consecutive switching instants, i.e., $t \in [t_l,\, t_{l+1})$, two possible scenarios should be taken into account:

- When $V(t) \geq \mathcal{B}$, it follows that $\dot{V}(t) \leq -\xi\kappa_{\sigma(t_{i+1}^-)} V(t)$, i.e., $V(t)$ is decreasing at an exponential rate.

- When $V(t) < \mathcal{B}$, it follows that $V(t)$ may be increasing.

In light of this, we consider two cases for the initial condition: $V(t_0) \geq \mathcal{B}$ (*case 1*); $V(t_0) < \mathcal{B}$ (*case 2*).

Case 1: we assume $V(t) \geq \mathcal{B}$ for $t \in [t_0,\, t_0 + T_1)$, where T_1 represents the time instant before $V(t)$ enters into the bound $\mathcal{B}$. This means that $V(t)$ is decreasing at an exponential rate for $t \in [t_0,\, t_0 + T_1)$. Denote the number of all switching intervals by N_1 in the whole time interval $[t_0,\, t_0 + T_1)$. Denote the number of intervals that subsystem p, $p \in \Omega$, is active by $N_{1p}(t)$, and the number of all switching intervals by $N_1(t)$ in the time interval $[t_0,\, t]$ for $t \in [t_0, t_0 + T_1)$. Therefore, when $t \in [t_0, t_0 + T_1)$, it follows from (14) and (18) that

$$
\begin{aligned}
V(t) &\leq V(t_{N_1(t)}) \\
&\leq \mu_{\sigma(t^-_{N_1(t)-1})} \exp\left(-\xi \kappa_{\sigma(t_{N_1(t)})}\left(t_{N_1(t)} - t_{N_1(t)-1}\right)\right) V(t_{N_1(t)-1}) \\
&\leq \mu_{\sigma(t^-_{N_1(t)-1})} \exp\left(-\kappa_{\sigma(t_{N_1(t)})}\left(t_{N_1(t)} - t_{N_1(t)-1}\right)\right) \mu_{\sigma(t^-_{N_1(t)-2})} \exp\left(-\xi \kappa_{\sigma(t_{N_1(t)-1})}\left(t_{N_1(t)-1} - t_{N_1(t)-2}\right)\right) V(t_{N_1(t)-2}) \\
&\;\;\vdots \\
&\leq \mu_{\sigma(t^-_{N_1(t)-1})} \exp\left(-\xi \kappa_{\sigma(t_{N_1(t)})}\left(t_{N_1(t)} - t_{N_1(t)-1}\right)\right) \mu_{\sigma(t^-_{N_1(t)-2})} \exp\left(-\xi \kappa_{\sigma(t_{N_1(t)-1})}\left(t_{N_1(t)-1} - t_{N_1(t)-2}\right)\right) \cdots \\
&\quad\; \mu_{\sigma(t_0)} \exp\left(-\xi \kappa_{\sigma(t_0)}(t_1 - t_0)\right) V(t_0) \\
&\leq \prod_{p=1}^{N} \mu_p^{N_{1p}(t)} \exp\left(-\sum_{p=1}^{N} N_{1p}(t) \xi \kappa_p (t_{p_l+1} - t_{p_l})\right) V(t_0)
\end{aligned}
\tag{20}
$$

for all $l \in \{0,\, 1,\, 2,\, \cdots,\, N_{1p}(t)\}$. Substituting the MDDT condition $\tau_p = t_{p_l+1} - t_{p_l} > \ln \mu_p / \xi \kappa_p$ into (20) gives rise to $V(t) < V(t_0)$ for $t \in [t_0, t_0 + T_1)$. Moreover, since $V(t_0 + T_1) < \mathcal{B}$, according to (14) we have that $V(t_{N_1+1}) < \mu_{\sigma(t^-_{N_1+1})}\mathcal{B}$ at the next switching instant t_{N_1+1} after $t_0 + T_1$. This means that $V(t)$ may be no smaller than $\mathcal{B}$ after the instant t_{N_1+1}. In view of this, similarly, we assume $V(t) \geq \mathcal{B}$ when $t \in [t_{N_1+1}, t_0 + T_2)$. Denote the number of all switching intervals in the interval $[t_{N_1+1},\, t_0 + T_2)$ by N_2 . Then, substituting $V(t_0)$ with $V(t_{N_1+1})$ in (20), based on similar proof lines as (20), we have $V(t) < V(t_{N_1+1})$ for $t \in [t_{N_1+1},\, t_0 + T_2)$. Due to $V(t_0 + T_2) < \mathcal{B}$, according to (14) we have that $V(t_{N_1+N_2+2}) < \mu_{\sigma(t^-_{N_1+t_{N_2}+2})}\mathcal{B}$ at the next switching instant $t_{N_1+t_{N_2}+2}$ after $t_0 + T_2$. Using a similar analysis recursively, we can conclude that $V(t) \leq \mathcal{B}\alpha/\beta$ for $t \in [t_0 + T_1, \infty)$. This implies that once $V(t)$ enters the interval $[0,\, \mathcal{B}]$, it cannot exceed the bound $\mathcal{B}\alpha/\beta$ for any time later with MDDT (9). According to the definition of GUUB, the switched linear system (1) with controller (4) is GUUB with the adaptive law (7)–(8) and the switching law (9).

Next, we study the dynamics of the tracking error. Based on the aforementioned analysis about GUUB, it can be obtained that

$$
V(t) \leq \max\left\{V(t_0), \frac{\alpha}{\beta}\mathcal{B}\right\}, \quad \forall t > t_0. \tag{21}
$$

In addition, the fact that $e^T(t)P_{\sigma(t)}e(t) \geq \beta\|e(t)\|^2$ leads to

$$
\begin{aligned}
V(t) &= e^T(t)P_{\sigma(t)}e(t) + \sum_{p=1}^{N} \operatorname{tr}\left[\tilde{K}_p(t)M_p^{-1}\tilde{K}_p^T(t)\right] + \sum_{p=1}^{N} \operatorname{tr}\left[\tilde{L}_p^T(t)M_p^{-1}\tilde{L}_p(t)\right] \\
&\geq \beta\|e(t)\|^2.
\end{aligned}
\tag{22}
$$

Then, it follows from (21)–(22) that the tracking error is upper bounded in the following form

$$
\|e(t)\|^2 \leq \frac{1}{\beta} \max\left\{c_1,\, \frac{\alpha\left(\alpha\|\bar{d}\|^2 + c_2\right)}{\beta(1-\xi)\max_{p\in\Omega}\kappa_p}\right\} \tag{23}
$$

with $c_1 = V(t_0)$, and the tracking error is GUUB with an ultimate bound b with:

$$
b \in \left[0,\, \frac{1}{\beta}\sqrt{\frac{\alpha\left(\alpha\|\bar{d}\|^2 + c_2\right)}{(1-\xi)\max_{p\in\Omega}\kappa_p}}\right]. \tag{24}
$$

Case 2: The same results (23) and (24) can be obtained following the proof lines from (20) to (22). This completes the proof. ■

Remark 3. The upper bound (11) and ultimate bound (12) of the tracking error indicate that the proposed methods prevent the tracking error in the closed-loop switched system from growing large over short switching intervals. The numerical example in Section 5 will show that a large overshoot can occur with MDADT when the interval between two consecutive switches is very short.

Remark 4. A quasi-time-dependent quadratic Lyapunov function was proposed recently in [36, 37], which has been shown to provide less conservative switching laws than classical Lyapunov functions for known switched systems. This could be a potentially useful tool to solve adaptive control of uncertain switched linear systems. Future work will be focusing on this direction.

For the case when the switching sequence is known, the following result is introduced.

Theorem 2. *With the adaptive law* (7)–(8) *and the switching law with MMDDT* (10)*, the GUUB stability of the unknown switched system* (1) *with controller* (4) *can be guaranteed. In addition, the tracking error is bounded as:*

$$\|e(t)\|^2 \leq \frac{1}{\beta}\max\left\{c_1,\ \max_{\substack{p,\ q\in\Omega\\ \mathcal{N}(p)=q}}\{\mu_{pq}\}\cdot\frac{(c_2+\alpha\|\bar{d}\|^2)}{\beta(1-\xi)\max_{p\in\Omega}\kappa_p}\right\} \tag{25}$$

where c_1 and c_2 are the same positive constants as in Theorem 1. In addition, the tracking error is GUUB with an ultimate bound b with:

$$b \in \left[0,\ \sqrt{\max_{\substack{p,\ q\in\Omega\\ \mathcal{N}(p)=q}}\{\mu_{pq}\}\cdot\frac{(c_2+\alpha\|\bar{d}\|^2)}{\beta(1-\xi)\max_{p\in\Omega}\kappa_p}}\right]. \tag{26}$$

Proof. The proof is similar with the proof of Theorem 1. The same Lyapunov function as (13) is adopted. The main difference arises from the relationship of the values between the Lyapunov function at switching instant t_{l+1}, which is expressed as follows:

$$V(t_{l+1}) \leq \frac{\bar{\lambda}_{\sigma(t_{l+1})}}{\underline{\lambda}_{\sigma(t_{l+1}^-)}}V(t_{l+1}^-) =: \mu_{\sigma(t_{l+1})\sigma(t_{l+1}^-)}V(t_{l+1}^-).$$

The dynamics of the Lyapunov function during the switching interval is identical with (15)–(19). Since the switching sequence is known, the maximum increase of the Lyapunov function at the switching instants is $\max_{p,\ q\in\Omega,\mathcal{N}(p)=q}$ instead of α/β as in the MDDT case. The rest of the proof follows the lines from (20) to (24) after substituting $\mu_{\sigma(t_{l+1}^-)}$ with $\mu_{\sigma(t_{l+1})\sigma(t_{l+1}^-)}$. We conclude that the adaptive law (7)–(8) and the switching law with MMDDT (10) lead to GUUB stability with bounds (25) and (26). ■

Remark 5. Note that stable closed-loop switched system can automatically guarantee stability of the switched reference model (2). We provide a brief proof of stability of the switched reference model using MDDT switching law. A Lyapunov function $V = x_{\mathrm{m}}^T P_p x_{\mathrm{m}}$ is studied: the decreasing rate is upper bounded by $1+\kappa_p$ according to (6), and the increment at the switching instants is $\alpha/\underline{\lambda}_p$. Therefore, if the switching interval is larger than $\ln\left(\alpha/\underline{\lambda}_p\right)/(1+\kappa_p)$, then the switched reference model is exponentially stable. It is observed that the proposed switching law (9) has larger switching intervals than $\ln\left(\alpha/\underline{\lambda}_p\right)/(1+\kappa_p)$. In light of this, we can say that the switched reference model is stable based on MDDT or MMDDT switching laws.

Remark 6. If there exists a common positive definite matrix P satisfying (6) for all $A_{\mathrm{m}p}$ and $d(\cdot)\equiv 0$, the tracking error tends to zero asymptotically using the adaptive law (7) with $\delta_p = 0$ for all $p \in \Omega$ and with arbitrarily fast switching. The interested reader is referred to [23] for more details.

The following corollary to *Theorem 2* can be established.

Corollary 1. *Consider two consecutive switching instants t_{p_l} and t_{p_l+1}, $l \in \mathbb{N}^+$, with $\sigma(t_{p_l}) = p$ and $\sigma(t_{p_l+1}) = q$, $p \times q \in \Omega \times \Omega$. If $\mu_{pq} \leq 1$ in* (10)*, then the switching interval $t_{p_l+1} - t_{p_l}$ can be as small as desired, i.e., the closed-loop switched system is GUUB and the tracking error is upper bounded as* (25) *with ultimate bound b as* (26) *under arbitrarily fast switches of the subsystem p when $\mathcal{N}(p) = q$, where $\mathcal{N}(p)$ denotes the index of subsystem to be switched on after subsystem p.*

PROOF. Since $\mu_{pq} \leq 1$ in (10), it follows $V(t_{l+1}) \leq V(t_{l+1}^-)$ at the switching instant t_{l+1}, which indicates the energy defined by the Lyapunov function is decreasing at the switching instant t_{l+1}. Considering that the Lyapunov function is non-increasing in the interval between two consecutive switching instants, τ_{pq} is allowed to be arbitrarily small. Therefore, the closed-loop systems are GUUB with arbitrarily fast switches of the subsystem p when $\mathcal{N}(p) = q$.

Remark 7. Note that Corollary 1 does not guarantee asymptotic stability under arbitrarily fast switches, unless a common Lyapunov function exists as discussed in Remark 6. Consider two subsystems p and q, for which the condition $\mu_{pq} \leq 1$ is satisfied: the system can switch arbitrarily fast from p to q. On the other hand, if the switching signal at switching instant t_{p_l+1} switches from q to p, we have $\mu_{qp} \geq 1$, which leads to GUUB since the Lyapunov function may increase at switching instant t_{p_l+1}.

5. Example

In this section, a highly maneuverable aircraft technology (HiMAT) vehicle [38, 39, 40] is adopted to illustrate the proposed adaptive control method. The adaptive control approach is utilized to design a closed-loop controller and switching signals for the unstable longitudinal dynamics. The switched linear system is red$\dot{x} = A_p x + B_p u + d$, $p \in \Omega$, and considers the following three modes:

$$A_1 = \begin{bmatrix} -0.8435 & 0.97505 & -0.0048 \\ 8.7072 & -1.1643 & 0.0026 \\ 0 & 1 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.1299 & -0.092 & -0.0107 & -0.0827 \\ -7.6833 & -4.7974 & 4.8178 & -5.7416 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -1.8997 & 0.98312 & -0.00073 \\ 11.720 & -2.6316 & 0.00088 \\ 0 & 1 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.2436 & -0.1708 & -0.00497 & -0.1997 \\ -46.206 & -31.604 & 22.396 & -31.179 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -1.2206 & 0.99411 & -0.00084 \\ -64.071 & -1.8876 & 0.00046 \\ 0 & 1 & 0 \end{bmatrix}, \quad B_3 = \begin{bmatrix} -0.0662 & -0.0315 & -0.0141 & -0.0749 \\ -27.333 & -13.163 & 11.058 & -26.878 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

5.1. Design of reference model

Three LQR controllers $u = K_p^* x$ with $Q = \mathrm{diag}([1\ 1\ 5])$, $R = \mathrm{diag}([1\ 1\ 1\ 1])$ are adopted to design the reference model, i.e., $\dot{x}_\mathrm{m} = A_{\mathrm{m}p} x_\mathrm{m} + B_{\mathrm{m}p} r = (A_p + B_p K_p^*) x_\mathrm{m} + B_p r$, $p \in \Omega$. The nominal parameters and the system matrices of reference model are:

$$K_1^* = \begin{bmatrix} 0.6219 & 0.7469 & 1.4508 \\ 0.3969 & 0.4671 & 0.9013 \\ -0.3174 & -0.4621 & -0.9483 \\ 0.4534 & 0.5572 & 1.0902 \end{bmatrix}, \quad L_1^* = I_{4\times 4}, \quad A_{\mathrm{m}1} = \begin{bmatrix} -0.9949 & 0.7939 & -0.3562 \\ -2.1076 & -14.5691 & -26.2966 \\ 0 & 1 & 0 \end{bmatrix}$$

$$K_2^* = \begin{bmatrix} 0.1984 & 0.6793 & 1.5202 \\ 0.1368 & 0.4646 & 1.0392 \\ -0.0642 & -0.3289 & -0.7527 \\ 0.1431 & 0.4585 & 1.0212 \end{bmatrix}, \quad L_2^* = I_{4\times 4}, \quad A_{\mathrm{m}2} = \begin{bmatrix} -1.9997 & 0.6484 & -0.7487 \\ -7.6710 & -70.3615 & -151.7803 \\ 0 & 1 & 0 \end{bmatrix}$$

$$K_3^* = \begin{bmatrix} -0.6674 & 0.6397 & 1.4517 \\ -0.3220 & 0.3081 & 0.6995 \\ 0.3287 & -0.2599 & -0.6292 \\ -0.6423 & 0.6288 & 1.4175 \end{bmatrix}, \quad L_3^* = I_{4\times 4}, \quad A_{\mathrm{m}3} = \begin{bmatrix} -1.1228 & 0.8986 & -0.2163 \\ -20.6916 & -43.2036 & -93.9421 \\ 0 & 1 & 0 \end{bmatrix}.$$

5.2. Robust adaptive control

Let $\kappa_1 = 0.25, \kappa_2 = 0.5, \kappa_3 = 0.4$. Solving (6) gives rise to the following positive definite matrices:

$$P_1 = \begin{bmatrix} 0.7337 & -0.0162 & -0.3781 \\ -0.0162 & 0.0549 & 0.0800 \\ -0.3781 & 0.0800 & 2.3960 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.5225 & -0.0028 & -0.0517 \\ -0.0028 & 0.0092 & 0.0132 \\ -0.0517 & 0.0132 & 1.9764 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0.7942 & -0.0063 & -0.3177 \\ -0.0063 & 0.0167 & 0.0241 \\ -0.3177 & 0.0241 & 2.4767 \end{bmatrix}.$$

Then, the bounds of DT, MDDT, MMDDT are obtained as shown in Table 1, which shows that a bigger class of switching signals based on MDDT is obtained than the class of switching signals based on DT. Moreover, when the switching sequence is known, MMDDT leads to even less conservative switching signals than MDDT and DT. We

Table 1: Comparison of three switching laws

Switching strategies	**DT**	**MDDT**	**MMDDT**
Switching sequences	Unknown	Unknown	Known in advance
Switching signals	$\tau_D^* = 23.7$ $\mu = 278.3$ $\kappa = 0.25$	$\tau_1^* = 16.3$, $\tau_2^* = 11.8$, $\tau_3^* = 13.2$ $\mu_1 = 48.6$, $\mu_2 = 278.3$, $\mu_3 = 154.1$ $\kappa_1 = 0.25$, $\kappa_2 = 0.5$, $\kappa_3 = 0.4$	$\tau_{13}^* = 16.3$, $\tau_{32}^* = 12.6$ $\tau_{21}^* = 11.8$, $\tau_{23}^* = 10$ $\mu_{13} = 48.6$, $\mu_{32} = 120.3$, $\mu_{21} = 272.4$ $\kappa_1 = 0.25$, $\kappa_2 = 0.5$, $\kappa_3 = 0.4$

design switching signals based on DT, MDADT, MDDT, and MMDDT as shown in Fig.1–4, respectively. Consider the adaptive gains $S_1 = S_2 = S_3 = 10I_{4\times4}$, the leakage rates $\delta_1 = \delta_2 = \delta_3 = 0.05$, the initial conditions $x(0) = [0\ 0\ 0]^T$, $x_{\mathrm{m}} = [2\ 2\ 1]^T$, $K_p(0) = 0.8K_p^*$, $L_p(0) = 0.8L_p^*$, the disturbance $d(t) = [0.2\sin(10t)\ 0.15e^{-t}\ 0.1\cos(\pi t)]^T$, and the reference input $r(t) = [2\sin(t)\ \cos(t)\ 0.5\sin(0.5t)\ 0]^T$. The tracking errors based on the four switching signals are shown in Fig. 5–8, respectively. It can be observed that the tracking errors are upper bounded, and an ultimate bound is 0.8, which is verified by the results in this work. Moreover, comparing Fig. 5 and Fig. 6, the fast switchings of MDADT negatively impact the transient performance of the tracking error.

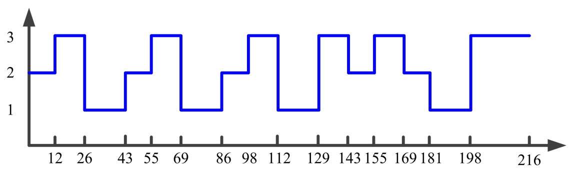

Figure 1: Switching signal based on MDDT

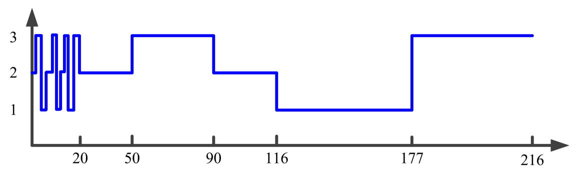

Figure 2: Switching signal based on MDADT

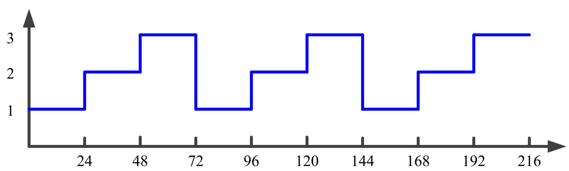

Figure 3: Switching signal based on DT

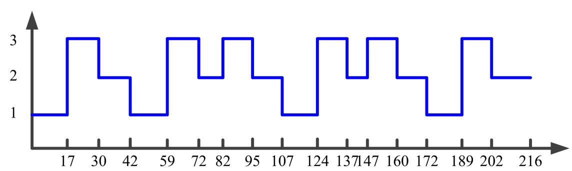

Figure 4: Switching signal based on MMDDT

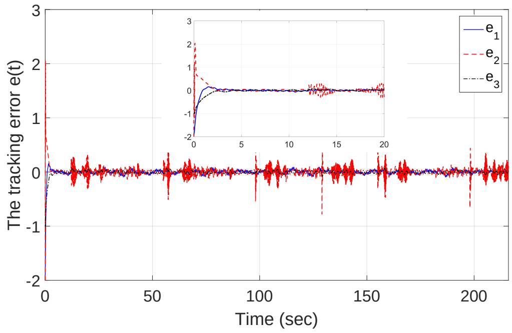

Figure 5: The tracking error based on MDDT with a enlarged detail in [0,20]

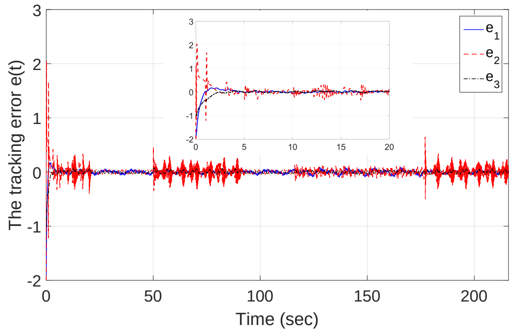

Figure 6: The tracking error based on MDADT with a enlarged detail in [0,20]

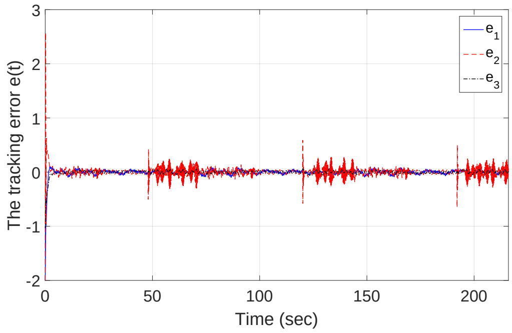

Figure 7: The tracking error based on DT

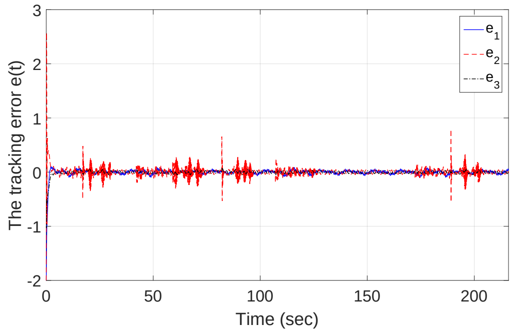

Figure 8: The tracking error based on MMDDT

6. Conclusion

In this paper, a robust adaptive tracking control problem for uncertain switched systems subject to disturbances has been studied. An adaptive law with leakage approach has been proposed to overcome the assumption, adopted till now in literature, of knowing the bounds of the uncertainty set. Moreover, switching laws based on the mode-dependent dwell time and the mode-mode-dependent dwell time have been developed, which can allow faster switching as compared to switching laws based on the dwell time. Global uniform ultimate boundedness of the closed-loop switched system based on the proposed methods can be guaranteed. The upper bound and the ultimate bound of the tracking error have been derived. Finally, an example of highly maneuverable aircraft technology has demonstrated the effectiveness of the proposed robust adaptive tracking control methods.

Acknowledgement

This work was supported in part by China Scholarship Council (CSC), File No. 20146160098.

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