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Discrete Stochastic Modelling of ATM-Traffic with Circulant Transition Matrices: A Time Domain Approach

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Abstract

In this report a new fast time domain approach for the identification of ATM-traffic is proposed. The traffic is measured and characterised by its first and second order statistic moments. A Markov Modulated Poisson Process (MMPP) is used to capture the information in these two statistic moments.

Since the identification of a general MMPP is time consuming because of the large computational requirements, a *circulant MMPP* is used to reduce the computational cost. A circulant MMPP is an MMPP with a circulant transition matrix.

The main advantages of this approach are the avoidance of inverse eigenvalue problem and the decoupling of the two statistic moments. Since ATM-traffic is highly correlated one can expect slowly decaying autocorrelations, which slows down the time domain identification. Therefore the autocorrelation is rewritten as a sum of exponentials using subspace-identification for stochastic linear time invariant systems. The identification of the second order statistics is decoupled from the first order statistics and uses 0/1 knapsack solvers and unconstrained optimisation.

Keywords: Markov modulated Poisson process, mixture distributions, Asynchronous Transfer Mode, broadband ISDN traffic identification

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1 Introduction

Asynchronous Transfer Mode (ATM) is a protocol for packet switched broadband ISDN networks. Its main characteristic is the combination of the advantages of the classic circuit mode and packet mode traffic. While circuit mode traffic offers high bit rates at the expense of working with fixed bandwidths and packet mode traffic uses bandwidth when necessary at rather low transmission rates, ATM offers both high bit rates and efficient use of available bandwidth. Therefore ATM uses the principle of *statistical multiplexing*, which is very efficient for variable bit rate applications: it is assumed that not every user uses his maximal assigned bit rate, so that more clients can be allocated to a channel than can be processed when every user consumes his maximum bit rate [1].

Buffers are placed in the nodes of the ATM-network to absorb the largest part of the fluctuations in the traffic. Figure 1 gives a schematic representation of a server with finite buffer length. The aggregated arrival pattern $\{a_k\}_{k=1}^{\infty}$ is the sum of the arrival patterns of the individual clients. Each client demands a certain negotiated level of *Quality of Service* (QoS), e.g. Cell Loss Ratio, Cell Variation Delay Tolerance, Peak Cell Rate, Minimum Cell Rate. The following question is important: given the currently processed traffic, is it possible to allocate a new client to the channel so that the new client obtains his level of QoS, without loss of the previously negotiated QoS of the other clients? The new client will certainly increase the load on the server and the QoS will certainly decrease.

The amount of this decline is calculated with *queueing theory*. This could be done by simulation, but this approach is rejected because of the computational requirements. To obtain a swift Connection Admission Control (CAC), a better approach is to use a *mathematical model* of the compound arrival pattern and use this model in queueing analysis. Figure 2 gives an overview how the identification procedure fits in the general framework of CAC. In this report a new time domain approach is proposed for the identification of such a mathematical model.

2 Mathematical background

The main purpose of the mathematical model is to increase the speed of the queueing analysis. Since queueing analysis cannot be performed on arbitrary model structures, there are strong restrictions on the model structure to use, so that recently developed and accurate identification algorithms in traditional system identification cannot be used straightforwardly [2].

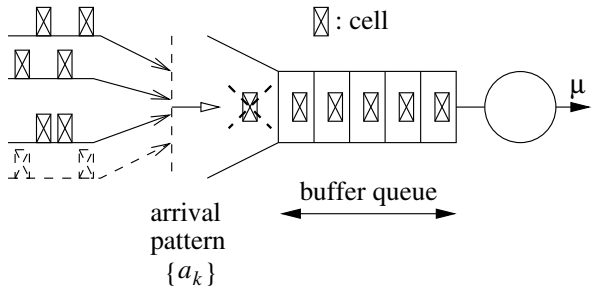

Figure 1: Schematic representation of a server with finite buffer length. The traffic is composed of the traffic of the individual clients. Most of the fluctuations are absorbed in the buffer. No cells are lost as long as the buffer size exceeds the length of the queue. This length and the losses are calculated by queueing analysis and depend on the traffic and the server rate μ.

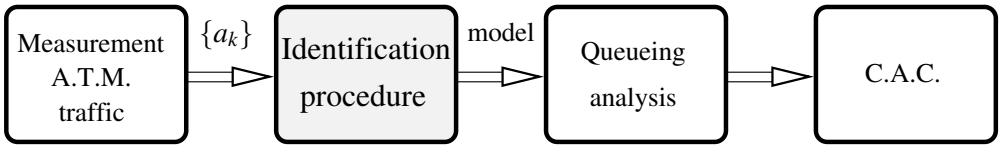

Figure 2: Representation of the identification procedure in the general framework of Connection Admission Control (CAC): first the ATM-traffic is measured, then the mathematical model is identified to perform the queueing analysis, which is used for the CAC. This report focuses on the identification of the mathematical model.

2.1 Model choice

In [3] it is illustrated that the most important features of the traffic are the second and first order statistic moments (autocorrelation and probability distribution function). *Markov Modulated Poisson Processes* (MMPP) have the sufficient flexibility to capture these two moments.

A general MMPP has two parameters: a transition matrix P and a Poisson parameter vector λ. The element $\lambda_i \geq 0$ of λ is the Poisson parameter of the Poisson process associated with state i of the Markov chain. The transition matrix P belongs to $\mathbb{P}^{N\times N}$, the set of all stochastic matrices of dimension N so that $\sum_{i=1}^{N} p_{ij} = 1$. The element p_{ij} of P stands for the conditional transition probability from state i to state j. If $s_i(k)$ denotes the probability that the Markov chain is in state i at time k, then:

$$\begin{bmatrix} s_1(k+1) & \dots & s_i(k+1) & \dots & s_N(k+1) \end{bmatrix} = \begin{bmatrix} s_1(k) & \dots & s_i(k) & \dots & s_N(k) \end{bmatrix} \begin{bmatrix} p_{11} & \dots & p_{1i} & \dots & p_{1n} \\ p_{21} & \ddots & \ddots & \vdots & \vdots \\ \vdots & & & & \vdots \\ p_{n1} & \dots & p_{ni} & \dots & p_{nn} \end{bmatrix}.$$

In steady state, the probabilities of the different states will not change any more. This state probability is given by $s(\infty) \equiv \pi$, such that $\pi = \pi \cdot P$. This means that the vector π^T is the left eigenvector of P associated with the eigenvalue 1.

Figure 3 represents an MMPP of order 4. It consists of a stochastic transition matrix P and a vector λ with the Poisson parameters. P describes the transition probabilities between the 4 states. The Poisson parameter λ_i of the Poisson process characterises the number of emitted cells when the Markov chain is in that state i.

In addition to the first paragraph of this section, it is shown in [4] that the low frequencies of the arrival pattern affect the queueing analysis the most, which is in fact easy to comprehend. Because the identification of the MMPP is only necessary to speed up the CAC, only the information that influences this queueing analysis needs to be captured by the MMPP. This implies that only the low frequency part of the autocorrelation and the distribution function are important.

2.2 Properties of MMPP's

An MMPP can also completely be described by its statistic moments. Since only the first two moments are important for the identification, only these moments of the MMPP are given here. The probability $f(x)$ and cumulative $F(x)$ distribution functions are $(x \in N)$:

$$f(x) = Pr\{a_k = x\} = \sum_{i=1}^{N} \pi_i f_{\lambda_i}(x) = \sum_{i=1}^{N} \pi_i e^{-\lambda_i} \frac{\lambda_i^x}{x!} \tag{1}$$

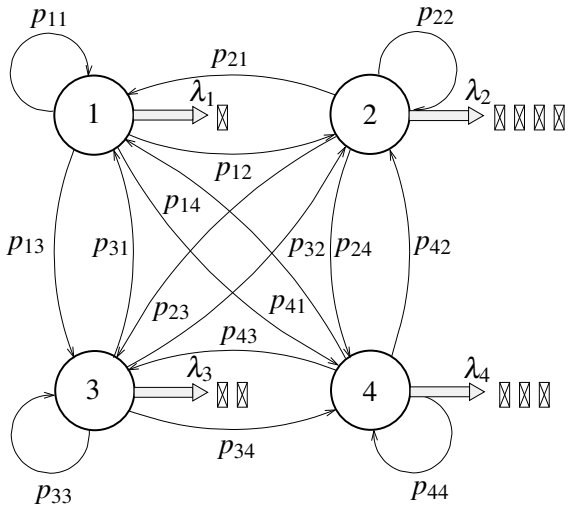

Figure 3: Markov chain of order 4: when the Markov chain is in e.g. state 3, the number of ATM-cells is described by the probability or cumulative distribution function of the associated Poisson process with λ_3, i.e. f_{λ_3} or F_{λ_3}.

$$F(x) = Pr\{a_k \leq x\} = \sum_{i=1}^{N} \pi_i F_{\lambda_i}(x) = \sum_{i=1}^{N} \pi_i e^{-\lambda_i} \sum_{j=0}^{x} \frac{\lambda_i^j}{j!} \quad . \tag{2}$$

The autocorrelation of an MMPP is[1]:

$$R(0) \equiv E(a_k a_k) = \pi \lambda_{\mathrm{d}} e + \pi \lambda_{\mathrm{d}}^2 e = \overline{\lambda^2} + \overline{\lambda} \tag{3}$$

$$R(n) \equiv E(a_k a_{k+n}) = \pi \lambda_{\mathrm{d}} P^n \lambda_{\mathrm{d}} e \qquad (n > 0) \tag{4}$$

$$R(\infty) \equiv \overline{a_k}^2 = \overline{\lambda}^2 \quad . \tag{5}$$

These properties will be used to identify the MMPP.

2.3 The inverse eigenvalue problem

The transition matrix P is a stochastic matrix, which means that $\sum_{i=1}^{N} p_{ij} = 1, \forall j$ and that each element $p_{ij} \geq 0$. These restrictions implicate that there are sets of eigenvalues which cannot be of a stochastic matrix. There are two theorems that give information on the existence of a solution:

1. The theorem of Karpelevič [5, 6] formulates necessary conditions, saying that an eigenvalue of a stochastic matrix cannot be in a region between a curve with points of contact on the unity circle and the unity circle. This curve is represented in Figure 4 for a stochastic matrix of dimension 4. However the theorem gives no information about combinations of eigenvalues.

2. Sufficient conditions are described in [7] for sets of real eigenvalues. This limits the applicability of this test, together with the fact that the existence of a set is not excluded.

[1] $\overline{x}$ denotes the weighted average of x, while e is a column vector of length N, containing all 1's and the index $_{\mathrm{d}}$ in λ_{d} stands for a diagonal matrix with the elements of λ on the main diagonal. E denotes the expectational operator.

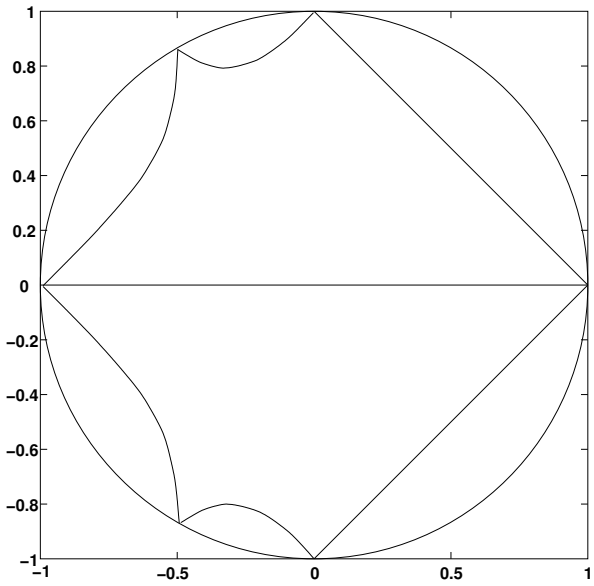

Figure 4: Theorem of Karpelevič for a stochastic matrix of dimension 4. There cannot exist an eigenvalue of a stochastic matrix between the curve and the unit circle. The first point of contact anticlockwise on the unit circle starting from $(1,0)$ is $e^{\frac{2\pi\sqrt{-1}}{N}}$. The curve between $(1,0)$ and this point of contact is a straight line.

Thinking of these two theorems, it is not straightforward to impose an eigenvalue set, which describes the dynamics of the arrival pattern $\{a_k\}$, on an MMPP.

To avoid the inverse eigenvalue problem, a good approach is to start from an MMPP with its restrictions and adapt it so that it resembles the given autocorrelation and distribution function.

2.4 Circulant matrices

Analysis of the above equations (2)–(6) shows that the numerical complexity of the computations is very high: the calculation of equation (4) requires $\mathcal{O}(N^3)$ operations, while the computation of the equations (1) and (2) requires large computational requirements because of the calculation of the Poisson distributions, as shown in [8][2]. Hence, this model class is inappropriate in an optimisation procedure, which is chosen in this report to avoid the inverse eigenvalue problem.

A restricted model class can simplify the computations, at the expense of the flexibility of the model class. Circulant stochastic matrices have a special structure which reduces the computations [9]:

$$P = \begin{bmatrix} p_1 & p_2 & \cdots & p_n \\ p_n & p_1 & \cdots & p_{n-1} \\ \vdots & \vdots & \ddots & \vdots \\ p_2 & p_3 & \cdots & p_1 \end{bmatrix} \tag{6}$$

Because of this structure, their eigenvalue decomposition can be written as a Fast Fourier Transform[3]

[2]This report is available by anonymous ftp from `ftp.esat.kuleuven.ac.be` in the directory `pub/SISTA/decock/reports/97-90.ps.gz`

[3]X^* denotes the complex conjugate of X.

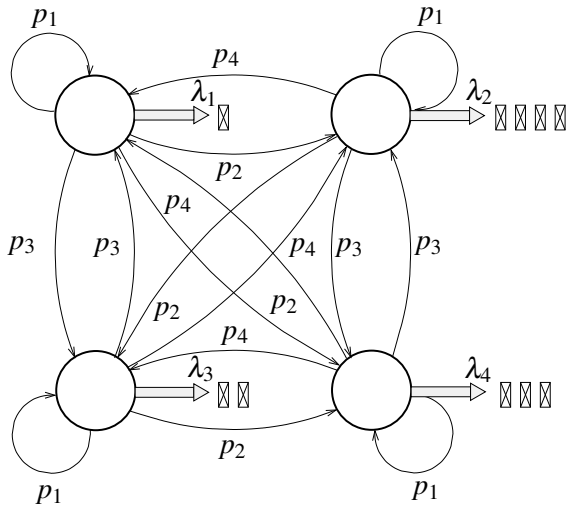

Figure 5: Schematic representation of a circulant MMPP of order 4, where the transition probabilities of Figure 3 are replaced by the appropriate transition probabilities of the circulant MMPP. Notice that due to the symmetric structure, the numbers of the chain are only necessary for the corresponding λ_i.

(`fft`) ([9]): $P \equiv F^* \gamma_\mathrm{d} F$, where $F(i,j) = \frac{1}{\sqrt{N}} \exp(-\frac{1}{N} 2\pi\sqrt{-1}(i-1)(j-1))$. This simplifies π into $\left(\frac{1}{N} \quad \ldots \quad \frac{1}{N}\right)$, which one can also understand regarding the stochastic circulant transition matrices as a subclass of doubly stochastic matrices [11]. The most important simplification occurs in equation (4):

$$\begin{aligned} R(n) \equiv E(a_k a_{k+n}) &= \pi\overline{\lambda} P^n \overline{\lambda} e \\ &= \frac{1}{N} \lambda^T F^* \gamma_\mathrm{d}^n F \lambda \\ &= \frac{1}{N} \mathtt{fft}(\lambda)^* \frac{1}{\sqrt{N}} \gamma_\mathrm{d}^n \mathtt{fft}(\lambda) \frac{1}{\sqrt{N}} \\ &= \frac{1}{N^2} \mathtt{fft}(\lambda)^* \gamma_\mathrm{d}^n \mathtt{fft}(\lambda) \ . \end{aligned} \tag{7}$$

This makes that the autocorrelation of a circulant MMPP is much easier to calculate than the autocorrelation of a (general) MMPP.

3 Identification procedure

The identified MMPP must approximate both the autocorrelation and the distribution function of the data. In this report the cumulative distribution function is chosen for reason of compact and global representation of the data.

A first possibility to solve the identification problem is to minimise a combined cost function, which is a weighted sum of the difference between the distribution functions of the data F_d and the model F_m and of the difference between the autocorrelations of the data R_d and of the model _m, as described in [10]. The distribution function however has to be calculated frequently, which delays the iterative optimisation process. When dealing with ATM-traffic, which is highly correlated, "long" or slowly decaying autocorrelations are to be expected, which also slows down the optimisation procedure. Therefore two improvements are proposed in this report.

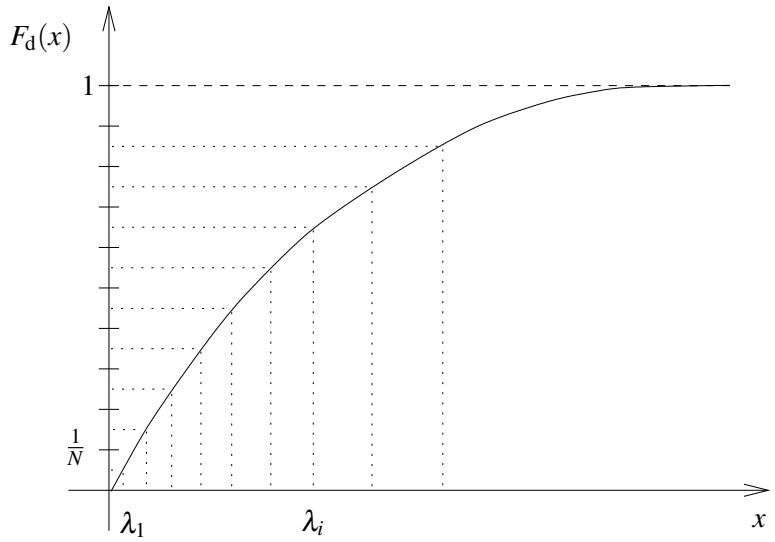

Figure 6: Graphical representation of the identification procedure of the first order statistic moment. The cumulative distribution function of the data F_{d} is approximated by a staircase function of Poisson processes with weight $\frac{1}{N}$.

Since the identification of both stochastic moments can be decoupled, the first order statistics are matched first. This determines the Poisson parameters λ. In a second step, the autocorrelation is calculated as a sum of exponentials, of which both the amplitudes and the poles have to be matched. This is done respectively by integer problem programming and by unconstrained optimisation.

3.1 First order statistics

Remember that because circulant matrices are a subclass of doubly stochastic matrices, the identification of the distribution function no longer depends on the transition matrix P, since $\pi_i = \frac{1}{N}$.

The cumulative distribution function can be approximated by a staircase function, which was also used in [3]. One also notices in equations (3) and (5) that both $R(0)$ and $R(\infty)$ are independent of P in the case of circulant matrices. The MMPP must be consistent with both the distribution function and the autocorrelation. Therefore two degrees of freedom are reserved in the identification procedure to match $R(0)$ and $R(\infty)$. This approach is illustrated in Figure 6. The two reserved λ_i's are calculated by solving a quadratic equation. Although it may seem that this approach is a rather rude approximation, it is proved in [12] that it is impossible to analytically reconstruct a given Poisson distribution with clustered Poisson parameters.

Besides the very fast identification procedure, one of the major advantages of this approach is that $R(\infty) = \overline{a_k}^2$ is correctly estimated and so is also the mean of the distribution function, which is a very important feature in the queueing analysis.

3.2 Second order statistics

The autocorrelation is the second characteristic of the data that captures relevant information for the queueing analysis. It describes the dynamics of the system. A slowly decaying autocorrelation means highly correlated traffic and increases the load on the buffer. Slowly decaying autocorrelations are highly probable in ATM-traffic, due to the high correlation. Despite the avoidance of eigenvalue

calculation by using circulant matrices, long autocorrelations still mean too long calculations in an iterative process.

Decomposition of the autocorrelation $R(n)$ in a sum of exponentials is a very compact representation and therefore very well suited in an iterative identification process. The decomposition is done by stochastic subspace identification [13, 14, 15] and results in:

$$R(n) = A_0 + \sum_{i=1}^{N_s} A_i b_i^n \quad , \tag{8}$$

where $A_0 = R(\infty)$, because of the stability of the system. Comparison of equation (7) with equation (8) leads to the following identity:

$$\frac{1}{N^2}\mathtt{fft}(\lambda)^* \gamma_d^n \mathtt{fft}(\lambda) \equiv A_0 + \sum_{i=1}^{N_s} A_i b_i^n \quad . \tag{9}$$

This equation means that both the (summated) amplitudes and the roots of both systems must be the same. As a result of the model restriction the order N of the circulant MMPP is typically much larger than the order of the power series N_s, since higher order models give better approximations of the first order statistics.

In the case of circulant matrices, the amplitude of the exponentials depends only on λ, which is already determined by the first order calculus. The matching is done in two consecutive steps. The first problem can be reformulated as an integer programming problem to allocate the amplitudes $\frac{1}{N^2}|\mathtt{fft}(\lambda)_i|^2$ to the amplitudes A_i. Secondly, a circulant stochastic transition matrix is searched for, so that the poles γ_i approximate the corresponding b_i. In the next two paragraphs a detailed description of each step is given. An overview of the total identification algorithm is given in Figure 7.

3.2.1 Integer problem

The first step consists of finding disjunct subsets C_i from a given set of numbers $C = \{\frac{1}{N}|\mathtt{fft}(\lambda)_i|^2 : i = 1,\ldots,N\}$ so that the sum of the numbers in this subset is as close as possible to a given number $G = A_i$. This step can be stated as an integer programming problem [16].

Let G be an amplitude A_i of the pole b_i and C be the set $\{\frac{1}{N}|\mathtt{fft}(\lambda)_i|^2 : i = 1,\ldots,N\}$, then the problem can be formulated as two 0/1 integer knapsack problems. One (P1) for giving the closest approximation which is lower than G and the other (P2) for finding the closest approximation which is larger than G. Figure 8 gives a detailed description of both knapsack problems. The second problem needs only to be solved if the first solution is not within a certain percentage of the given number G. Otherwise the best of both solutions is chosen.

As the 0/1 knapsack problem is known to be NP-complete, no polynomial solution algorithm exists. Therefore it is solved by an implicit enumeration approach called branch an bound. Using the branch and bound implementation of Fayard and Plateau [17], the problem can be solved in about 1 second for a typical set of 64 numbers[4].

This knapsack solver is used in an iterative process. First the problem for the largest given number $G = A_i$ is solved. Then the subset of the current solution C_1 is removed from the initial set $C = \{\frac{1}{N}|\mathtt{fft}(\lambda)_i|^2 : i = 1,\ldots,N\}$. Then the knapsack solver is applied to the second largest A_i, now with the reduced set. Although it is obvious that one obtains a sub-optimal solution, this approach is much faster than the optimal solution and also puts the largest weight on the most important exponentials.

[4] A Pentium 66 MHz was used to do the calculation.

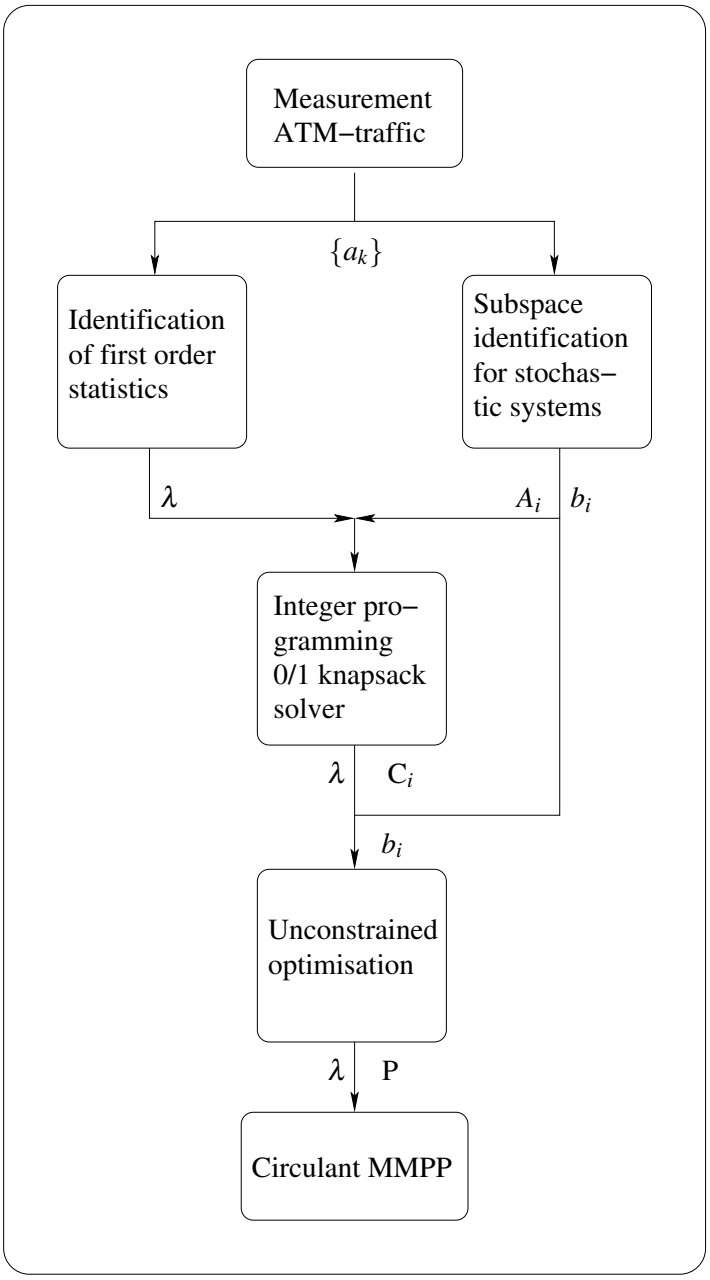

Figure 7: Overview of the identification method: the identification of the first order statistics is decoupled from the identification of the second order statistics. Subspace-identification determines the autocorrelation as a sum of exponentials (A_i, b_i). The 0/1 knapsack problem calculates disjunct subsets C_i from C, given λ and A_i. In the final step unconstrained optimisation is used to calculate a circulant stochastic transition matrix with poles b_i at the amplitudes of C_i in the diagonal form of equation (7). The Poisson parameters λ and the transition matrix P describe completely the circulant MMPP.

G	=	a given number;			
C	=	a given set of numbers;			
c_i	$\in$	C;			
x_i	=	decision variable for P1, = 1 if c_i is in the subset, = 0 otherwise;			
y_i	=	decision variable for P2, = 0 if c_i is in the subset, = 1 otherwise;			
MAX		$\sum_i (c_i x_i)$ (P1)	MAX		$\sum_i (c_i y_i)$ (P2)
S.T.		$\sum_i c_i x_i \leq G$	S.T.		$\sum_i c_i y_i \leq \sum_i c_i - G$
		$x_i \in \{0,1\}$			$y_i \in \{0,1\}$

Figure 8: Interpretation of the integer programming problem as two 0/1 knapsack problems, see the appendix for proof.

3.2.2 Optimisation problem

The second problem is to determine a circulant transition matrix so that the poles γ_i approximate the corresponding poles b_i of the selected amplitudes in the diagonal form of equation (7).

The problem can be reformulated as an unconstrained optimisation problem by using the parametrisation described in [10].

As cost function the sum of the squares of the differences between the given poles and real poles is minimised. Therefore the function `leastsq` in the Matlab optimisation toolbox is used. The Levenberg-Marquardt method was used [18], since one might expect ill conditioned problems, remembering the results of [2].

3.2.3 Example

As an example a third order (general) MMPP is identified with a (circulant) MMPP of order 64. Data were produced by simulating the original MMPP. About 100,000 points $\{a_k\}$ were generated, the latter half was used for the identification procedure to avoid the influence of the initial state. A third order original model was used since it is illustrated in [2, 8, 10] that higher order models do not necessarily have more complex dynamics, only the first order statistics are influenced. This is also the reason for the apparently large order of the identified model.

Using an HP 9000 Model 712/80 workstation with 64 MB internal memory and clock speed 80 MHz, all computations (except the knapsack solver) took about 60 seconds, starting from raw data. The cumulative distribution function and autocorrelation of the original model are given in Figures 9 and 10 (full line). The cumulative distribution function and autocorrelation of the identified circulant MMPP are also shown on the same figures (dashed lines).

One notices that the autocorrelations match almost perfectly and that the distribution function is quite well approximated. The main reason for the "difference" is that the original MMPP has distinct, widely spread Poisson parameters, which results in a staircase distribution function. In such cases, some Poisson parameters in the identified model are chosen in between the original Poisson parameters.

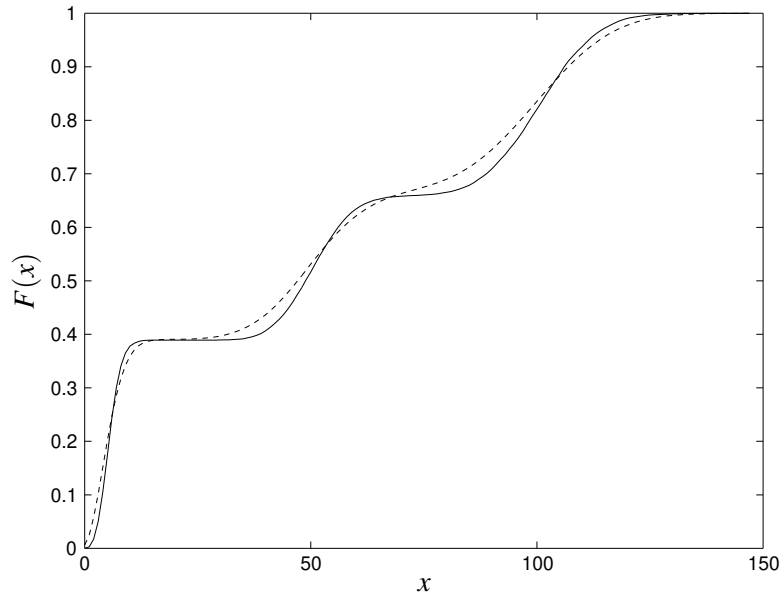

Figure 9: Comparison of the cumulative distribution function of the original MMPP (full line) and the identified (circulant) MMPP (dashed line).

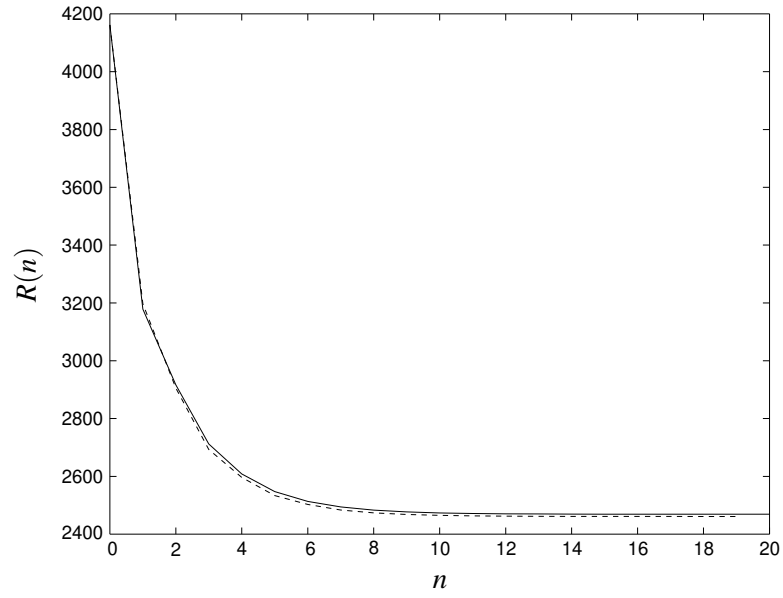

Figure 10: Comparison of the autocorrelation of the original MMPP (full line) and the identified (circulant) MMPP (dashed line).

4 Conclusion

In this report a time domain approach was presented to identify a circulant MMPP, based on the autocorrelation and the cumulative distribution function. It was explained that these models are important in the queueing analysis used in the Connection Admission Control of (packet switched) ATM-networks.

To avoid the inverse eigenvalue problem, optimisation algorithms must be used and models with low computational cost were searched for. The use of circulant transition matrices reduces the computational cost of the eigenvalue decomposition to the calculation of a Fast Fourier Transform and decouples the matching of the first and second order statistic moment since the left eigenvector corresponding to eigenvalue 1 of a circulant transition matrix only depends on the model order.

To further reduce the number of calculations, a quasi staircase approximation of the cumulative distribution function was used, with attention to the beginning of the autocorrelation and the mean of the distribution. The autocorrelation was approximated by subspace-identification as a sum of exponentials, the amplitudes were allocated by integer programming and the poles were identified by unconstrained optimisation.

Further research of the authors is focused on the queueing analysis and the comparison of circulant MMPP's with (general) MMPP's.

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Appendix

A Derivation of the 0/1 knapsack problem (P1)

In (P1) the positive difference s_1 between G and the sum of the numbers in the subset is minimised, or in other words the closest approximation which is lower than G is looked for. The derivation is given below in Figure 11.

$$\begin{aligned} \text{MIN} \quad & s_1 \\ \text{S.T.} \quad & \textstyle\sum_i c_i x_i + s_1 = G \\ & s_1 \geq 0 \\ & x_i \in \{0,1\} \end{aligned}$$

Substitute s_1, according to the equality.

$$\begin{aligned} \text{MIN} \quad & G - \textstyle\sum_i c_i x_i \\ \text{S.T.} \quad & G - \textstyle\sum_i c_i x_i \geq 0 \\ & x_i \in \{0,1\} \end{aligned}$$

Since G is a constant, it can be deleted from the objective function and the minimisation problem is turned into a maximisation problem. This leads to a 0/1 knapsack problem.

$$\begin{aligned} \text{MAX} \quad & \textstyle\sum_i c_i x_i \qquad \text{(P1)} \\ \text{S.T.} \quad & \textstyle\sum_i c_i x_i \leq G \\ & x_i \in \{0,1\} \end{aligned}$$

This is a 0/1 knapsack problem.

Figure 11: Derivation of the first knapsack problem (P1).

B Derivation of the 0/1 knapsack problem (P2)

In the second problem (P2) the positive difference s_2 between the sum of the numbers in the subset and G is minimised, in other words the closest approximation which is larger than G is looked for. The derivation of this knapsack problem is given in Figure 12.

$$\begin{aligned} \text{MIN} \quad & s_2 \\ \text{S.T.} \quad & \textstyle\sum_i c_i x_i - s_2 = G \\ & s_2 \geq 0 \\ & x_i \in \{0,1\} \end{aligned}$$

Substituting s_2 and deleting the constant in the objective function gives:

$$\begin{aligned} \text{MIN} \quad & \textstyle\sum_i c_i x_i \\ \text{S.T.} \quad & \textstyle\sum_i c_i x_i \geq G \\ & x_i \in \{0,1\} \end{aligned}$$

Substitute (x_i) by $(1-y_i)$.

$$\begin{aligned} \text{MIN} \quad & \textstyle\sum_i c_i (1-y_i) \\ \text{S.T.} \quad & \textstyle\sum_i c_i (1-y_i) \geq G \\ & y_i \in \{0,1\} \end{aligned}$$

or,

$$\begin{aligned} \text{MIN} \quad & \textstyle\sum_i c_i - \sum_i c_i y_i \\ \text{S.T.} \quad & \textstyle\sum_i c_i - \sum_i c_i y_i \geq G \\ & y_i \in \{0,1\} \end{aligned}$$

or,

$$\begin{aligned} \text{MAX} \quad & \textstyle\sum_i c_i y_i \qquad \text{(P2)} \\ \text{S.T.} \quad & \textstyle\sum_i c_i y_i \leq c_i - G \\ & y_i \in \{0,1\} \end{aligned}$$

This is again a 0/1 knapsack problem.

Figure 12: Derivation of the second knapsack problem (P2).

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