

Technical report bds:98-01

Optimal traffic light control for a single intersection*

B. De Schutter

If you want to cite this report, please use the following reference instead:

B. De Schutter, "Optimal traffic light control for a single intersection," *Proceedings of the 1999 American Control Conference*, San Diego, California, pp. 2195–2199, June 1999.

Control Systems Engineering
Faculty of Information Technology and Systems
Delft University of Technology
Delft, The Netherlands
phone: +31-15-278.51.19 (secretary)
fax: +31-15-278.66.79
Current URL: <http://www.dcsc.tudelft.nl>

*This report can also be downloaded via http://pub.deschutter.info/abs/98_01.html

Optimal Traffic Light Control for a Single Intersection

Bart De Schutter

Control Engineering Lab, Fac. of Information Technology and Systems
 Delft University of Technology, P.O.Box 5031, 2600 GA Delft, The Netherlands
 b.deschutter@its.tudelft.nl

Abstract

We consider a traffic light controlled intersection. First we construct a model that describes the evolution of the queue lengths (as continuous variables) in each lane. Next, we discuss how optimal and suboptimal traffic light switching schemes (with possibly variable cycle lengths) for this system can be determined. We also show that for a special class of objective functions suboptimal traffic light switching schemes can be computed very efficiently.

1 Introduction

As the number of vehicles and the need for transportation grow, cities around the world face serious road traffic congestion problems. On the short term the most effective measures in the battle against traffic congestion seem to be a selective construction of new roads and a better control of traffic through traffic management. Traffic light control can be used to augment the flow of traffic in urban environments by providing a smooth circulation of the traffic or by using “green waves”, or to regulate the access to highways or main roads (ramp metering).

In this paper we study the optimal traffic light control problem for an intersection of two two-way streets¹. We derive an approximate model that describes the evolution of the queue lengths as a continuous function of time. Starting from this model we can then compute the traffic light switching scheme that minimizes a criterion such as average queue length, worst case queue length, average waiting time, . . . , thereby augmenting the flow of traffic and diminishing the effects of traffic congestion. We show that for a special class of objective functions an optimal traffic light switching scheme can be computed very efficiently. The main difference of the approach presented in this paper and most other existing methods is that we allow the green-amber-red cycle time to vary from one cycle to another. Furthermore, we use an optimization over a fixed number of switch-overs instead of an optimization over a fixed number of time steps.

¹Our derivation can easily be extended to an intersection of more than two streets and streets with more than two lanes.

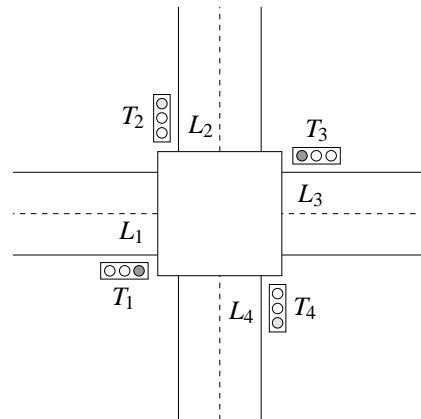


Figure 1: A traffic light controlled intersection of two two-way streets.

Table 1: The traffic light switching scheme.

Period	T_1, T_3	T_2, T_4
$t_0 - t_1 - \delta_a$	red	green
$t_1 - \delta_a - t_1$	red	amber
$t_1 - t_2 - \delta_a$	green	red
$t_2 - \delta_a - t_2$	amber	red
$t_2 - t_3 - \delta_a$	red	green
$t_3 - \delta_a - t_3$	red	amber
\vdots	\vdots	\vdots

2 The set-up and the model of the system

We consider an intersection of two two-way streets (see Figure 1). There are four lanes L_1, L_2, L_3 and L_4 , and on each corner of the intersection there is a traffic light (T_1, T_2, T_3 and T_4). For each traffic light there are three subsequent phases: green, amber, and red². We assume that the duration of the amber phase is fixed and equal to δ_a .

Let t_0, t_1, t_2, \dots be the main switching time instants, i.e., the time instants at which the traffic lights switch from one main phase to another (see Table 1). Define $\delta_k = t_{k+1} - t_k$. Let

²This is an extension of the work we reported in [2] where we only considered two phases: green and red.

$l_i(t)$ be the queue length (i.e., the number of cars waiting) in lane L_i at time instant t . In reality $l_i(t)$ will be an integer valued function and the arrival and departure rates will vary as a function of time. As a consequence, the exact model for the evolution of the queue lengths is not very amenable to mathematical analysis. Therefore, we introduce some assumptions that will result in a much simpler (approximate) model that can be analyzed very easily and for which we can efficiently compute optimal traffic light switching schemes (see Section 3).

We make the following assumptions:

- the queue lengths are continuous variables,
- in each main phase the average arrival and departure rates of the cars are constant³,
- for each lane the average departure rate during a green phase is greater than or equal to the average departure rate during the subsequent amber phase.

The first two assumptions deserve a few remarks:

- Designing optimal traffic light switching schemes is only useful if the arrival and departure rates of vehicles at the intersection are high. In that case, approximating the queue lengths by continuous variables only introduces small errors. Furthermore, there is also some uncertainty and variation in time of the arrival and departure rates, which makes that in general computing the exact optimal traffic light switching scheme is utopian. Moreover, in practice we are more interested in quickly obtaining a good approximation of the optimal traffic light switching scheme than in spending a large amount of time to obtain the exact optimal traffic light switching scheme.
- If we keep in mind that one of the main purposes of the model that we shall derive, is the design of optimal traffic light switching schemes, then assuming that the average arrival and departure rates are constant in each phase is not a serious restriction since we can approximate time-varying arrival and departure rates by piecewise constant functions⁴. Moreover, we can also use a *moving horizon strategy*: we compute the optimal traffic light switching scheme for, say, 10 cycles, based on a prediction of the average arrival and departure rates (using historical data and data measured during the previous cycles) and we apply this scheme during the first of the 10 cycles, meanwhile we update our estimates of the arrival and departure

³This is an extension of the work reported in [3] where we required the average arrival and departure rates to be constant whereas in this paper these rates may vary from phase to phase.

⁴In order to determine the average rates for each green-amber or red phase, we could first assume that all main phases have equal length. Then we compute the optimal scheme and use the result to get better estimates of the average rates for the main phases, which can then be used as the input for another optimization run. If necessary we could repeat this process a few times.

rates and compute a new optimal scheme for the next 10 cycles, and so on.

Let $\bar{\lambda}_i^{(k)}$ be the average arrival rate of vehicles in lane L_i in time interval (t_k, t_{k+1}) . Let $\bar{\mu}_i^{(k)}$ (respectively $\bar{\kappa}_i^{(k)}$) be the average departure rate in lane L_i in time interval (t_k, t_{k+1}) when the traffic light is green (respectively amber) and the queue length is larger than 0 (i.e., when there are cars waiting or arriving at lane L_i).

Let us now write down the equations that describe the relation between the queue lengths at the main switching time instants. Consider lane L_1 . When the traffic light T_1 is red, there are arrivals at lane L_1 and no departures. Hence,

$$\frac{dl_1(t)}{dt} = \bar{\lambda}_1^{(2k)}$$

for $t \in (t_{2k}, t_{2k+1})$ and

$$l_1(t_{2k+1}) = l_1(t_{2k}) + \bar{\lambda}_1^{(2k)} \delta_{2k} .$$

In the time interval (t_{2k+1}, t_{2k+2}) the traffic light T_1 is green and there are arrivals and departures at lane L_1 . Since the net queue growth rate is $\bar{\lambda}_1^{(2k+1)} - \bar{\mu}_1^{(2k+1)}$ and since the queue length cannot be negative, we have

$$\frac{dl_1(t)}{dt} = \begin{cases} \bar{\lambda}_1^{(2k+1)} - \bar{\mu}_1^{(2k+1)} & \text{if } l_1(t) > 0 \\ 0 & \text{if } l_1(t) = 0 \end{cases}$$

for $t \in (t_{2k+1}, t_{2k+2} - \delta_a)$. We can write down a similar expression for the amber phase $(t_{2k+2} - \delta_a, t_{2k+2})$. So

$$\begin{aligned} l_1(t_{2k+2} - \delta_a) &= \max\left(l_1(t_{2k+1}) + \right. \\ &\quad \left. (\bar{\lambda}_1^{(2k+1)} - \bar{\mu}_1^{(2k+1)}) (\delta_{2k+1} - \delta_a), 0\right) \\ l_1(t_{2k+2}) &= \max\left(l_1(t_{2k+2} - \delta_a) + \right. \\ &\quad \left. (\bar{\lambda}_1^{(2k+1)} - \bar{\kappa}_1^{(2k+1)}) \delta_a, 0\right) \\ &= \max\left(l_1(t_{2k+1}) + (\bar{\lambda}_1^{(2k+1)} - \bar{\mu}_1^{(2k+1)}) \right. \\ &\quad \left. \delta_{2k+1} + (\bar{\mu}_1^{(2k+1)} - \bar{\kappa}_1^{(2k+1)}) \delta_a, \right. \\ &\quad \left. (\bar{\lambda}_1^{(2k+1)} - \bar{\kappa}_1^{(2k+1)}) \delta_a, 0\right) . \end{aligned}$$

Note that we also have

$$l_1(t_{2k+1}) = \max\left(l_1(t_{2k}) + \bar{\lambda}_1^{(2k)} \delta_{2k}, 0\right)$$

since $l_1(t) \geq 0$ for all t .

We can write down similar equations for $l_2(t_k)$, $l_3(t_k)$ and $l_4(t_k)$. So if we define

$$\mathbf{x}_k = [l_1(t_k) \quad l_2(t_k) \quad l_3(t_k) \quad l_4(t_k)]^T ,$$

we obtain

$$\mathbf{x}_{2k+1} = \max(\mathbf{x}_{2k} + \mathbf{b}_1^{(2k)} \delta_{2k} + \mathbf{b}_2^{(2k)}, \mathbf{b}_3^{(2k)}) \quad (1)$$

$$\mathbf{x}_{2k+2} = \max(\mathbf{x}_{2k+1} + \mathbf{b}_1^{(2k+1)} \delta_{2k+1} + \mathbf{b}_2^{(2k+1)}, \mathbf{b}_3^{(2k+1)}) \quad (2)$$

for $k = 0, 1, 2, \dots$ and for appropriately defined vectors $\mathbf{b}_1^{(k)}$, $\mathbf{b}_2^{(k)}$ and $\mathbf{b}_3^{(k)}$.

3 Optimal traffic light control

3.1 Problem statement

From now on we assume that the average arrival and departure rates in each main phase are known. For a given integer N we want to compute an optimal sequence t_0, t_1, \dots, t_N of switching time instants that minimizes a criterion such as:

- (weighted) average queue length over all queues:

$$J_1 = \sum_{i=1}^4 \frac{w_i}{t_N - t_0} \int_{t_0}^{t_N} l_i(t) dt, \quad (3)$$

- (weighted) average queue length over the worst queue:

$$J_2 = \max_i \left(\frac{w_i}{t_N - t_0} \int_{t_0}^{t_N} l_i(t) dt \right), \quad (4)$$

- (weighted) worst case queue length:

$$J_3 = \max_{i,t} (w_i l_i(t)), \quad (5)$$

- (weighted) average waiting time over all queues:

$$J_4 = \sum_{i=1}^4 \frac{w_i}{t_N - t_0} \left(\sum_{k=0}^{N-1} \int_{t_k}^{t_{k+1}} \frac{l_i(t)}{\bar{\lambda}_i^{(k)}} dt \right), \quad (6)$$

- (weighted) average waiting time over the worst queue:

$$J_5 = \max_i \left(\frac{w_i}{t_N - t_0} \sum_{k=0}^{N-1} \int_{t_k}^{t_{k+1}} \frac{l_i(t)}{\bar{\lambda}_i^{(k)}} dt \right), \quad (7)$$

where $w_i > 0$ for all i .

We can impose some extra conditions such as minimum and maximum durations for the green times, maximum queue lengths, and so on. This leads to the following problem:

$$\text{minimize } J \quad (8)$$

subject to

$$\delta_{\min, g, 1}^{(2k+1)} \leq \delta_{2k+1} - \delta_a \leq \delta_{\max, g, 1}^{(2k+1)} \quad \text{for } k \in \beta_N \quad (9)$$

$$\delta_{\min, g, 2}^{(2k)} \leq \delta_{2k} - \delta_a \leq \delta_{\max, g, 2}^{(2k)} \quad \text{for } k \in \alpha_N \quad (10)$$

$$\mathbf{x}_k \leq \mathbf{x}_{\max}^{(k)} \quad \text{for } k \in \gamma_N \quad (11)$$

$$\mathbf{x}_{2k+1} = \max(\mathbf{x}_{2k} + \mathbf{b}_1^{(2k)} \delta_{2k} + \mathbf{b}_2^{(2k)}, \mathbf{b}_3^{(2k)}) \quad \text{for } k \in \alpha_N \quad (12)$$

$$\mathbf{x}_{2k+2} = \max(\mathbf{x}_{2k+1} + \mathbf{b}_1^{(2k+1)} \delta_{2k+1} + \mathbf{b}_2^{(2k+1)}, \mathbf{b}_3^{(2k+1)}) \quad \text{for } k \in \beta_N \quad (13)$$

where $\delta_{\min, g, i}^{(k)}$ and $\delta_{\max, g, i}^{(k)}$ are the minimum and maximum green time in lane L_i in (t_k, t_{k+1}) , $(\mathbf{x}_{\max}^{(k)})_i$ is the maximum queue length in L_i in (t_k, t_{k+1}) , $\alpha_N = \{0, 1, \dots, \lfloor \frac{N-1}{2} \rfloor\}$, $\beta_N = \{0, 1, \dots, \lfloor \frac{N}{2} \rfloor - 1\}$ and $\gamma_N = \{1, 2, \dots, N\}$.

Now we discuss some methods to solve problem (8)–(13). Consider (12) for an arbitrary index k . It is easy to verify [3] that this equation is equivalent to:

$$\mathbf{x}_{2k+1} - \mathbf{x}_{2k} - \mathbf{b}_1^{(2k)} \delta_{2k} - \mathbf{b}_2^{(2k)} \geq \mathbf{0} \quad (14)$$

$$\mathbf{x}_{2k+1} - \mathbf{b}_3^{(2k)} \geq \mathbf{0} \quad (15)$$

$$\sum_{i=1}^4 (\mathbf{x}_{2k+1} - \mathbf{x}_{2k} - \mathbf{b}_1^{(2k)} \delta_{2k} - \mathbf{b}_2^{(2k)})_i \cdot (\mathbf{x}_{2k+1} - \mathbf{b}_3^{(2k)})_i = 0. \quad (16)$$

We can repeat this reasoning for (13) and for each k . So if we define

$$\mathbf{x}^* = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_N \end{bmatrix} \quad \text{and} \quad \delta^* = \begin{bmatrix} \delta_0 \\ \delta_1 \\ \vdots \\ \delta_{N-1} \end{bmatrix},$$

we finally get a problem of the following form:

$$\text{minimize } J \quad (17)$$

subject to

$$\mathbf{A}\mathbf{x}^* + \mathbf{B}\delta^* + \mathbf{c} \geq \mathbf{0} \quad (18)$$

$$\mathbf{x}^* + \mathbf{d} \geq \mathbf{0} \quad (19)$$

$$\mathbf{E}\mathbf{x}^* + \mathbf{F}\delta^* + \mathbf{g} \geq \mathbf{0} \quad (20)$$

$$(\mathbf{A}\mathbf{x}^* + \mathbf{B}\delta^* + \mathbf{c})^T (\mathbf{x}^* + \mathbf{d}) = 0. \quad (21)$$

The system (18)–(21) is a special case of an Extended Linear Complementarity Problem (ELCP) [1]. In [1] we have developed an algorithm to compute a parametric description of the complete solution set of an ELCP. Once this parametric description is obtained, we can compute for which combination of the parameters the objective function J reaches a global minimum. However, since the general ELCP is an NP-hard problem [1], the ELCP based approach is not feasible if the number of switching cycles N is large. If N is large, we could consider a small number N_s of switching cycles, compute the optimal switching strategy with the method given above, implement the first step(s) of this strategy, and recompute the optimal switching strategy for the next N_s switching cycles. This approach is called the multi-ELCP approach. Note that although this is feasible in practice, it will only give a suboptimal solution.

We can also consider problem (8)–(13) as a constrained optimization problem in δ^* where the constraints (11)–(13) are considered as nonlinear constraints; alternatively these

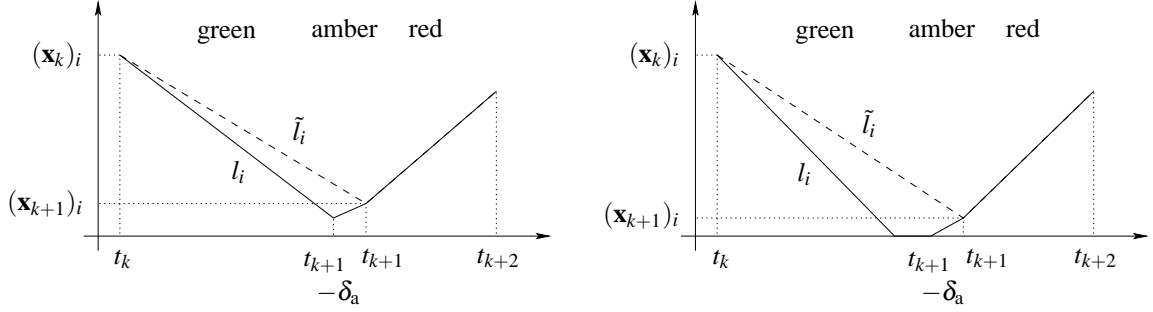


Figure 2: The functions l_i (full line) and \tilde{l}_i (dashed line). During the red phase the functions l_i and \tilde{l}_i coincide. The left plot shows a situation in which the queue length does not become 0 during the green phase, whereas the right plot shows a situation where the queue length becomes 0 during the green phase.

constraints can also be taken into account by adding an extra penalty term for values of \mathbf{x}_k that exceed $\mathbf{x}_{\max}^{(k)}$ to the objective function. The major disadvantage of these two approaches is that in general the minimization routine will only return a local minimum. Our computational experiments have shown that several initial starting points are necessary to obtain the global minimum.

3.2 The relaxed problem and suboptimal solutions

Now we make another approximation that will result in suboptimal traffic light switching schemes that can be computed very efficiently and that approximate the exact optimum very well.

For given \mathbf{x}_0 and t_0 , we define the function $\tilde{l}_i(\cdot, \mathbf{x}^*, \delta^*)$ as the piecewise-linear function that interpolates in the points $(t_k, l_i(t_k))$ for $k = 0, 1, \dots, N$. The approximate objective functions \tilde{J}_i are defined as in (3)–(7) but with l_i replaced by \tilde{l}_i . It is easy to verify that the values of J_3 and \tilde{J}_3 coincide. Now let $l \in \{1, 2, 4, 5\}$. The value of J_l and \tilde{J}_l is determined by the surface under the functions l_i and \tilde{l}_i respectively. If the duration of the amber phase is zero and if the queue lengths never become zero, then the functions l_i and \tilde{l}_i and the values of J_l and \tilde{J}_l coincide (cf. Figure 2). In practice, the length of the amber phase will be small compared to the length of the green or red phases. Furthermore, an optimal traffic light switching scheme implies the absence of long periods in which no cars wait in one lane while in the other lanes the queue lengths increase. So if we have an optimal traffic light switching scheme, then the periods during which the queue length in some lane is equal to 0 are in general short. As a consequence, for traffic light switching schemes in the neighborhood of the optimal scheme \tilde{J}_l will be a good approximation of J_l .

Let \mathcal{P} be the problem (8)–(13). We define the “relaxed” problem $\tilde{\mathcal{P}}$ corresponding to the original problem \mathcal{P} as:

$$\text{minimize } J \quad (22)$$

subject to

$$\delta_{\min, g, 1}^{(2k+1)} \leq \delta_{2k+1} - \delta_a \leq \delta_{\max, g, 1}^{(2k+1)} \quad \text{for } k \in \beta_N \quad (23)$$

$$\delta_{\min, g, 2}^{(2k)} \leq \delta_{2k} - \delta_a \leq \delta_{\max, g, 2}^{(2k)} \quad \text{for } k \in \alpha_N \quad (24)$$

$$\mathbf{x}_k \leq \mathbf{x}_{\max}^{(k)} \quad \text{for } k \in \gamma_N \quad (25)$$

$$\mathbf{x}_{2k+1} \geq \mathbf{x}_{2k} + \mathbf{b}_1^{(2k)} \delta_{2k} + \mathbf{b}_2^{(2k)} \quad \text{for } k \in \alpha_N \quad (26)$$

$$\mathbf{x}_{2k+1} \geq \mathbf{b}_3^{(2k)} \quad \text{for } k \in \alpha_N \quad (27)$$

$$\mathbf{x}_{2k+2} \geq \mathbf{x}_{2k+1} + \mathbf{b}_1^{(2k+1)} \delta_{2k+1} + \mathbf{b}_2^{(2k+1)} \quad \text{for } k \in \beta_N \quad (28)$$

$$\mathbf{x}_{2k+2} \geq \mathbf{b}_3^{(2k+1)} \quad \text{for } k \in \beta_N. \quad (29)$$

Note that compared to the original problem we have replaced (12)–(13) by relaxed equations of the form (14)–(15) without taking (16) into account. In general it is easier to solve $\tilde{\mathcal{P}}$ since the set of feasible solutions of $\tilde{\mathcal{P}}$ is convex, whereas the set of feasible solutions of \mathcal{P} corresponds to an ELCP and is thus in general not convex [3].

Proposition 3.1 *If J is a strictly monotonous function of \mathbf{x}^* — i.e., if for any δ^* with positive components and for all $\tilde{\mathbf{x}}^*, \hat{\mathbf{x}}^*$ with $\mathbf{0} \leq \tilde{\mathbf{x}}^* \leq \hat{\mathbf{x}}^*$ and with $\tilde{x}_j^* < \hat{x}_j^*$ for at least one index j , we have $J(\tilde{\mathbf{x}}^*, \delta^*) < J(\hat{\mathbf{x}}^*, \delta^*)$ — then any solution of the relaxed problem $\tilde{\mathcal{P}}$ is also a solution of the original problem \mathcal{P} .*

Proof: The proof is similar to that of Proposition 3.2 of [3]. The main difference is that in [3] we assumed that the average arrival and departure rates were constant during the whole period (t_0, t_N) , whereas in this paper we allow them to vary from one main phase to another. ■

Using a reasoning similar to the one of [3] it can be shown that \tilde{J}_1 and \tilde{J}_4 are strictly monotonous functions of \mathbf{x}^* , i.e., they satisfy the conditions of Proposition 3.1. Furthermore, if all the δ_k ’s are equal then \tilde{J}_1 and \tilde{J}_4 are linear, strictly monotonous functions of \mathbf{x}^* , which implies that problem (8)–(13) then reduces to a linear programming problem, which can be solved efficiently using a simplex or interior point method. In [3] we also discuss some extensions of the basic model such as: an amber duration that is a variable or that depends on k ; streets consisting of more than two lanes; different average departure rates for vehicles that turn left, right or go straight ahead; and intersections of networks.

Table 2: The values of the objective function J_4 (up to 3 decimal places) and the CPU time (up to 2 decimal places) needed to compute the (sub)optimal switching interval vectors of the example of Section 4.

δ^*	$J_4(\delta^*)$	CPU time
δ_{ELCP}^*	364.944	1114.46
δ_{con}^*	364.944	177.37
δ_{pen}^*	367.760	155.85
δ_{mul}^*	366.715	14.69
$\tilde{\delta}^*$	364.944	0.88
δ_{lin}^*	378.372	0.63

For more information on traffic modeling and traffic light control the interested reader is referred to [4, 5, 6, 7, 8].

4 Example

The following example illustrates that using the approximations introduced in the Section 3.2 for the objective functions J_4 leads to good suboptimal solutions that can be computed very efficiently. All times will be expressed in seconds and all rates in vehicles per second.

Consider the intersection of Figure 1 with the following data: $\bar{\lambda}_1^{(k)} = 0.22$, $\bar{\lambda}_2^{(k)} = 0.13$, $\bar{\lambda}_3^{(k)} = 0.19$, $\bar{\lambda}_4^{(k)} = 0.12$, $\bar{\mu}_1^{(k)} = \bar{\mu}_3^{(k)} = 0.5$, $\bar{\mu}_2^{(k)} = \bar{\mu}_4^{(k)} = 0.4$, $\bar{\kappa}_1^{(k)} = \bar{\kappa}_3^{(k)} = 0.05$, $\bar{\kappa}_2^{(k)} = \bar{\kappa}_4^{(k)} = 0.03$, $\mathbf{x}_0 = [22 \ 18 \ 15 \ 14]^T$, $\delta_a = 3$, $\delta_{\text{min},g,1}^{(k)} = \delta_{\text{min},g,2}^{(k)} = 6$, $\delta_{\text{max},g,1}^{(k)} = \delta_{\text{max},g,2}^{(k)} = 60$, $\mathbf{x}_{\text{max}} = [25 \ 15 \ 25 \ 15]^T$. Let $\mathbf{w} = [2 \ 1 \ 2 \ 1]^T$. We want to compute a traffic light switching sequence t_0, t_1, \dots, t_7 that minimizes the weighted average waiting time J_4 . We have computed an optimal solution δ_{ELCP}^* obtained using the ELCP method, a solution δ_{con}^* using constrained optimization with nonlinear constraints, a solution δ_{pen}^* using constrained optimization with a penalty function, a multi-ELCP solution δ_{mul}^* with $N_s = 3$, a solution $\tilde{\delta}^*$ that minimizes the approximate objective function \tilde{J}_4 and a linear programming solution δ_{lin}^* . In Table 2 we have listed the value of the objective function J_4 for the various switching interval vectors and the CPU time needed to compute the switching interval vectors on a SUN Ultra 10 300 MHz workstation with the optimization routines called from MATLAB and implemented in C or Fortran. The CPU time values listed in the table are average values over 10 experiments.

If we look at Table 2 then we see that if we take the trade-off between optimality and efficiency into account, then the $\tilde{\delta}^*$ solution is clearly the most interesting.

5 Conclusions

We have derived a model that describes the evolution of the queue lengths at a traffic light controlled intersection of two streets. We have shown how an optimal traffic light switching scheme for the given system can be determined. In general this leads to a minimization problem over the solution set of an Extended Linear Complementarity Problem. For objective functions that depend strictly monotonously on the queue lengths at the traffic light switching time instants, the optimal traffic light switching scheme can be computed very efficiently. We have derived approximate objective functions for which this property holds. Moreover, if the objective function is linear, the problem reduces to a linear programming problem.

Topics for further research include: extension of our approach to models with integer queue lengths, extension to centralized traffic light control for networks of intersections, and development of other efficient algorithms to compute optimal traffic light switching schemes.

References

- [1] B. De Schutter and B. De Moor, "The extended linear complementarity problem," *Mathematical Programming*, vol. 71, no. 3, pp. 289–325, Dec. 1995.
- [2] B. De Schutter and B. De Moor, "Optimal traffic light control for a single intersection," in *Proceedings of the 1997 International Symposium on Nonlinear Theory and its Applications (NOLTA'97)*, Honolulu, Hawaii, pp. 1085–1088, Nov.–Dec. 1997.
- [3] B. De Schutter and B. De Moor, "Optimal traffic light control for a single intersection," *European Journal of Control*, vol. 4, no. 3, pp. 260–276, 1998.
- [4] N.H. Gartner, J.D.C. Little, and H. Gabbay, "Simultaneous optimization of offsets, splits, and cycle time," *Transportation Research Record*, no. 596, pp. 6–15, 1976.
- [5] J.H. Lim, S.H. Hwang, I.H. Suh, and Z. Bien, "Hierarchical optimal control of oversaturated urban traffic networks," *International Journal of Control*, vol. 33, no. 4, pp. 727–737, Apr. 1981.
- [6] A.D. May, *Traffic Flow Fundamentals*. Englewood Cliffs, New Jersey: Prentice-Hall, 1990.
- [7] E.S. Park, J.H. Lim, I.H. Suh, and Z. Bien, "Hierarchical optimal control of urban traffic networks," *International Journal of Control*, vol. 40, no. 4, pp. 813–829, Oct. 1984.
- [8] M.G. Singh and H. Tamura, "Modelling and hierarchical optimization for oversaturated urban road traffic networks," *International Journal of Control*, vol. 20, no. 6, pp. 913–934, 1974.