Exercise Set 7 (2009)

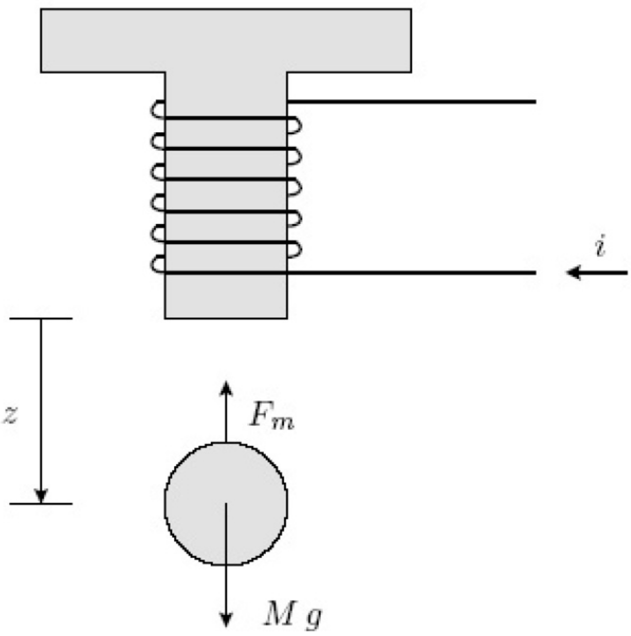

Figure 1: Magnetic levitation system.

Consider the magnetic levitation system as depicted in Figure 1. A simple nonlinear model of this system is given by

$$M\frac{d^2}{dt^2}z = Mg - k\frac{i^2}{z^2}, \quad L\frac{d}{dt}i + Ri = v$$

where $M = 0.005$, $g = 9.81$, $k = 0.0003$, $L = 0.1$, $R = 2$ (in SI units) and where the voltage v is the system's input while the ball position z is the system's output.

a) With $v_0 = 2$ determine the equilibrium (z_0, i_0) of the system with $z_0 > 0$.

b) Determine a function $f(z, i)$ such that the system can be exactly represented as

$$\frac{d}{dt}\begin{pmatrix} z - z_0 \\ \dot{z} \\ i - i_0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ \sqrt{g}f(z,i) & 0 & -\alpha f(z,i) \\ 0 & 0 & -\frac{R}{L} \end{pmatrix}\begin{pmatrix} z - z_0 \\ \dot{z} \\ i - i_0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L} \end{pmatrix}(v - v_0).$$

This motivates to consider the parameter-dependent system

$$\dot{x} = \begin{pmatrix} 0 & 1 & 0 \\ \sqrt{g}\,\delta(t) & 0 & -\alpha\,\delta(t) \\ 0 & 0 & -\frac{R}{L} \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L} \end{pmatrix} u, \quad z = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} x \tag{1}$$

where $\delta(t)$ varies in the interval defined by a 50%-deviation from the nominal (mid-point) value $\delta_0 = f(z_0, i_0)$.

c) Let us denote the LTI system that results from (1) for $\delta(t) = \delta_0$ by $z = Gu$. The goal is to design a stabilizing controller $u = K(z - r)$ such that the output z tracks the reference signal r.

Use H_∞-synthesis for the generalized plant

$$e = Gu - r, \quad y = Gu - r$$

(the un-weighted open-loop interconnection with generalized disturbance r, control input u, to-be-controlled output $\mathrm{col}(u, e)$ and measured output y) and the weights

$$w_e(s) = \frac{2s + 100}{5s + 1} \quad \text{and} \quad w_u(s) = 8 * 10^{-5}$$

imposed on the tracking error e and the control input u in order to design K. Analyze the resulting LTI controlled closed-loop interconnection. (Provide relevant plots of frequency and step responses with conclusions about the quality of the design).

Remark. The controller has a fast stable pole. For the subsequent exercises replace the dynamics of this fast pole by a direct feedthrough term (residualization). Figure 2 shows my controller before (blue) and after (green) residualization.

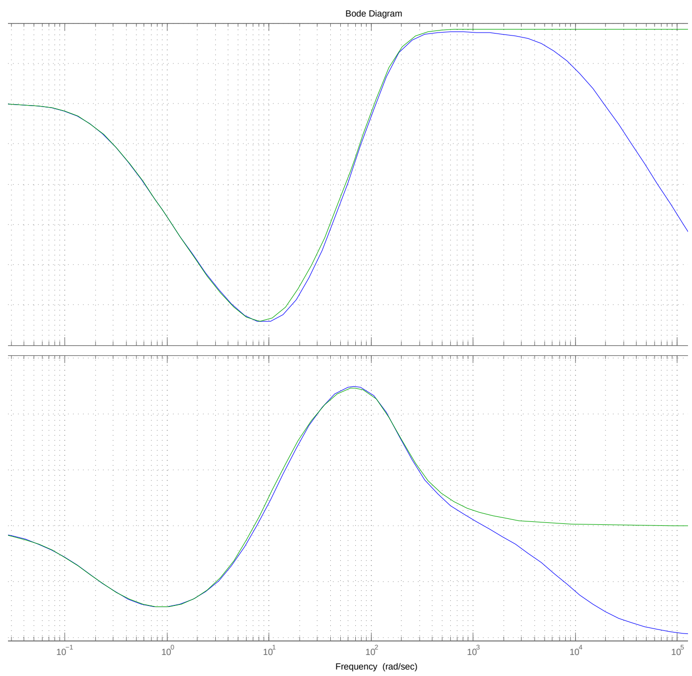

Figure 2: Designed H_∞-controller.

d) For the designed H_∞-controller compute the quadratic stability margin of the (un-weighted) controlled closed-loop system. What do you conclude about the allowed values of $f(z(t), i(t))$ along a trajectory without endangering stability?

Hint: Explore and use the command `quadstab`.

e) With a reference signal $r(t) = a\sin(\omega t)$ for various values a and ω, confirm the previous stability guarantees by nonlinear simulation, and investigate the possibility to destabilize the controlled system.

   Hint: To save time you can make use of my simulation files `si.m` and `funcll.m`.

f) The command `pdlstab` allows to verify robust stability by affine parameter-dependent Lyapunov functions for $|\delta(t)| \leq r$ and $|\dot{\delta}(t)| \leq v$. Code your own bisection algorithm in order to draw the curve which displays the trade-off between v and r.

   Optional: Compute the stability margin if δ is parametric and time-invariant.