SWITCHING CONTROL OF UNCERTAIN SYSTEMS

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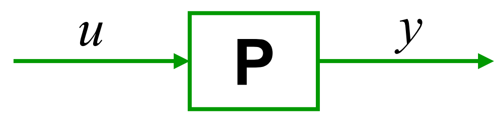
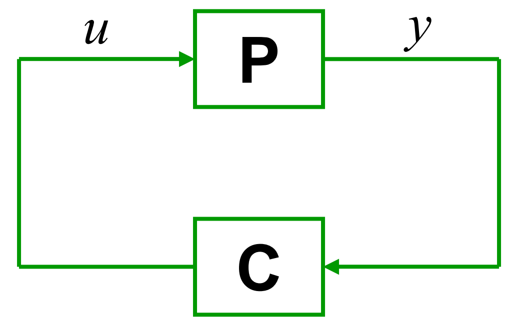
HYBRID CONTROL

Plant:

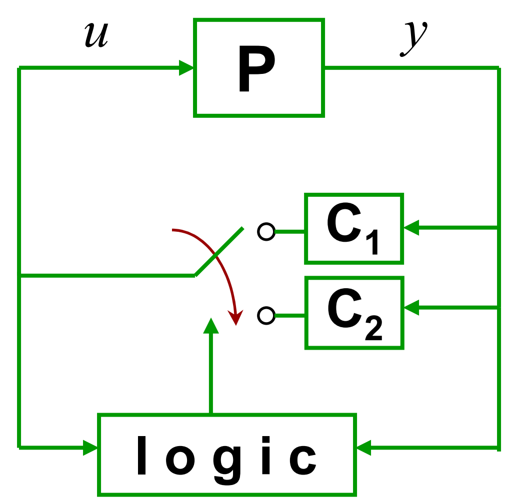


$$\dot{x} = f(x, u)$$
$$y = h(x)$$

Classical continuous feedback paradigm:



But logical decisions are often necessary:



The closed-loop system is hybrid

REASONS for SWITCHING

- Nature of the control problem
- Sensor or actuator limitations
- Large modeling uncertainty
- Combinations of the above

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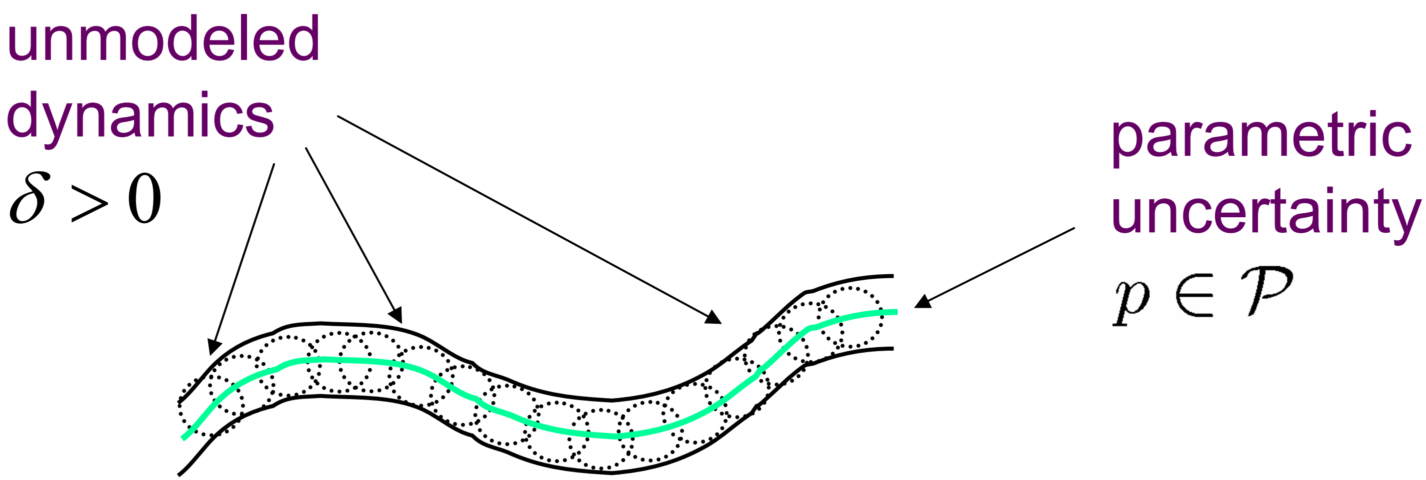
MODELING UNCERTAINTY



$$\mathcal{F} = \bigcup_{p \in \mathcal{P}} \mathcal{F}_p$$

Also, noise n and disturbance d

Adaptive control (continuous tuning)

vs. supervisory control (switching)

EXAMPLE

Scalar system:

$$\boxed{\dot{y} = y^2 + p^* u}$$

$$\mathcal{P} = [-10, -0.1] \cup [0.1, 10]$$

$p^* \in \mathcal{P}$, otherwise unknown

(purely parametric uncertainty)

$$u_p = -\frac{1}{p^*}(y^2 + y) \;\Rightarrow\; \dot{y} = -y \qquad \text{stable } \checkmark$$

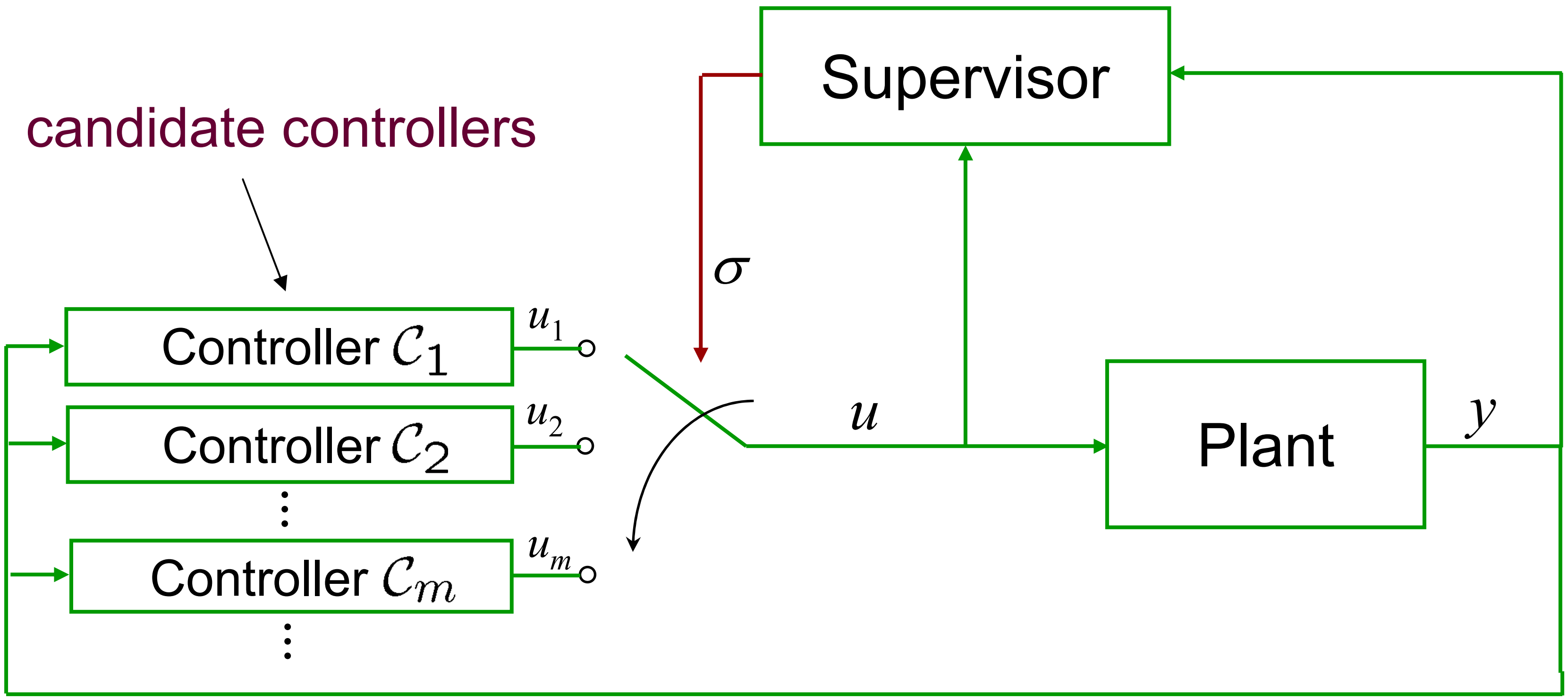
not implementable

Controller family: $u_q = -\frac{1}{q}(y^2 + y), \;\; q \in \mathcal{P}$

Could also take $u_q, \;\; q \in \mathcal{Q} = \{-1, 1\}$

controller index set

SUPERVISORY CONTROL ARCHITECTURE



σ – switching signal, takes values in $\mathcal{Q}$

$\mathcal{C}_\sigma$ – switching controller

TYPES of SUPERVISION

- Prescheduled (prerouted)
- Performance-based (direct)
- Estimator-based (indirect)

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OUTLINE

- Basic components of supervisor
- Design objectives and general analysis
- Achieving the design objectives
- Examples

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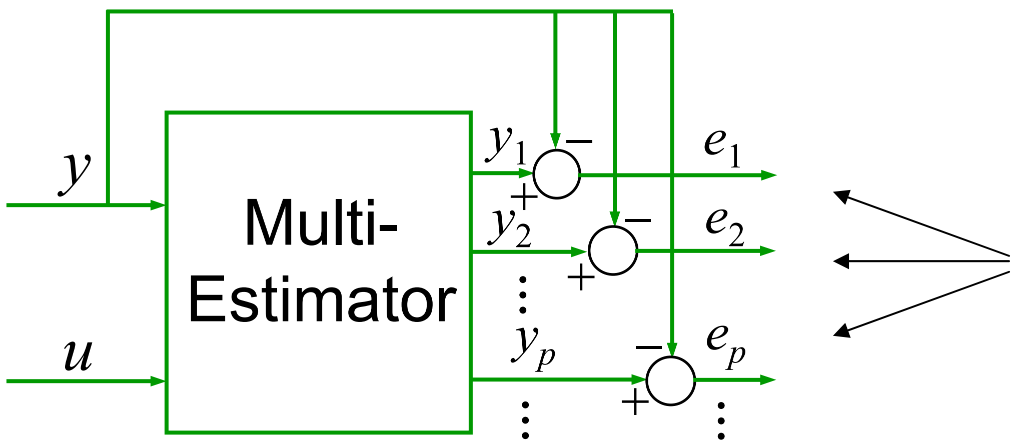
SUPERVISOR



estimation errors:

$$e_p := y_p - y, \;\; p \in \mathcal{P}$$

Want e_{p^*} to be small

Then e_p small indicates $p = p^*$ likely

EXAMPLE

$$\dot{y} = y^2 + p^* u$$

Multi-estimator:

$$\dot{y}_p = -(y_p - y) + y^2 + pu, \ \ p \in \mathcal{P}$$

$$e_p = y_p - y, \ \ p \in \mathcal{P}$$

$$\Downarrow$$

$\dot{e}_{p^*} = -e_{p^*}$ => $e_{p^*} \to 0$ exp fast $\forall\, u$

EXAMPLE

$$\dot{y} = y^2 + p^* u - d$$

disturbance

Multi-estimator:

$$\dot{y}_p = -(y_p - y) + y^2 + pu, \quad p \in \mathcal{P}$$

$$e_p = y_p - y, \quad p \in \mathcal{P}$$

$$\Downarrow$$

$\dot{e}_{p^*} = -e_{p^*} + d \;\; => \;\; e_{p^*} \to d$ exp fast $\;\forall\, u$

STATE SHARING

$$\dot{y}_p = -(y_p - y) + y^2 + pu, \quad p \in \mathcal{P}$$

Bad! Not implementable if $\mathcal{P}$ is infinite

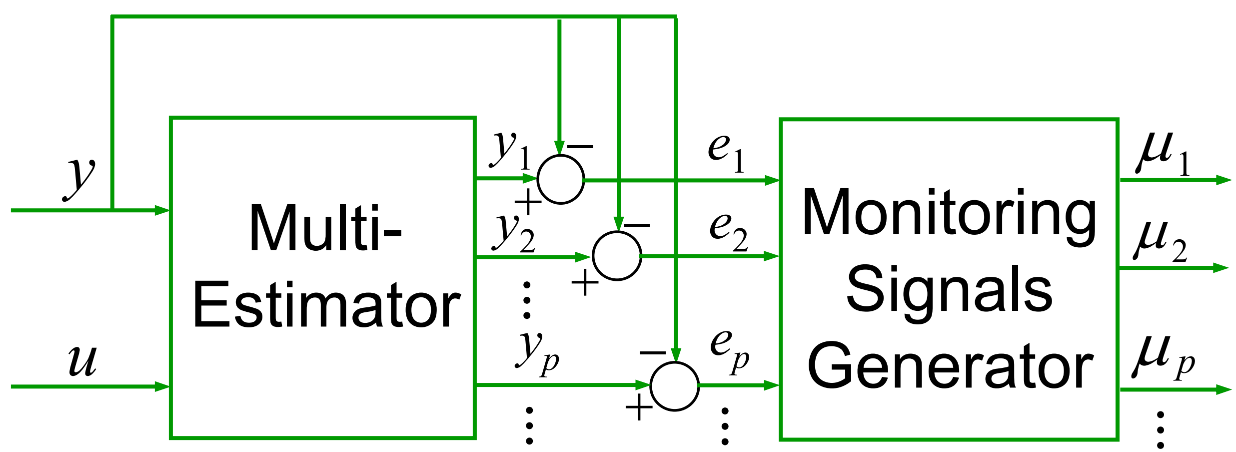
The system

$$\begin{aligned} \dot{z}_1 &= -z_1 + y + y^2 \\ \dot{z}_2 &= -z_2 + u \\ y_p &= z_1 + pz_2, \quad p \in \mathcal{P} \end{aligned}$$

produces the same signals

$$\dot{y}_p = \dot{z}_1 + p\dot{z}_2 = -z_1 + y + y^2 - pz_2 + pu = -y_p + y + y^2 + pu$$

SUPERVISOR



Examples:

$$\mu_p(t) = \int_0^t |e_p(\tau)|^2 d\tau \Leftrightarrow \dot{\mu}_p = |e_p|^2, \ \mu_p(0) = 0$$

$$\mu_p(t) = \int_0^t e^{-\lambda(t-\tau)} |e_p(\tau)|^2 d\tau \Leftrightarrow \dot{\mu}_p = -\lambda \mu_p + |e_p|^2, \ \mu_p(0) = 0$$

EXAMPLE

Multi-estimator:

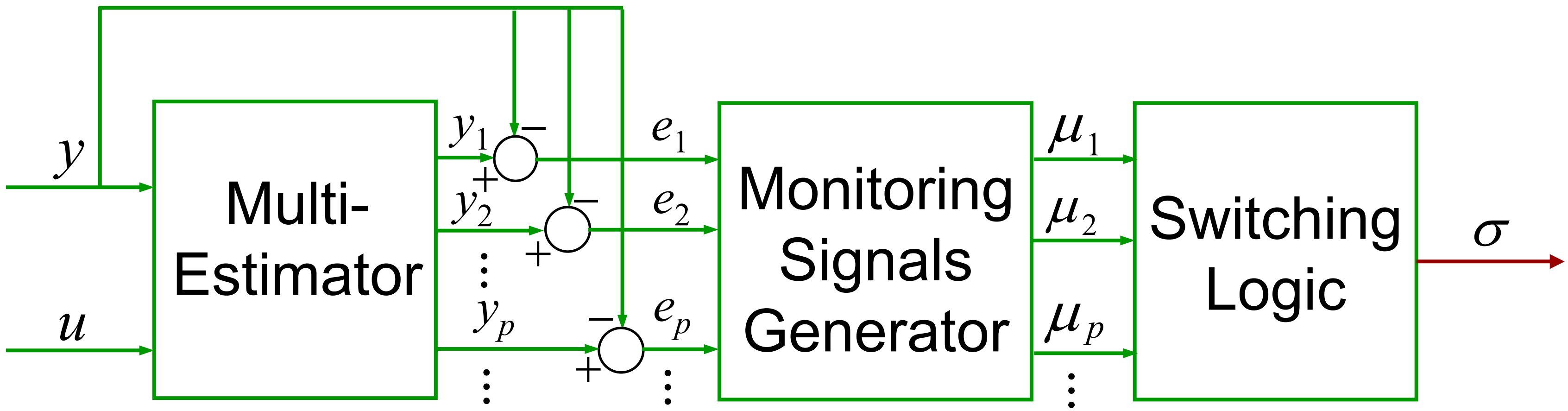
$$\begin{aligned}\dot z_1 &= -z_1 + y + y^2\\ \dot z_2 &= -z_2 + u\\ y_p &= z_1 + p z_2,\ \ p \in \mathcal{P}\end{aligned}$$

$\boxed{\dot\mu_p = e_p^2}$ – can use state sharing

$$e_p^2 = (z_1 + p z_2 - y)^2 = (z_1 - y)^2 + 2 p z_2 (z_1 - y) + p^2 z_2^2$$

$$\begin{aligned}\dot\eta_1 &= (z_1 - y)^2\\ \dot\eta_2 &= 2 z_2 (z_1 - y)\\ \dot\eta_3 &= z_2^2\\ \mu_p &= \eta_1 + p\eta_2 + p^2\eta_3,\ \ p \in \mathcal{P}\end{aligned}$$

SUPERVISOR



Basic idea: $\sigma(t) = \arg\min_{p \in \mathcal{P}} \mu_p(t)$

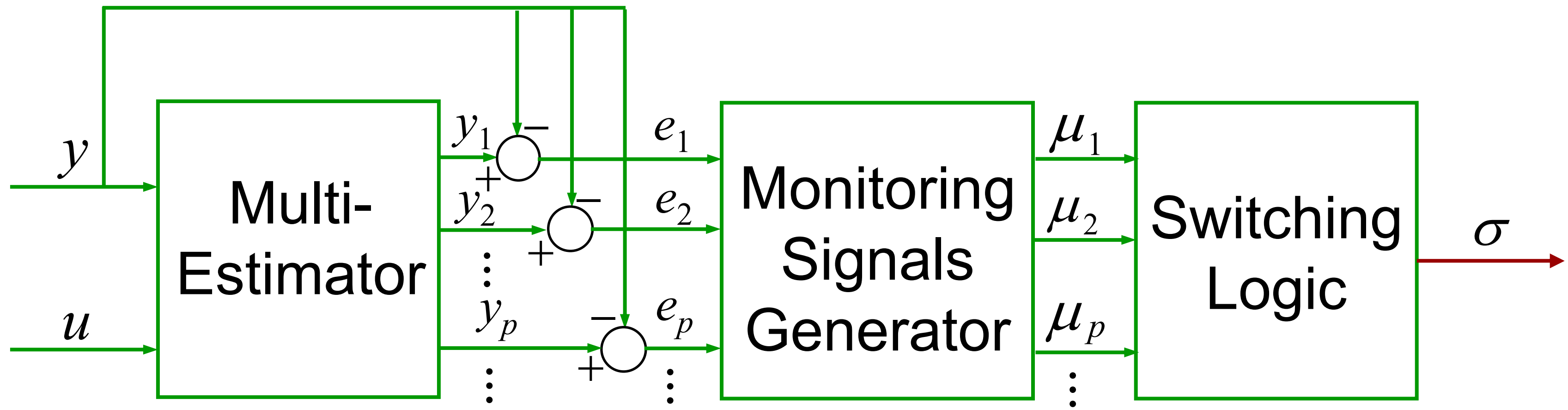
assume $\mathcal{Q} = \mathcal{P}$

Justification? Plant $\in \mathcal{F} = \bigcup_{p \in \mathcal{P}} \mathcal{F}_p$, controllers: $\mathcal{C}_q,\ q \in \mathcal{P}$

μ_p small => e_p small => plant likely in $\mathcal{F}_p$ => $\mathcal{C}_p$ gives stable closed-loop system

("certainty equivalence")

SUPERVISOR



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μ_p small => e_p small $\not\Rightarrow$ plant likely in $\mathcal{F}_p$ => $\mathcal{C}_p$ gives stable closed-loop system

only know converse!

Need: e_p small => $\mathcal{C}_p$ gives stable closed-loop system

This is detectability w.r.t. e_p

DETECTABILITY

Linear case:

$$\dot{x} = A_q x$$ plant in closed loop with $\mathcal{C}_q$

$$e_q = C_q x$$ view as output

Want this system to be detectable

$$\Updownarrow$$

$$\boxed{e_q \to 0 \Rightarrow x \to 0}$$

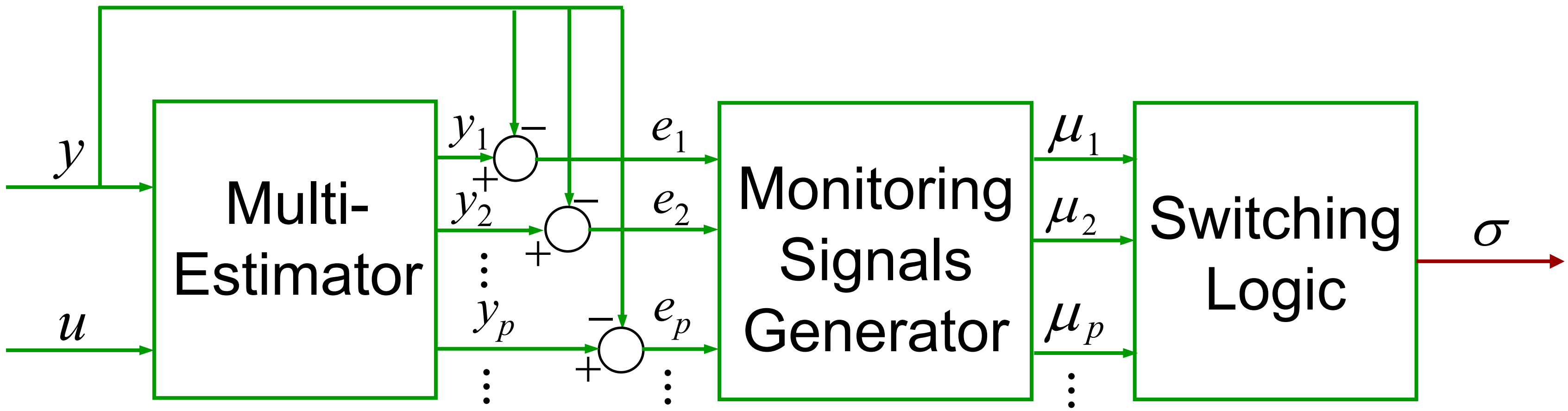
$$\Updownarrow$$

"output injection" matrix

$\exists\, K_q : A_q - K_q C_q$ is Hurwitz

$$\underbrace{\dot{x} = (A_q - K_q C_q)x}_{\text{asympt. stable}} + K_q e_q$$

SUPERVISOR



We know: e_{p^*} is small

Switching logic (roughly): $\sigma(t) = \arg\min_{p \in \mathcal{P}} \mu_p(t)$

This (hopefully) guarantees that e_σ is small

Need: e_σ small => stable closed-loop switched system

This is switched detectability

DETECTABILITY under SWITCHING

Switched system:

$$\dot{x} = A_\sigma x \quad \longleftarrow \text{ plant in closed loop with } \mathcal{C}_\sigma$$

$$e_\sigma = C_\sigma x \quad \longleftarrow \text{ view as output}$$

Want this system to be detectable: $e_\sigma \to 0 \Rightarrow x \to 0$

Assumed detectable for each frozen value of σ

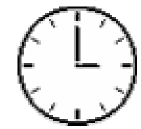
Output injection:

$$\underbrace{\dot{x} = (A_\sigma - K_\sigma C_\sigma)x}_{\text{need this to be asympt. stable}} + K_\sigma e_\sigma$$

Thus σ needs to be "non-destabilizing":

- switching stops in finite time
- slow switching (on the average)

SUMMARY of BASIC PROPERTIES



Multi-estimator:

1. At least one estimation error (e_{p^*}) is small

- $e_{p^*} \to 0 \ \forall u$ when $n = 0,\ d = 0,\ \delta = 0$
- e_{p^*} is bounded for bounded n & d

Candidate controllers:

2. For each $\mathcal{C}_q$, closed-loop system is detectable w.r.t. e_q

Switching logic:

3. e_σ is bounded in terms of the smallest e_p

4. Switched closed-loop system is detectable w.r.t. e_σ provided this is true for every frozen value of σ

conflicting: for 3, want to switch to $\arg\min_p \mu_p(t)$
for 4, want to switch slowly or stop

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The above assumes $\mathcal{Q} = \mathcal{P}$

Otherwise need a controller assigment map $\chi : \mathcal{P} \to \mathcal{Q}$

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Analysis:

$$\left.\begin{array}{l} 1 + 3 \ \Rightarrow\ e_\sigma \text{ is small} \\ 2 + 4 \ \Rightarrow\ \text{detectability w.r.t. } e_\sigma \end{array}\right\} \Rightarrow \text{state is small} \ \checkmark$$

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MULTI-ESTIMATOR

Linear systems: general results available

$$\int_0^t e^{2\lambda\tau} e_{p^*}^2(\tau)\,d\tau \le B_1 e^{2\lambda t} + C_1 + \delta_1 \int_0^t e^{2\lambda\tau} u^2(\tau)\,d\tau$$

come from n & d bounds

system constants

come from δ

$$|e_{p^*}(t)| \le B_2 + C_2 e^{-\lambda t} + \delta_2 e^{-\lambda t} \sqrt{\int_0^t e^{2\lambda\tau} u^2(\tau)\,d\tau}$$

In particular, $e_{p^*} \to 0$ exp fast when $n = 0$, $d = 0$, $\delta = 0$

MULTI-ESTIMATOR

Nonlinear systems: special classes, n, d, δ are 0

Plant:

$$\dot{x} = f(x, u, p^*)$$
$$y = x$$

Multi-estimator:

$$\dot{y}_p = A_p(y_p - y) + f(y, u, p)$$

A_p Hurwitz, $p \in \mathcal{P}$

$e_{p^*} \to 0$ exp fast ✓

Plant:

$$\dot{x} = A_{p^*}x + f(C_{p^*}x, u, p^*)$$
$$y = C_{p^*}x$$

A_{p^*} Hurwitz

Multi-estimator:

$$\dot{x}_p = A_p x_p + f(y, u, p)$$
$$y_p = C_p x$$

A_p Hurwitz, $p \in \mathcal{P}$

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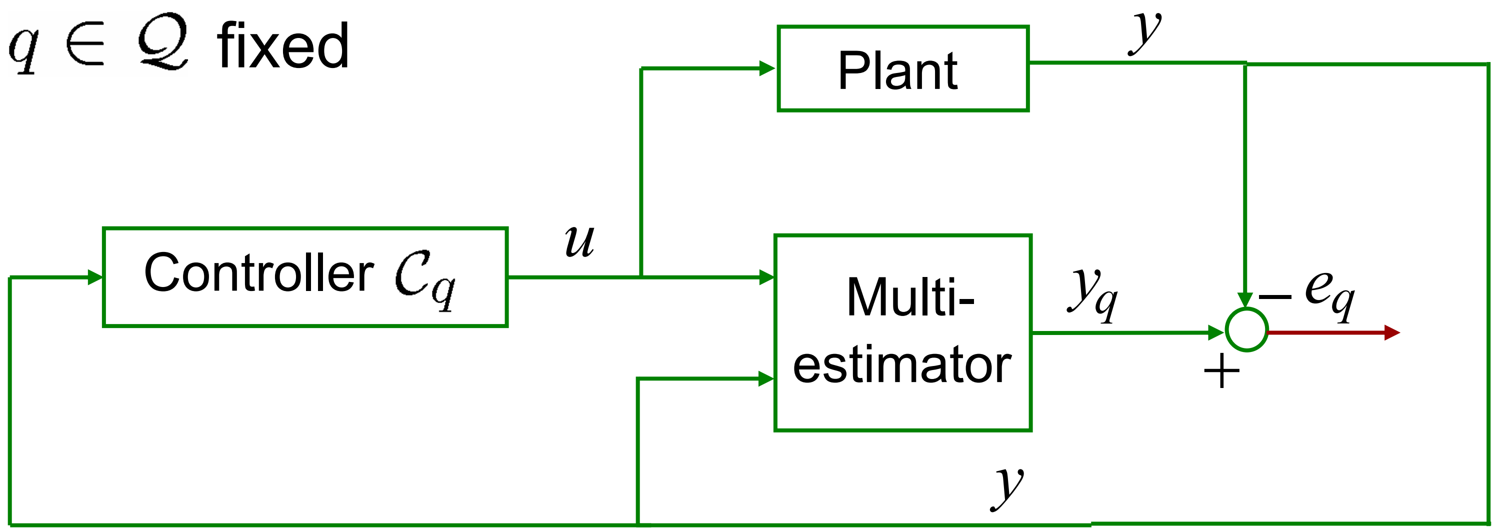
CANDIDATE CONTROLLERS

$q \in \mathcal{Q}$ fixed



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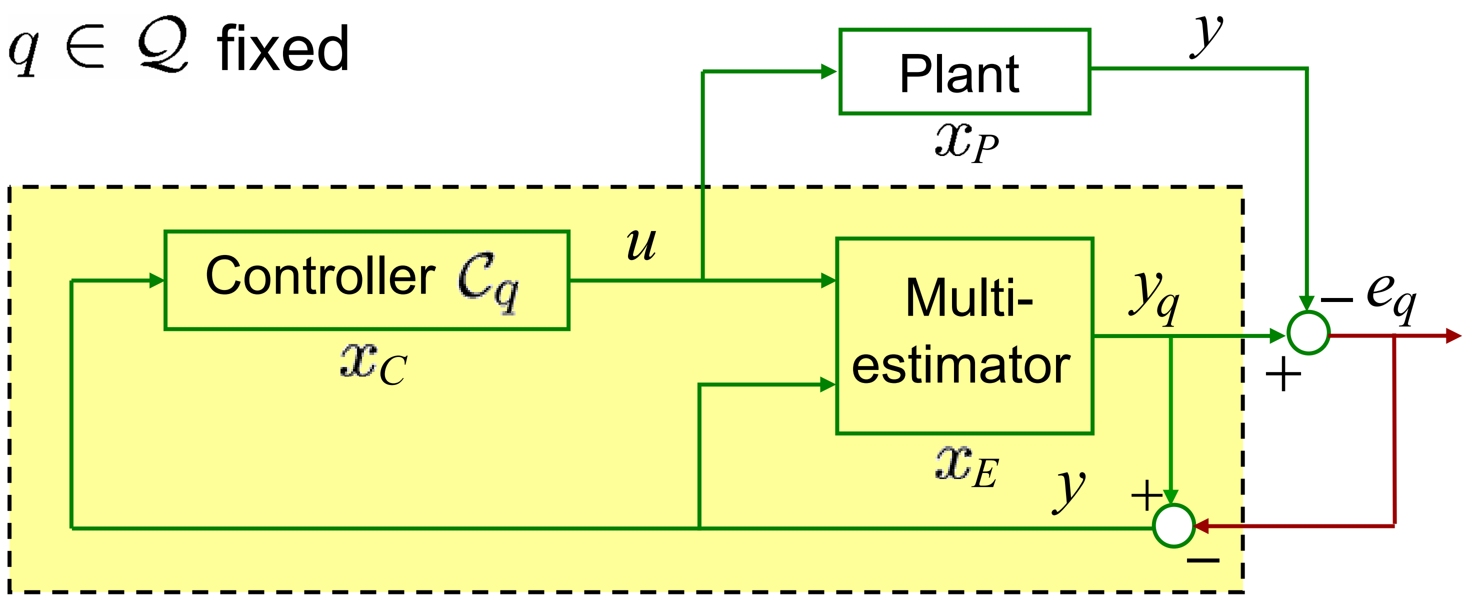


Linear: overall system is detectable w.r.t. e_q if

i. system inside the box is stable

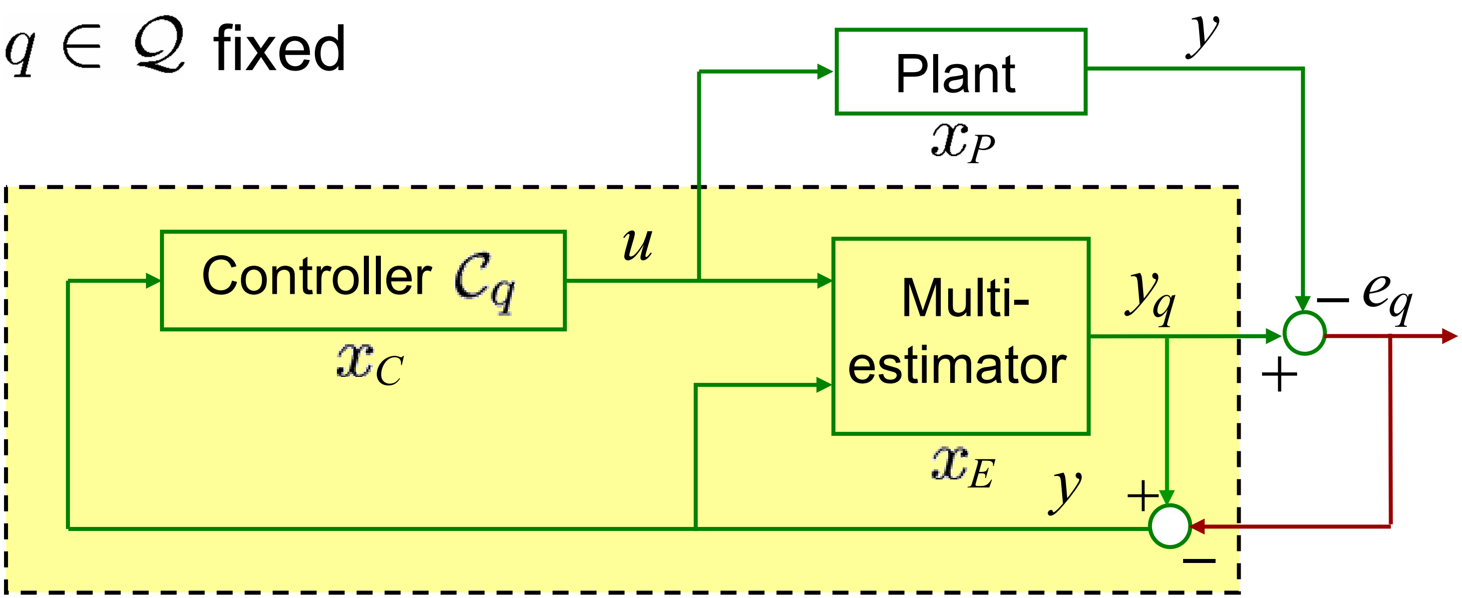
ii. plant is detectable

Need to show: $e_q \to 0 \Rightarrow x_P, x_C, x_E \to 0$

$$e_q \to 0 \underset{\text{i}}{\Rightarrow} x_C, x_E \to 0 \Rightarrow u, y_q \to 0 \Rightarrow y = y_q - e_q \to 0 \underset{\text{ii}}{\Rightarrow} x_P \to 0$$

CANDIDATE CONTROLLERS

$q \in \mathcal{Q}$ fixed



Linear: overall system is detectable w.r.t. e_q if

i. system inside the box is stable

ii. plant is detectable

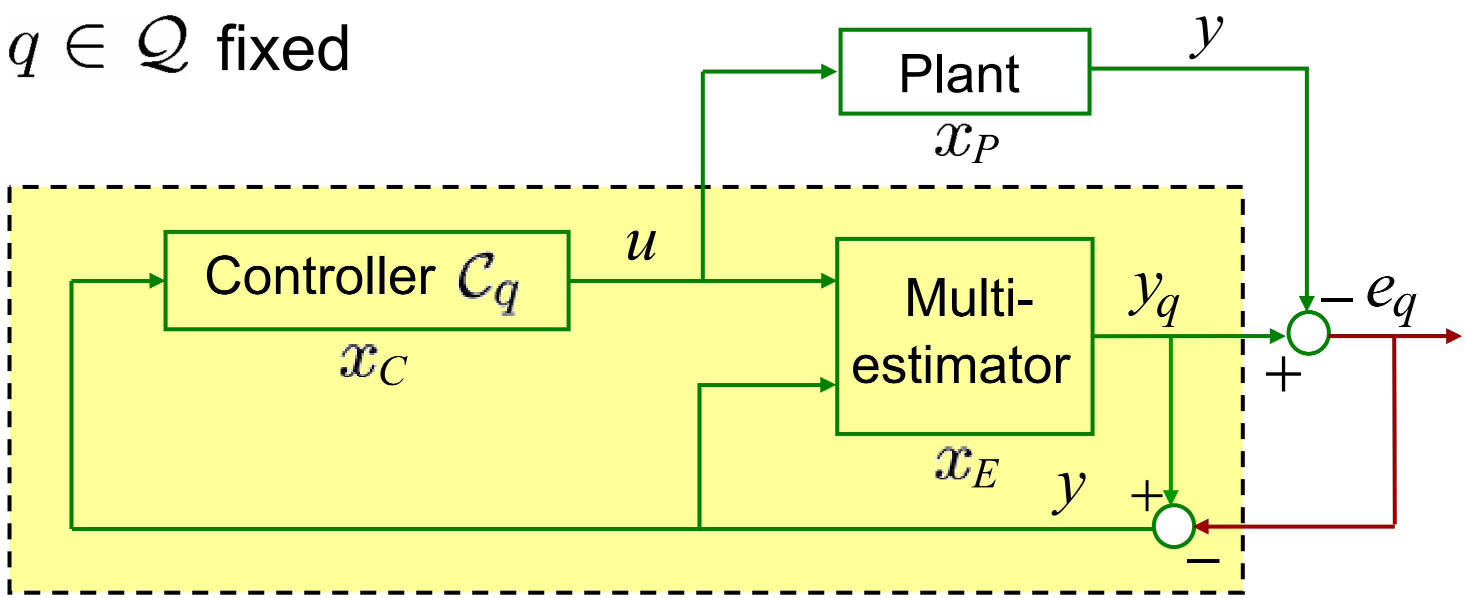
Nonlinear: same result holds if stability and detectability are interpreted in the ISS sense:

external signals

$$|x(t)| \le \beta(|x(0)|,t) + \gamma\left(\|v\|_{[0,t]}\right), \quad \beta \in \mathcal{KL}, \gamma \in \mathcal{K}$$

CANDIDATE CONTROLLERS

$q \in \mathcal{Q}$ fixed



Linear: overall system is detectable w.r.t. e_q if

i. system inside the box is stable

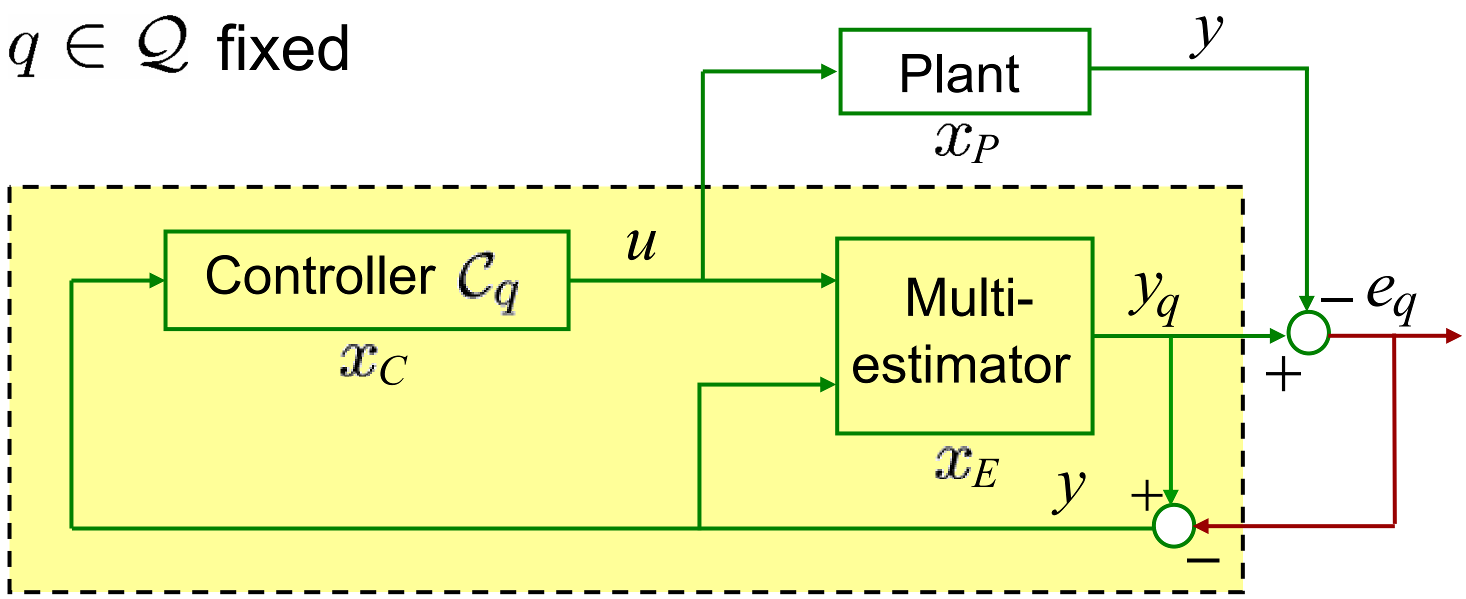
ii. plant is detectable

Nonlinear: same result holds if stability and detectability are interpreted in the integral-ISS sense:

$$|x(t)| \leq \beta(|x(0)|, t) + \int_0^t \gamma(|v(\tau)|)d\tau, \;\; \beta \in \mathcal{KL}, \gamma \in \mathcal{K}$$

CANDIDATE CONTROLLERS

$q \in \mathcal{Q}$ fixed



Linear: overall system is detectable w.r.t. e_q if

i. system inside the box is output-stable

ii. plant is minimum-phase

Nonlinear version also possible

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Candidate controllers:

2. For each $\mathcal{C}_q$, closed-loop system is detectable w.r.t. e_q

Switching logic:

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conflicting: for 3, want to switch to $\arg\min_p \mu_p(t)$
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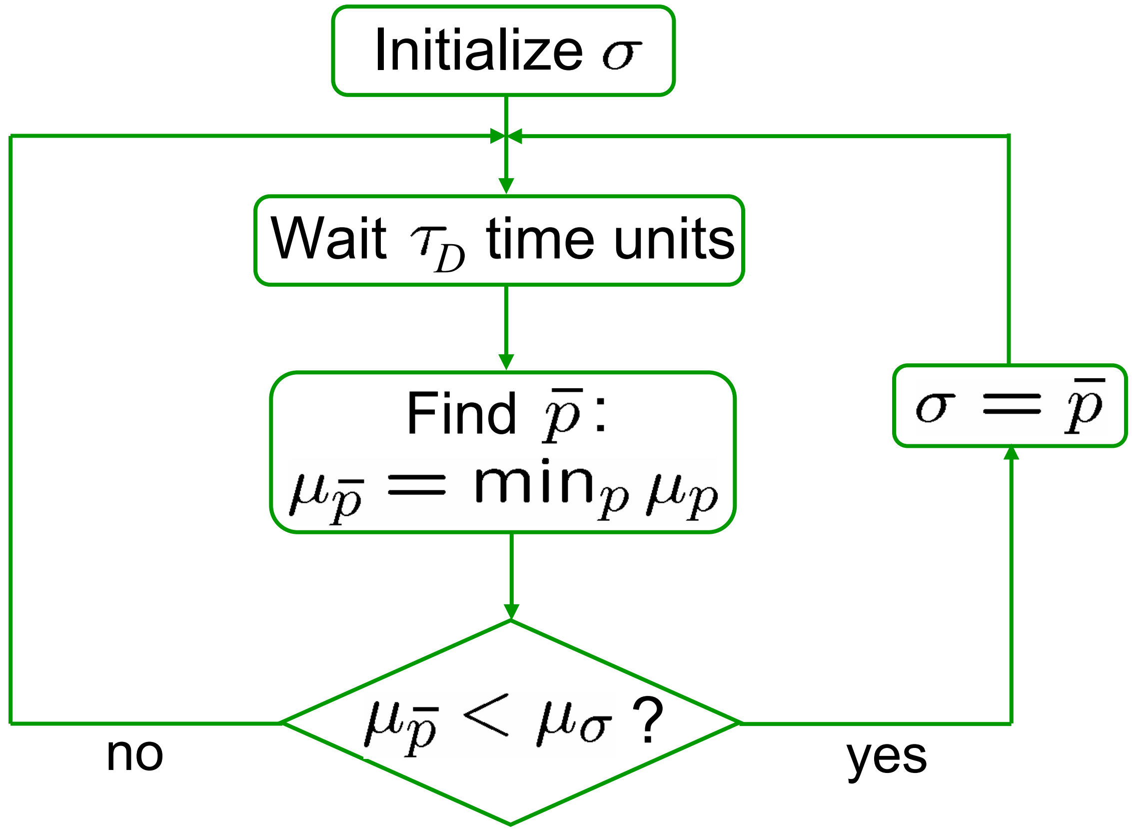
DWELL-TIME SWITCHING LOGIC

$\mu_p,\ p \in \mathcal{P}$ – monitoring signals $\tau_D > 0$ – dwell time



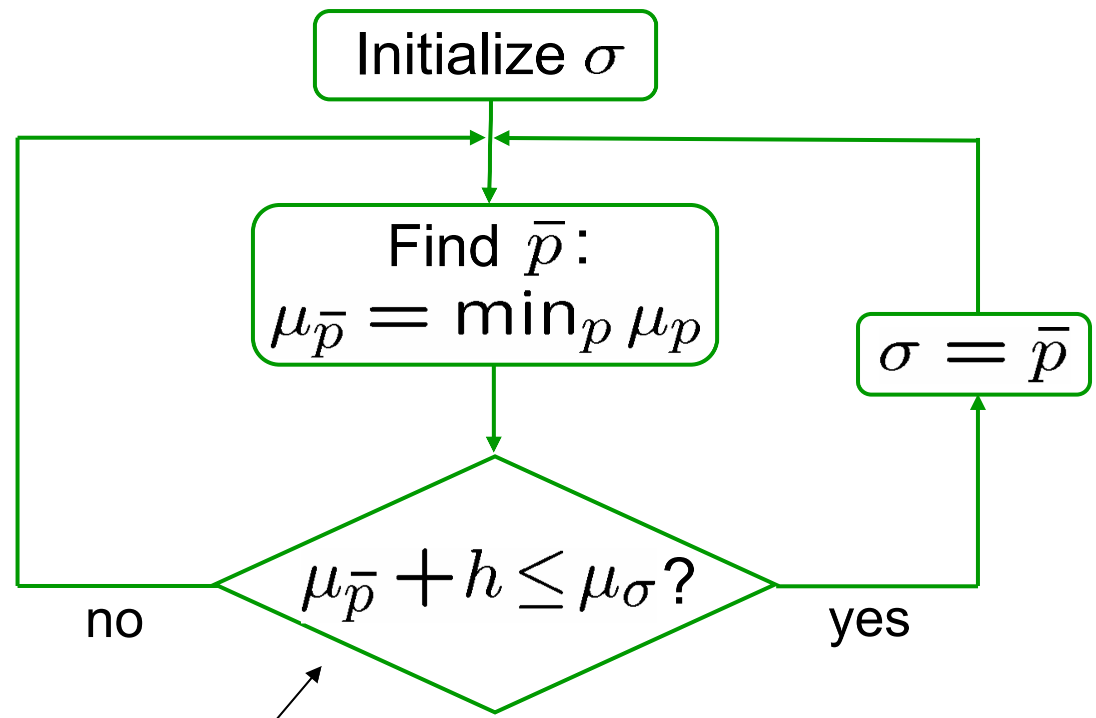
Detectability is preserved if τ_D is large enough ✓

Obtaining a bound on e_σ in terms of e_{p^*} is harder

Not suitable for nonlinear systems (finite escape)

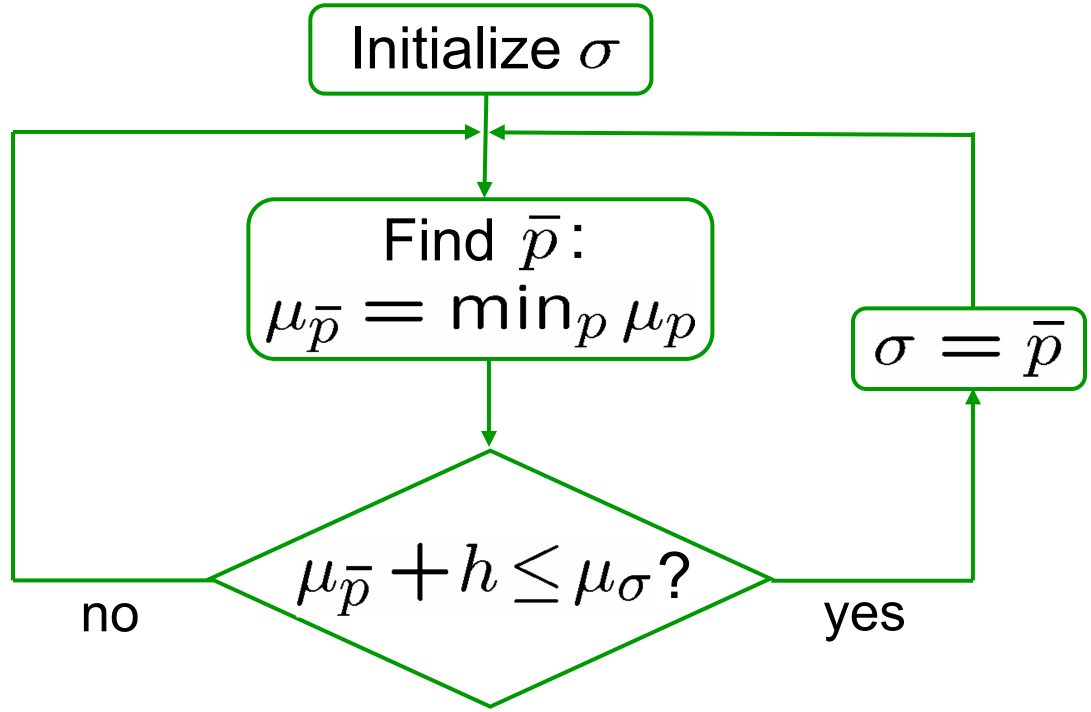
HYSTERESIS SWITCHING LOGIC

$\mu_p,\ p \in \mathcal{P}$ – monitoring signals $\qquad h > 0$ – hysteresis constant



or $(1+h)\mu_{\bar{p}} \le \mu_\sigma$
(scale-independent)

HYSTERESIS SWITCHING LOGIC



$\mathcal{P}$ finite, $\mu_p \uparrow$, μ_{p^*} bounded => switching stops in finite time

This applies to $\delta, n, d = 0$, $e_{p^*} \to 0$ exp fast, $\mu_p = \int |e_p|^2$

Linear, $\delta = 0$, n, d bounded => average dwell time $\tau_{AD}(h)$

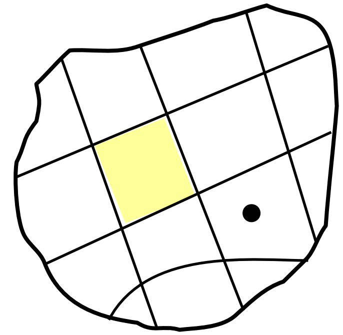
$$\int |e_\sigma|^2 \le |\mathcal{P}|(1+h) \int |e_p|^2$$

HYSTERESIS SWITCHING LOGIC

$\mathcal{P}$ continuum

Partition $\mathcal{D}_q,\ q \in \mathcal{Q}$

finite controller index set



Initialize σ

Find $\bar{p} \in \mathcal{D}_{\bar{q}}$:
$\mu_{\bar{p}} = \min_{p \in \mathcal{P}} \mu_p$

$\mu_{\bar{p}} + h \leq \min_{p \in \mathcal{D}_\sigma} \mu_p$?

no

yes

$\sigma = \bar{p}$

Similar results hold

OUTLINE

- Basic components of supervisor
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LINEAR OBSERVER-BASED DESIGN

Plant: $\dot{x} = A_{p^*}x + B_{p^*}u$ $\quad p^* \in \mathcal{P}$ – finite set

$y = C_{p^*}x$ $\quad$ stabilizable & detectable

Multi-estimator: $\dot{x}_p = (A_p + L_pC_p)x_p + B_pu - L_py$

$y_p = C_px_p$

$A_p + L_pC_p$ Hurwitz, $p \in \mathcal{P}$

Candidate controllers: $u_p = K_px_p$

$A_p + B_pK_p$ Hurwitz, $p \in \mathcal{P}$

Monitoring signals: $\dot{\mu}_p = |e_p|^2, \; \mu_p(0) = 0, \; p \in \mathcal{P}$

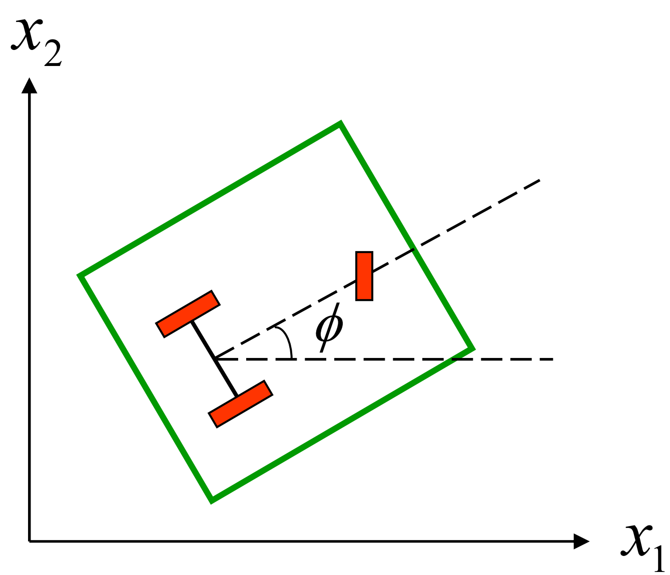
Switching logic: hysteresis

Switching stops, all signals converge to 0 ✓

REASONS for SWITCHING

- Nature of the control problem
- Sensor or actuator limitations
- Large modeling uncertainty
- Combinations of the above

PARKING PROBLEM



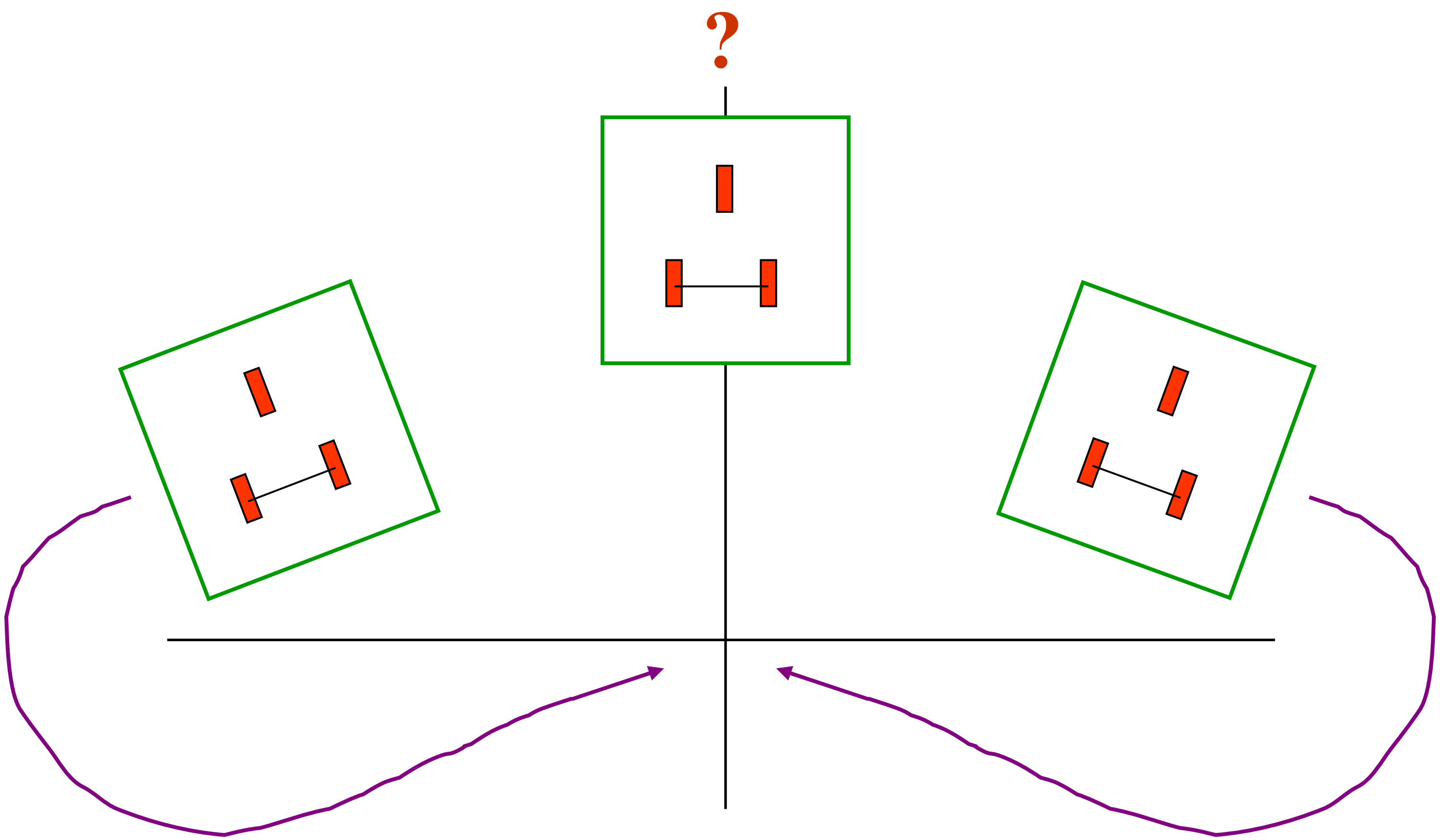
$$\dot{x}_1 = p_1 w_1 \cos\phi$$

$$\dot{x}_2 = p_1 w_1 \sin\phi$$

$$\dot{\phi} = p_2 w_2$$

Unknown parameters p_1, p_2 correspond to the radius of rear wheels and distance between them

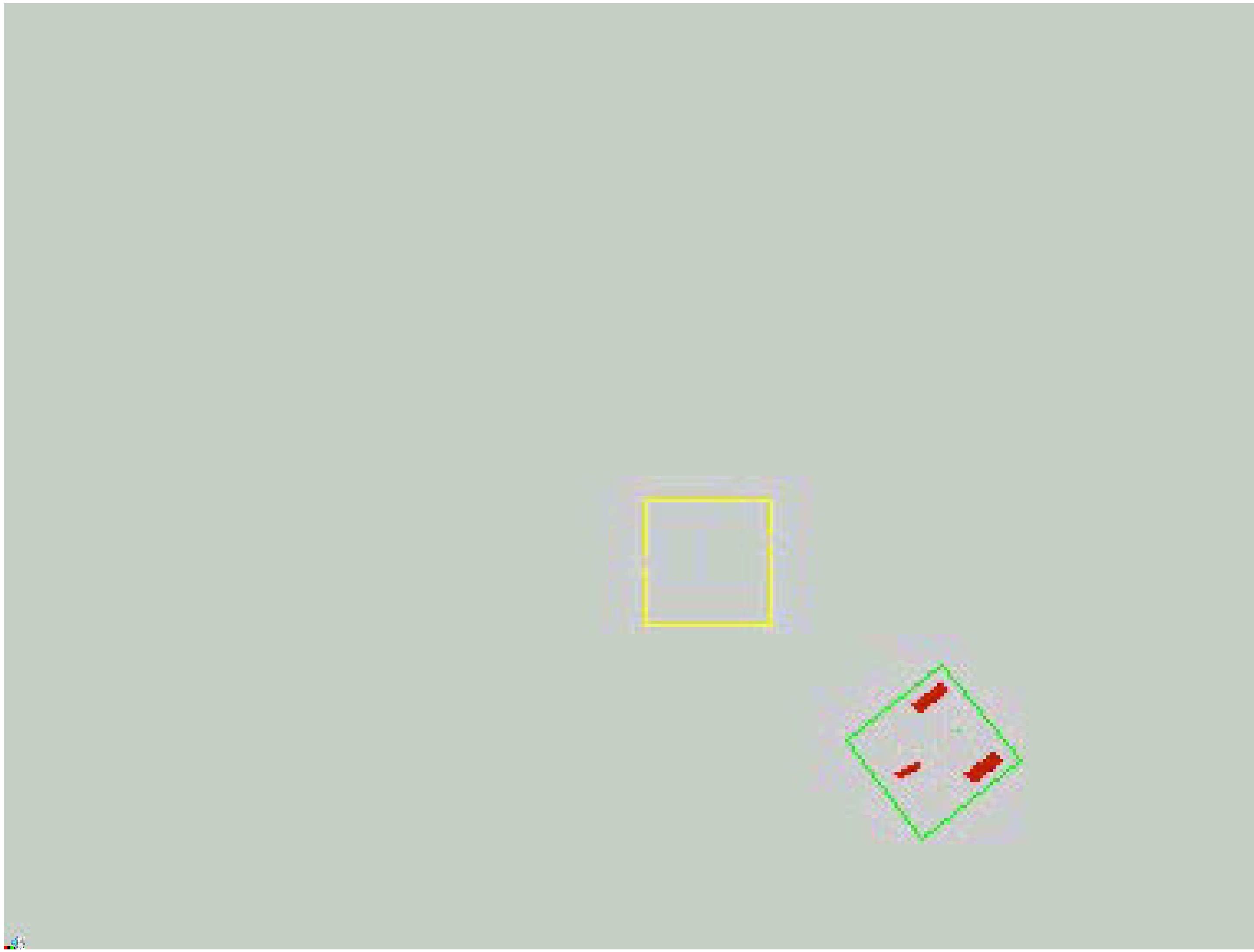
NONHOLONOMY: OBSTRUCTION to STABILIZATION



Solution: move away first

Hybrid candidate controllers!

SIMULATION



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Morse, Hespanha