

# Nonlinear Control for Magnetic Bearings in Deployment Test Rigs: Simulation and Experimental Results

J.M.A. Scherpen<sup>§1</sup>, B. van der Kerk<sup>§</sup>, J.B. Klaassens<sup>§</sup>,  
M. Lazeroms<sup>§</sup>, S.Y. Kan<sup>§§</sup>

<sup>§</sup> Fac. of Inf. Techn. & Syst., Dept. of Electrical Eng.,  
Delft University of Technology, P.O. Box 5031,  
2600 GA Delft, The Netherlands.  
Email: J.M.A.Scherpen@et.tudelft.nl  
Phone: +31-15-278 5768, Fax: +31-15-278 6679

<sup>§§</sup> Fokker Space B.V., P.O. Box 32070,  
2303 DB Leiden, The Netherlands

**In: Proc. CDC'98, Tampa, Fl, USA, pp. 2613–2618**

## Abstract

In this paper three control schemes for a test set-up of a magnetic bearing system for deployment rigs of solar arrays are described. The air gap of the magnet has to be controlled to a constant value independent of the deployment of the solar array. The deployment of the rig has been modeled as a variation in the load of the magnetic bearing. The considered controller design procedures are linear PD, nonlinear PD, and sliding mode. For all three schemes simulations are presented in order to make a comparison. However, the experiments with a test set-up have only been done for the linear PD and the sliding mode control scheme, since they were simple to implement. The results suggest to take the nonlinearities into account, since the nonlinear control schemes give better performance.

**Keywords:** magnetic bearing, solar array, (nonlinear) PD control, sliding mode control.

## 1 Introduction

Large solar arrays are the generators for electric power in space applications. The solar array is launched folded and is unfolded in space. Before launch the deployability must be verified. This test is performed using a deployment test rig. The precise simulation of the deployment motion on the ground is difficult because of the friction at low-speed movement. The test facility has to support the weight of the solar array without disturbing the deployment motion. Several methods have been used such as air-bearings and ball-bearing. The remaining limitations of the existing solutions forced the necessity to other solutions.

This paper presents the methods of control of an asymmetrical active magnetic bearing for the introduced application. The advantages of magnetic bearings are:

- insensible to seams in or dust on the rail,
- large allowable disturbances because of the large air gap (2 mm),
- avoidance of lubrication,
- usable in vacuum.

The active magnetic bearing uses electro-magnetic force to provide contactless support for the solar array under test. The magnetic bearing is made up with one single magnet producing an attracting force counteracting the weight of the deployment test rig. The force is operating in one direction only while the solar array may float without any external force in the other directions. The purpose of the controller is to maintain the device under test at a constant distance to the support (read magnet) of the test rig. The magnetic bearings have the advantage of being contactless, allow low-speed movements, provide active vibration control and adjust the stiffness of the suspending solar array. It is necessary to design a robust controller for the bearing system. The control problem is complicated due to the inherent nonlinearities associated with the electro-mechanical dynamics. Many controllers are based on the PID strategy. Modern control theory provide means for advanced controllers. This paper is focused on the sliding mode control [5], [9], also introduced for a specific magnetic suspension system in [7]. Sliding mode control is robust in the presence of parameter uncertainties and disturbances. It is possible to obtain a stable trajectory for a time varying system that switches between two unstable modes. The results are compared with the results of a simple linear PD controller.

---

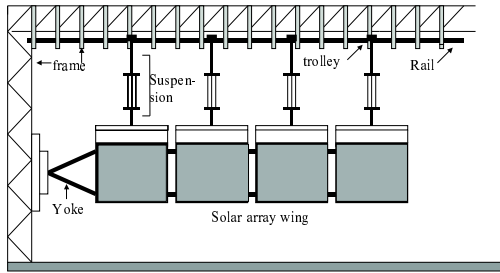
<sup>1</sup>Corresponding author

The magnet is controlled through their coil current. The design of the power electronic system have to guarantee that the amplifier never reaches saturation. Therefore noise induced by the applied sensor have to be included in the design of the controller.

The paper is organized as follows. It start with a description of the applied asymmetrical active magnetic bearing. Next, the design of the controllers are explained. The paper is focusing on the linear and nonlinear PD controllers and the sliding mode controller. Finally, the results of simulation for all controller schemes, and the results of experimental work for the linear PD and sliding mode scheme are compared.

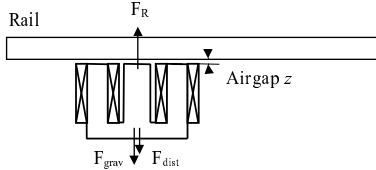
## 2 Magnetic levitation

Figure 1 shows the experimental set-up of a deployment test rig with a fully deployed solar array wing. The magnetic bearings are used in the trolleys in order to create a frictionless motion along the rail. For this application we have build



**Figure 1:** Deployment test rig with a fully deployed solar array wing

an experimental test set-up of one bearing, where our goal is to control the air gap of the magnet to a constant small value (see Figure 2) in order to create a frictionless motion of the trolleys along the rail. We used electromagnets in the magnetic bearing that are E-type cores with coils around each side-pole. Their design must be optimized in order to get a minimum weight for the magnet. This is important to reduce the robustness problems due to inertia. With the electromagnet it is possible to build a magnetic bearing. The bearing consists of the magnet with two coils independently driven by a power amplifier and a rail as shown in Figure 2. In our test set-up we have modeled the employment of



**Figure 2:** Forces acting on the magnetic bearing

the solar array rigs as a variation in the load, i.e., the mass on which the gravitational field applies.  $F_{grav}$  represents the gravitational force (N),  $F_R$  represents the force generated by the magnet(N),  $z$  is the air gap (m),  $i$  the input current of the magnet (A), and  $m$  the weight of the magnet (kg), which varies as a result from the employment of the solar array

rig.  $A$  is the cross-sectional area of the central magnetic pole ( $m^2$ ),  $\mu_0$  is the permeability of vacuum (H/m), and  $N$  the number of windings of the coil. We have the following relation for the magnet:

$$m \frac{d^2}{dt^2} z(t) = F_{grav} - F_R \quad \text{with}$$

$$F_{grav} = mg, \quad F_R = K_{mag} \frac{i(t)^2}{z(t)^2} \quad \text{with} \quad K_{mag} = \frac{\mu_0 N^2 A}{4}$$

The dynamic equation of the system now can be described by the following nonlinear ordinary differential equation:

$$\frac{d^2}{dt^2} z(t) = g - \frac{1}{m} K_{mag} \frac{i(t)^2}{z(t)^2} \quad (1)$$

In order to control the position of the magnetic bearing we need a sensor that measures the position between the magnet and the rail. For this application, we have used an optical sensor. The sensor has a linear transfer characteristic over a distance of 20 mm and the bandwidth is about 3 kHz. The operation range of 20 mm is much more than needed (2-3 mm). There exists noise on the sensor signal from which the influence is studied in the controller designs.

## 3 Controller design

As a result of the experimental work in the Fokker Space Test Laboratory for solar arrays the deployment of the solar array rig can be modeled as a large possible variation in the mass. We want to compensate for this mass variation by a simple, but effective controller design procedure. Our design goal is to develop a controller scheme that controls the air gap of the magnet to a constant value. We consider three different controller design schemes, i.e., linear PD control, nonlinear PD control, and sliding mode control. First a standard linear PD controller based on linearization of the model with a constant load is considered. From this controller we may expect that it is not able to compensate for the modeled load variation, since it is not a robust control scheme. We nevertheless consider the scheme because it is simple, and easy to implement, and we want to compare the results with the other control design schemes that are based on the nonlinear model. The second controller design scheme is a nonlinear PD-control scheme, see e.g. [10, 11], and the third controller design scheme is based on the sliding mode control concept (e.g., [8]). This concept is selected because this scheme is able to handle uncertainties in the model, and the simple scheme often results in an easily implementable controller.

### 3.1 Linear PD controller

In order to calculate the gains for the proportional and derivative action of the linear PD controller, the system has to be linearized around a setpoint  $(z_0, i_0)$ . This setpoint is the steady state situation of the system. Linearization of

system (1) around  $z_0 = 0.002$  (the desired air gap of the magnet), with mass  $m = 8$ , and thus with  $i_0 = 4.2$  yields

$$m \frac{d^2}{dt^2} \bar{z}(t) = K_z \bar{z}(t) - K_i \bar{i}(t)$$

where

$$K_z = 2K_{mag} \frac{i_0^2}{z_0^3} = 39239, \quad K_i = 2K_{mag} \frac{i_0}{z_0^2} = 18.7.$$

By standard PD controller design, we obtain as proportional action  $P = 9700$ , and as derivative action  $D = 49$ .

### 3.2 Nonlinear PD controller

The PD controller designed in the previous paragraph has a constant gain for the proportional and derivative action. The actual operating region of a linear PD controller is quite small, since it is completely based on linearization of the model. The idea of a nonlinear PD controller is to make the gains variable, in order to get different values when the system has a large steady state error and/or a large velocity error. See also [10, 11]. By considering the transient response, some important observations can be made:

- The proportional term dominates when the position error is large. This stems from the observation that if the position error is large the velocity error is close to zero.
- The damping term dominates if the position error is small. This stems from the observation that if the position error is small, the velocity error is large.
- If the position error starts decreasing again, the velocity error is close to its peak.

Based on these observations, we can design a nonlinear PD controller. In [10] it is found that the functions to replace the derivative and proportional gain are the following:

$$D(z, \dot{z}) = \frac{D_1}{1 + \beta \exp(\alpha \dot{z} \bar{z})} + D_0 \quad (2)$$

$$P(z, \dot{z}) = \frac{P_1}{1 + \beta \exp(\alpha \dot{z} \bar{z})} + P_0 \quad (3)$$

where  $D_0, D_1, P_0, P_1, \alpha$ , and  $\beta$  are constant parameters,  $\dot{z}$  is the velocity error, and  $\bar{z}$  the position error, i.e.,  $\dot{z} = \dot{z}_0 - \dot{z} = -\dot{z}$ , and  $\bar{z} = z_0 - z = 0.002 - z$

The maximum and minimum of the function for  $D$  and  $P$  are defined by  $D_1, P_1$ , and  $D_0, P_0$  respectively. At the setpoint, the velocity and position error are zero and thus determines  $\beta$  if the value of the nonlinear function is closer to its maximum (smaller  $\beta$ ) or its minimum (larger  $\beta$ ). The value of parameter  $\alpha$  has influence on sensitivity of the nonlinear function for  $\bar{z}$  and  $\dot{z}$ . The influence of  $\alpha$  is larger if  $\alpha$  itself is larger, and lower if its value is lower. The value of  $\beta$  is set in such way that the proportional and derivative gain of the nonlinear PD controller in the setpoint are the same as the proportional and derivative gain of the linear PD controller. The most important feature of the nonlinear

PD controller is that it creates a damping characteristic that will heavily damp any undesirable movement. In the case of the magnetic bearing the undesirable movement is expected to occur in case of a deployment of the solar array rig. A similar reasoning holds for the stiffness characteristic that is created by the nonlinear gains.

The stability of this nonlinear PD controller can be easily proven by choosing the proper Lyapunov function, see e.g. [10, 11]. So it seems that this nonlinear PD control would be a good choice. However, implementing this controller by analogue techniques is complicated because of the exponential in the gains. For that reason, we also considered a more simple nonlinear PD control scheme that has similar damping characteristics. With a saturation function it is possible to create such damping characteristics. The following controller gains are proposed:

$$D(z, \dot{z}) = \frac{D_1}{1 + \beta \text{sat}(\alpha \dot{z} \bar{z})} + D_0 \quad (4)$$

$$P(z, \dot{z}) = \frac{P_1}{1 + \beta \text{sat}(\alpha \dot{z} \bar{z})} + P_0 \quad (5)$$

where

$$\text{sat}(x) = \begin{cases} -1, & x \leq -1 \\ x, & -1 < x < 1 \\ 1, & x \geq 1 \end{cases}$$

The values for  $P_0, P_1, D_0, D_1$ , and  $\beta$  are chosen such that

$$\begin{aligned} \frac{P_1}{1 + \beta} + P_0 &= P \\ \frac{D_1}{1 + \beta} + D_0 &= D \end{aligned}$$

where  $P$  and  $D$  are the gains from the linear PD controller. The value for  $\alpha$  can be used to tune the sensitivity of the controller. The search for a Lyapunov function in order to prove stability of this controller scheme has not been successful.

### 3.3 Sliding Mode controller

The sliding mode control technique is frequently used in applications of which the parameters of the model are not exactly known. Model imprecision may come from actual uncertainty about the system, or from the choice to use a simplified representation of the system's dynamics. In our application the uncertainty is caused by the load attached to the magnetic bearing and by the magnetic bearing itself.

The sliding mode control scheme is based on the principle that it is easier to control a 1st-order system, than it is to control general  $n$ th-order systems. Accordingly, a notational simplification is introduced, which allows  $n$ th-order problems to be replaced by 1st-order problems. For this transformed problem, almost perfect performance can be achieved in the presence of arbitrary parameter inaccuracies. Such performance, however, is obtained at the price of a very high control activity. The bandwidth of the whole

system must allow this high control activity. In our application the limited bandwidth of the system is limited by the power amplifier including the inductive load. Perfect performance cannot be achieved with sliding control, but we strive to a controller such that the performance of the system is within a acceptable range.

We rewrite equation (1) as

$$\ddot{z} = g + b(z) \cdot u \quad (6)$$

where

$$b(z) = -\frac{K_{mag}}{m \cdot z^2},$$

and the control input  $u$  is given by the square of the input current, i.e.,  $u = i^2$ . In order to apply the simple sliding mode control technique of [8] we transform the state to  $x = -z$ . Then the control gain  $b(x) = \frac{K_{mag}}{m \cdot x^2} > 0$ , which is state dependent and unknown, but since the bounds on the load of the magnet are known, we can also give bounds on  $b(x)$ . The mass varies between 1 kg and 16 kg, which implies that

$$0 < \frac{K_{mag}}{16x^2} =: b_{min}(x) \leq b(x) \leq b_{max}(x) := \frac{K_{mag}}{x^2} \quad (7)$$

Since the control input enters multiplicatively in the dynamics, it is natural to choose the estimate  $\hat{b}(x)$  of  $b(x)$  as

$$\hat{b}(x) = \sqrt{b_{min}(x) \cdot b_{max}(x)}$$

The bounds of equation (7) can be written in the form

$$\beta^{-1} \leq \frac{\hat{b}(x)}{b(x)} \leq \beta, \quad \beta = \sqrt{\frac{b_{max}(x)}{b_{min}(x)}} = 4$$

Since the control law is designed to be robust to the bounded uncertainty (7),  $\beta$  is called the gain margin of the design. In order to have the system track  $x(t) = x_d(t) = -0.002$ , (i.e.,  $\dot{x}_d(t) = 0$ ), a sliding surface  $S(t)$  is defined by  $s(x; t) = 0$ , with  $s = \dot{\tilde{x}} + \lambda \cdot \tilde{x}$  for  $\lambda > 0$ , and  $\tilde{x} = x - x_d = x + 0.002$ . Now we combine a control law that achieves  $\dot{s} = 0$  for the estimated dynamics with a discontinuous term across the surface  $s = 0$  in order to satisfy the sliding condition that makes sure that outside  $S(t)$  we have  $\frac{1}{2} \frac{d}{dt} s^2 \leq -\eta |s|$ , despite of the uncertainty in the control gain. In this construction we have the following constant to take into account:  $k \geq \beta \cdot \eta + (\beta - 1) |\dot{\hat{u}}|$ . By the amplifier and construction of the magnet, we would like to have as maximum for the control  $i = 10$  A. In order to fulfill this specification we take  $k = 340$ . The choice of  $\lambda$  is motivated by its relation to the error that is made. A large value of  $\lambda$  yields a small output error, but also a high control activity. A trade-off between these issues results in our choice of  $\lambda = 1000$ . Furthermore, our controller design was based on  $u = i^2$ , which implies that  $u$  has to be non-negative. Physically this means that the magnet is not able to push the rigs away. In the implementations we solve this by putting  $i = 0$  in case the controller design asks for a negative  $u$ . The gravity makes sure then that the rig is moving away from the magnet, but results in a slower convergence than the original design.

The discontinuous term in the control law causes chattering around the sliding surface. We avoid the chattering by smoothing out the control discontinuity in a thin boundary layer around the switching surface. Then finally our control law becomes:

$$i = \sqrt{\frac{g - 1000 \cdot \dot{x} - 340 \cdot \text{sat}(\dot{x} + 1000 \cdot (x + 0.002))}{\hat{b}(x)}}$$

where  $\text{sat}(y) = -1$  if  $y \leq -1$ ,  $\text{sat}(y) = y$  if  $-1 \leq y \leq 1$ , and  $\text{sat}(y) = 1$  if  $y \geq 1$ .

## 4 Simulation results

First we have compared the results of the previously designed controllers by simulations in MatLab/Simulink. In case of the nonlinear PD controller we have chosen to simulate the original control scheme with the exponential terms. The reason for that is that we did not perform the experimental tests with that controller yet, and complexity of the exponential did not give problems in the simulations. At least now we can make a comparison between the three original control schemes on simulation level. We have done simulations with and without a noise source added to the sensor signal, and filtered by a low-pass filter of 4 kHz. The change of load that we have simulated is a block wave signal of a period of 0.2 sec. and an amplitude of 70 N superimposed on an average load of 80 N, and is shown in Figure 3.

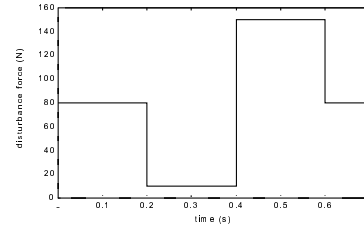


Figure 3: The disturbance force on the system

For each figure one can see that at  $t = 0$  the magnet is brought into its steady state position for an air gap of 0.002 m. Figure 4 shows the response of the system for a linear PD control, including noise acting on the system. Figure 5 shows the response of the system without the noise. The

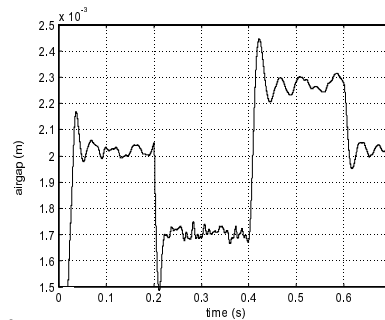
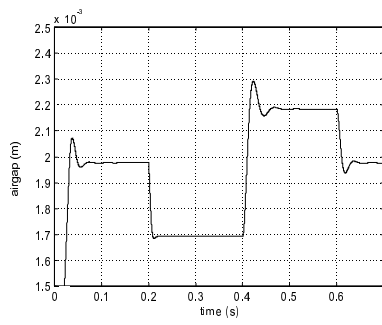
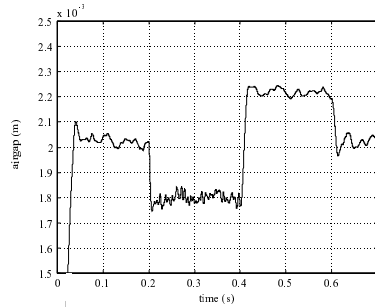


Figure 4: The air gap variation with linear PD control and noise on the sensor

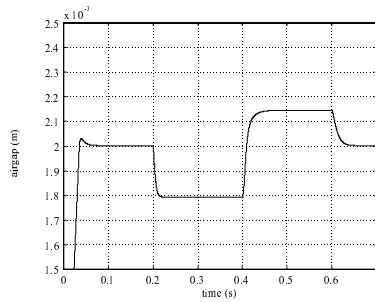


**Figure 5:** The air gap variation with linear PD control and no noise on the sensor

variation of the air gap around the set point of  $2 \cdot 10^{-3}$  m is in the order of  $0.5 \cdot 10^{-3}$  m. In Figure 6 the response of the system under nonlinear PD control, with noise acting on the sensor, is shown. Figure 7 shows the response of the nonlinear PD controlled system without noise. Comparing the



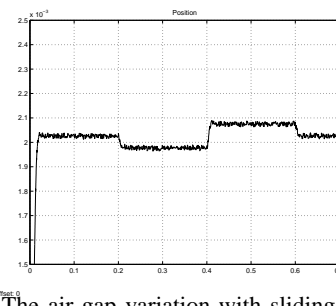
**Figure 6:** The air gap variation with nonlinear PD control and noise on the sensor



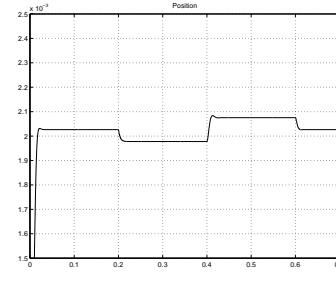
**Figure 7:** The air gap variation with nonlinear PD control and no noise on the sensor

simulation results for the linear and nonlinear PD control schemes, we see that the system under nonlinear PD control has no overshoot at all, while the steady state error is of a slightly smaller order. Hence, the nonlinear PD controller certainly shows better performance.

In Figure 8 the response of the system under sliding mode control, with noise acting on the sensor, is shown. Figure 9 shows the response of the sliding mode controlled system without noise. If we do not considering the initial variation which is due to settling time, the variation in the air gap for the sliding mode controlled system is of the order  $0.09 \cdot 10^{-3}$  m, which is much smaller than the variation for the linear PD controller. In the nonlinear PD case the order is  $0.2 \cdot 10^{-3}$  m, and hence the sliding mode controller shows



**Figure 8:** The air gap variation with sliding mode control and noise on the sensor



**Figure 9:** The air gap variation with sliding mode control and no noise on the sensor

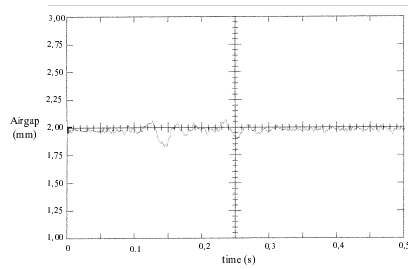
in these simulations the best results. This fits with our expectation, since in our sliding mode controller design we have explicitly taken the robustness against the mass variation into account.

## 5 Experimental Results

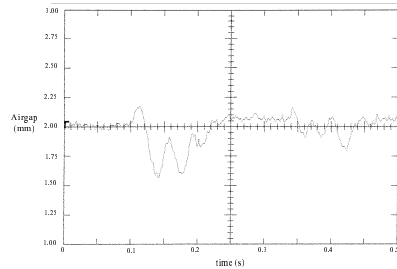
In our experiments we have performed two different tests for both controller schemes. These goals of these tests are to find out

- what is the maximum impulse-shaped load variation, and what is the systems response to this variation.
- what is the response of the system when an oscillating load variation is attached to the magnetic bearing.

In order to create an impulse-shaped load variation, a mass is attached to the magnet by a rope with no elasticity and falls from a certain height. This test method approximates quite well an impulse load variation. The test mass is 0.1 kg. During the test with the PD controlled system it appeared that the maximum height from which we could let the mass fall was 1 cm. Increasing this height made the system unstable. The results with the sliding mode controller were much better, since in that case, we could let the mass fall from a height of 0.1 m before the system became unstable. The responses are shown in Figure 10 and Figure 11. In the second test, the response to an oscillating load variation is considered. This is experimentally realized by a mass of 1.4 kg that is attached to the magnet by a spring, which achieves an oscillation frequency of 2.4 Hz. In Figure 12, and 13 the results of the experiment are shown. From the second test is can be concluded that the sliding mode controller has slightly better performance than the linear PD, since the variation of the air gap is just a little smaller for



**Figure 10:** The variation of the air gap with linear PD control with mass of 0.1 kg falling from a height of 0.01 m

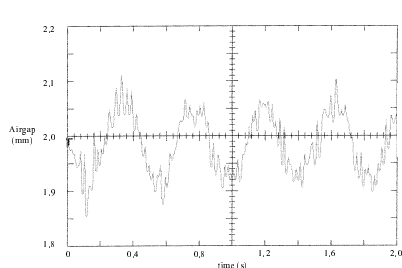


**Figure 11:** The variation of the air gap with sliding mode control with mass of 0.1 kg falling from a height of 0.1 m

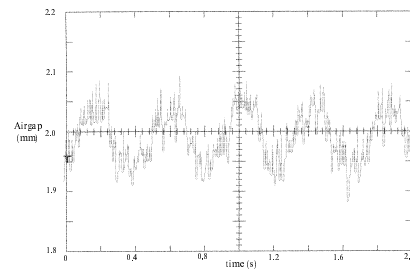
the sliding mode controller. It can be expected that with a larger load variation the sliding mode controller performs significantly better than the linear PD controller. A larger load variation probably results in an air gap variation out of the operating region of the PD controller.

## 6 Conclusions

In this paper we have considered three control schemes for a test set-up of a magnetic bearing system for deployment rigs of solar arrays. The air gap of the magnet has to be controlled to a constant value, and the deployment of the rig has been modeled as a variation in the load of the magnet. In the linear and nonlinear PD controller design it is not possible to take this variation into account. However, in case of a larger variation from the desired air gap, the nonlinear PD controller gives much better performance in the simulations. For the third controller scheme, the sliding mode controller the variation in load is explicitly considered, i.e., the controller is designed to take this model uncertainty into account. The experiments with the test set-up have been done for the linear PD and the sliding mode control scheme, since they were the easiest to implement. These tests show



**Figure 12:** The variation of the air gap with linear PD control with the oscillating load variation



**Figure 13:** The variation of the air gap with sliding mode control with the oscillating load variation

that the sliding mode control scheme indeed performs better than the linear PD controller.

Ongoing research deals with performing new test with this system, in order to improve the test set-up and the model. Furthermore, we study improvement of the design of the controllers, by considering some more nonlinear control schemes, i.e., by using nonlinear  $\mathcal{H}_\infty$  techniques, nonlinear adaptive control techniques, and passivity based control techniques. The simplicity of the controller schemes and the performance (also under disturbances) play an important role in this study. Besides, the sensor is a major source of noise affecting the performance of the controller. A further research on the technique of measuring the air gap is going on.

## Acknowledgment

The authors like to thank Jelle van Hengel for his help with some of the simulations, and Fokker Space BV for their enthusiastic support of this project.

## References

- [1] Auer, F., *Combined electro-magnetic suspension and propulsion for positioning with sub-micrometer accuracy*, Delft University Press, 1995.
- [2] De Miras, J., Charara, A., Caron, B., Nonlinear sliding mode control of electro-magnetic suspension, *13th Triennial IFAC World Congress, San Francisco, USA*, 1996, p463-468.
- [3] Horiuchi, Y., Inoue, M., Miyoshi, K., Sugimoto, T., Okamoto, T., Hariu, K., Development of Magnetically Suspended Sliders for deployable antenna test facility, *Fifth International Symposium on Magnetic Bearings, Kanazawa, Japan*, August 1996, p363-368.
- [4] Kerk, B., van der, Magnetic bearings in deployment test rigs for solar arrays, masters thesis, Delft University of Technology, 1997.
- [5] Lévine, J., Lottin, J., Ponsart, J., A nonlinear approach to the control of magnetic bearings, *IEEE transactions on control systems technology*, Vol. 4, No.5, September 1996, p524-544.
- [6] Neuijen, S., Research into the possibilities of using magnetic bearings in a deployment test rig for solar arrays, Laboratory of Control Engineering, masters thesis, Delft University of Technology, 1996.
- [7] McConnell, R., Gray, W.S., Sliding mode control of a magnetic suspension system using sampled-data, *Proc. 1994 Conf. Inf. Sc. Syst.*, Princeton, New Jersey, 1994, p426-430.

- [8] Slotine, J.E., Li, W., *Applied Nonlinear Control*, Prentice-Hall Inc., 1991.
- [9] Smith, R.D., Weldon, W.F., Nonlinear control of a rigid rotor magnetic bearing system: modelling and simulation with full state feedback", *IEEE Transaction on Magnetics*, Vol.31, pp.973-980, March 1995.
- [10] Xu, Y., Ma, D., Hollerback, J., Nonlinear proportional and derivative control for high disturbance rejection and high gain force control, *IEEE Int. Conf. on Rob. and Aut.*, Vol. 1, Atlanta 1993, pp. 752-759.
- [11] Xu, Y., Ma, D., Hollerback, J., A nonlinear PD controller for force and contact transient control, *IEEE Control Systems*, February 1995, pp. 15-21.