Compressed Markov parameter estimation in PBSID

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***Abstract*—**

I. INTRODUCTION

blablabla

II. THE COMPRESSED MARKOV PARAMTER ESTIMATION

First we define the stacked vector Y:

$$Y = \begin{bmatrix} y_{p+1}, & \cdots, & y_N \end{bmatrix},$$

In a similar way we can obtain the stacked vectors U, X. Further, we define the stacked matrix Z:

$$Z = \begin{bmatrix} \bar{z}_1, & \cdots, & \bar{z}_{N-p+1} \end{bmatrix}.$$

Using the VARX model structure and if the matrix $\Psi = \begin{bmatrix} Z^T & U^T \end{bmatrix}^T$ has full row rank, the Markov parameter set Ξ can be estimated by solving the following linear problem:

$$\min_{\Xi} \|Y - \Xi\Psi\|_F^2. \tag{1}$$

However, for a large window it is possible that the matrix $\Psi = \begin{bmatrix} Z^T & U^T \end{bmatrix}^T$ is singular. If that is the case, apply the Partial Least-Squares (PLS) method as follows:

$$\Psi = \begin{bmatrix} U & U_\perp \end{bmatrix} \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V \\ V_\perp \end{bmatrix}, \tag{2}$$

Define $\breve{\Xi} = \Xi U$, and $\breve{\Psi} = U^T\Psi = \Sigma V$, then the compressed Markov parameter set $\breve{\Xi}$ can be estimated by solving the following linear problem:

$$\min_{\breve{\Xi}} \left\|Y - \breve{\Xi}\breve{\Psi}\right\|_F^2. \tag{3}$$

III. OBTAINING THE EXTENDED OBSERVABILITY TIMES CONTROLLABILITY MATRIX

The approximation of the matrix $\widetilde{\Gamma\mathcal{K}}$, which can be fully constructed by the Markov parameter set Ξ and is given by[1]:

$$\widetilde{\Gamma\mathcal{K}}Z = \begin{bmatrix} \Xi_{(:,1:pm)} \\ \begin{bmatrix} O^{\ell\times m}, & \Xi_{(:,1:(p-1)m)} \end{bmatrix} \\ \begin{bmatrix} O^{\ell\times 2m}, & \Xi_{(:,1:(p-2)m)} \end{bmatrix} \\ \vdots \\ \begin{bmatrix} O^{\ell\times (f-1)m}, & \Xi_{(:,1:(p-f+1)m)} \end{bmatrix} \end{bmatrix} Z. \tag{4}$$

with compressed Markov parameters

$$\widetilde{\Gamma\mathcal{K}}Z = \begin{bmatrix} \breve{\Xi}U^T{}_{(:,1:pm)} \\ \begin{bmatrix} O^{\ell\times m}, & \breve{\Xi}U^T{}_{(:,1:(p-1)m)} \end{bmatrix} \\ \begin{bmatrix} O^{\ell\times 2m}, & \breve{\Xi}U^T{}_{(:,1:(p-2)m)} \end{bmatrix} \\ \vdots \\ \begin{bmatrix} O^{\ell\times (f-1)m}, & \breve{\Xi}U^T{}_{(:,1:(p-f+1)m)} \end{bmatrix} \end{bmatrix} Z \tag{5}$$

Now define:

$$Z = \begin{bmatrix} Z_{(1)} \\ Z_{(2)} \\ \vdots \\ Z_{(p)} \end{bmatrix}, \quad U = \begin{bmatrix} U_{(1)} \\ U_{(2)} \\ \vdots \\ U_{(p)} \end{bmatrix}$$

it becomes

$$\widetilde{\Gamma\mathcal{K}}Z = \begin{bmatrix} \breve{\Xi}\left(\sum_{i=1}^{p} U_{(i)}^T Z_{(i)}\right) \\ \breve{\Xi}\left(\sum_{i=1}^{p-1} U_{(i)}^T Z_{(i+1)}\right) \\ \breve{\Xi}\left(\sum_{i=1}^{p-2} U_{(i)}^T Z_{(i+2)}\right) \\ \vdots \\ \breve{\Xi}U_{(1)}^T Z_{(p)} \end{bmatrix} \tag{6}$$

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[1]For simplicity MATLAB notation is used.