A Decentralized Event-Based Approach for Robust Model Predictive Control

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Abstract. In this paper, we propose an event-based sampling policy to implement a constraint-tightening, robust MPC method. The proposed policy enjoys a computationally tractable design and is applicable to perturbed, linear time-invariant systems with polytopic constraints. In particular, the triggering mechanism is suitable for plants with no centralized sensory node as the triggering mechanism can be evaluated locally at each individual sensor. From a geometrical viewpoint, the mechanism is a sequence of hyper-rectangles surrounding the optimal state trajectory such that robust recursive feasibility and robust stability are guaranteed. The design of the triggering mechanism is cast as a constrained parametric-in-set optimization problem with the volume of the set as the objective function. Re-parameterized in terms of the set vertices, we show that the problem admits a finite tractable convex program reformulation and a linear program relaxation. Several numerical examples are presented to demonstrate the effectiveness and limitations of the theoretical results.

1. Introduction

Nowadays, networked control systems (NCSs) generally demand an array of compatibility and efficiency measures from control design methods, such as utilization under shared resources, applicability to mobile tasks, and compatibility with digital communication infrastructures [4]. Event-based control (EBC) is a class of strategies that aim to improve efficiency of NCSs in the context of communication and computation. In EBC, the dynamics determine the instance to update a control action (contrary to the traditional case where a control action is updated *periodically*) [17]. There are two options to implement such an event-based logic: embedded in the sensory system, the so-called *event-triggered control* [36] and [15], or embedded in the controller, the so-called *self-triggered control* [2] and [28]. The responsible entity to determine an update instance is known as the *triggering mechanism*. In particular, model predictive control (MPC) methods [11] have been the subject of many studies in order to be amended with an EBC mindset.

MPC methods are a class of *on-line* optimization-based control approaches. In these methods, a measure of system performance is optimized over a finite horizon while states and inputs are subject to certain constraints. When the underlying dynamics is uncertain, the specific term *robust* MPC (RMPC) is used for these methods in the literature [25]. We refer the interested reader to the survey papers [27] and [26] that discuss about different aspects of MPC.

Traditionally, the controller solves the corresponding optimization problem at every time step and produces as outcomes two sequences of *optimal* inputs and states. Then, the controller sends the first element of input sequence to the actuators and the remaining elements of the input sequence and the whole state sequence are discarded. These discarded predictions in a standard MPC setting can serve as a basis to design a triggering mechanism. Moreover, the computational burden of MPC methods is a major drawback, hindering their usage in practice. One thus hopes by employing an EBC approach to reduce the frequency at which the underlying optimization problem is solved. Notice that there are already some techniques in the MPC literature (the so-called *warm start* approaches [40]) that exploit the computed sequences at the previous step to speed up the computation process.

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There is also a big incentive to exploit the computed sequences of MPC methods in a class of NCSs, namely, *wireless sensor/actuator networks* (WSANs). In these systems, the most important concern is the energy efficiency, see e.g., [39, Section IV.B]. The main source of energy depletion in a wireless node is the transceiver (responsible for sending and receiving data). To reduce the frequency of data transfer, it is hence more efficient (energy-wise) to aggregate the data into a single packet (if possible) and transmit the resulting packet at once over the communication network [22] and [24].

Statement of contribution: In this paper, an event-triggered (ET) approach is proposed to implement an RMPC method on perturbed, linear time-invariant (LTI) systems. The RMPC method is originally introduced in [31]. The core idea behind the ET approach is to construct a sequence of *hyper-rectangles* around the optimal state sequence available from solving the RMPC problem. Then, these hyper-rectangles will be sent to the sensors. The optimal input sequence will also be transmitted to the actuators. Once the observed states at the sensory units leave these hyper-rectangles, a triggering happens and the states at the triggering instance will be transmitted to the controller. This procedure is then repeated in a sampled-data fashion. A key feature of the proposed ET approach is its ability to decide based on the local observation of each *individual* sensor, whether to trigger or not. This feature stems from the fact that the sets describing the triggering mechanism are hyper-rectangles. Hence, the conditions required for a triggering in different states of the system are independent of each other. This feature is in particular appealing to systems with *decentralized* (spatially dispersed) sensing units, including systems equipped with high-level (or supervisory) MPC methods, e.g., water treatment systems [35], HVAC systems [20], and commercial refrigeration systems [18] to name a few. Notice that the collocation of the triggering mechanism and the sensory units is physically impossible in such systems. Moreover, the addition of a *central* node (on which the triggering mechanism is placed on) to collect the sensory data comes at the price of extra communication bandwidth usage. On the theoretical side, the design of the ET approach is *decoupled* from the design of the underlying RMPC method. As a result, a fair comparison between the performances of the ET and standard implementations of the RMPC method becomes possible. This paper extends the results of the authors' previous work in [9] in multiple directions, in particular, by simplifying the triggering "law". The approach in [9] requires an "advanced" triggering mechanism that is responsible for (i) constructing certain input and state sequences, (ii) evaluating the satisfaction of MPC's constraints by these sequences, and (iii) comparing the values of the cost function based on the constructed sequences with the value function at the last triggering instance. The main contributions of the paper are summarized as follows.

- **Decoupled recursive feasibility and stability:** Given an RMPC method in place, we propose a set-theory-based, ET approach that preserves robust recursive feasibility and robust stability. The proposed approach is decoupled from the control synthesis process and does not require additional assumptions, such as extra conditions on eigenvalues of weighting matrices in the cost function or the need to define user-specified thresholds for the triggering mechanism (Theorem 4.1).
- **Decentralized applicability:** The proposed approach enjoys a decentralized triggering mechanism that only requires local sensory information (Definition 3.3).
- **Tractable convex program reformulation:** We show that a certain type of non-convex volume-maximization problem with set-based constraints that is deployed to design the triggering mechanism admits a finite tractable convex program (CP) reformulation (Theorem 4.4).
- **Suboptimal linear program relaxation:** Motivated by an approach in the literature, we further show that a linear program (LP) relaxation of the CP reformulation is possible (Theorem 4.5).

Literature review: In what follows, we first review several event-triggered, MPC approaches. We then close this section by giving a brief account of several computationally efficient approaches that are customized for MPC problems.

Related works: Let us first mention the shared properties of the references below: linear discrete-time models, event-triggering mechanisms, constrained MPC methods, minimal (to none) coupling of the parameters of the triggering mechanism and the considered MPC method, and a computationally viable approach to design the triggering mechanism.

To deal with practical issues such as a band-limited communication channel, a novel design approach for NCSs is proposed in [16]. They employ the notion of *moving horizon* [30] to design the estimator and controller. A remarkable character of their approach is its ability to decide *on-the-fly* which input channel should be updated (i.e., a certain type *input-channel* event-triggering control). In case of collocated controller and actuator units, an event-based estimator with a bounded covariance matrix is designed in [34]. While the estimator receives data via a Lebesgue sampling approach, it periodically updates the controller's information regarding the disturbances with a polytopic over-approximation of the covariance matrix. The authors of [7] propose an interesting transmission strategy for wireless sensor/controller communications with practical energy-aware provisions (the controller is collocated with the actuator system). Using some predefined thresholds for each state's sensor (i.e., an ℓ_1-type triggering mechanism), the controller is computed offline using an *explicit* MPC approach [6]. Based on a prescribed 2-norm ball around the optimal state trajectory, the authors in [23] propose a triggering mechanism for WSANs. They show that the approach is robustly stable to a set that is a function of the radius of threshold ball and the maximal 2-norm of disturbance. For linear, continuous-time dynamical systems affected by a Wiener process, a co-design method (i.e., simultaneous design of the scheduler and the controller) is proposed in [3]. The main idea is inspired by the notion of *rollout* from *dynamic programming* [8]. More importantly, the authors show that under some mild conditions, an event-based control approach outperforms a traditional control approach w.r.t. closed-loop performance/average transmission rate. (Notice that for most of the approaches in the literature including our paper such a guarantee is not provided.) A set theoretic triggering mechanism is introduced in [10] for systems with collocated controller and sensory units. The approach is inspired by the *tube-based* MPC proposed in [29]. By exploiting the known probability distribution of disturbance, they also guarantee an average sampling rate. However, their tube-contraction method requires a certain type of realization of a discrete-time system, see [10, Remark 8]. Demirel et al., introduce a sensor/actuator event-triggering mechanism for control systems with limited number of control messages (i.e., communication and computation resources are scarce) [12]. They relax the underlying combinatorial problem into a convex one by an appropriate definition of event thresholds. In [19], a packetized approach is proposed for input-affine, nonlinear systems with bounded additive disturbances in continuous-time. In the proposed approach, an RMPC controller (connected via a communication network to the plant) takes into account the mismatched uncertainties while an integral sliding-mode controller [37] (placed at the plant) counters the effect of the matched uncertainties.
Algorithmic viewpoint: An MPC optimization problem is computationally expensive by itself. Hence, the merit of an event-based policy of implementation would be lost if the mechanism demands a drastically higher computational effort compared to the underlying MPC problem. Dunn and Bertsekas in [13] exploit the structure of their problem to reduce the cubic complexity of computing a Newton step to a linear one. In [38], the authors use a specific *ordering* of decision variables to promote a sparse structure that decreases the cost of computing a control action. The authors in [32] employ a simple, gradient-based algorithm to solve an MPC problem while providing *a priori* computational complexity certificate.

The layout of the paper is as follows. The mathematical notions used in the paper are outlined in Section 2. Section 3 is devoted to the considered RMPC method. The main results regarding the event-based implementation policy are introduced in Section 4. Section 5 contains the technical proofs. Several numerical examples are presented in Section 6 to evaluate the effectiveness and limitations of the theoretical results. Finally, we present several future research directions in Section 7.

2. Notation and Preliminaries

We begin with a brief review of the mathematical preliminaries employed in the rest of the paper.

Notation: The set of non-negative integers is denoted by $\mathbb{Z}_{\geq 0}$. Given positive integers m and n, $\mathbb{R}^m$ and $\mathbb{R}^{m\times n}$ represent the m-dimensional Euclidean space and the space of $m \times n$ matrices with real entries, respectively. Given two integers i, j where $i \leq j$, $\{i : j\} := \{i, i+1, \dots, j\}$. For any pairs of vectors $a, b \in \mathbb{R}^n$, the inequality $a < (\leq) b$ is realized in a component-wise manner. Given a vector $v \in \mathbb{R}^n$ and a scalar $p \geq 1$, $\|v\|_p$ denotes the p-norm $\left(\sum_{i=1}^n (v^i)^p\right)^{1/p}$. Given a matrix $M \in \mathbb{R}^{m \times n}$, M_{ij} denotes the i-th row, j-th column entry of M. Moreover, the matrix $M^+ \in \mathbb{R}^{m \times n}$ is the matrix with entries $M^+_{ij} := \max\{0, M_{ij}\}$. The $n \times n$ zero and identity matrices are denoted by 0_n and I_n, respectively. Given a set $\mathcal{S} \subset \mathbb{R}^n$ and a matrix $M \in \mathbb{R}^{m \times n}$, the set $M\mathcal{S}$ denotes the set $\{c \in \mathbb{R}^m : \exists s \in \mathcal{S}, Ms = c\}$. Given a matrix $M \succ 0$ (i.e., positive definite), the squared weighted distance of a point $r \in \mathbb{R}^n$ from a closed set $\mathcal{S} \subset \mathbb{R}^n$ is defined as $d_M(r, \mathcal{S}) := \min_{s \in \mathcal{S}} \ ||r - s||_M^2 = \min_{s \in \mathcal{S}} \ (r - s)^\top M(r - s)$. Denote the projection of r onto $\mathcal{S}$ by $\Pi_M(r, \mathcal{S}) \in \text{argmin}_{s \in \mathcal{S}} d_M(r, \mathcal{S})$. Note that when $\mathcal{S}$ is also convex, the projection is unique. Given sets $\mathcal{C}$ and $\mathcal{D}$, the Pontryagin difference $\mathcal{C} \ominus \mathcal{D}$ and the Minkowski sum $\mathcal{C} \oplus \mathcal{D}$ are defined as $\mathcal{C} \ominus \mathcal{D} := \{c : \ c + d \in \mathcal{C}, \forall d \in \mathcal{D}\}$ and $\mathcal{C} \oplus \mathcal{D} := \{c + d : \forall c \in \mathcal{C}, \forall d \in \mathcal{D}\}$, respectively. The function $\text{sign}(\,\cdot\,)$ represents the standard sign function. Given a set $\mathcal{X} \in \mathbb{R}^n$ and an *extended* real-valued function $f : \mathcal{X} \to [-\infty, +\infty]$, the *effective domain* of f is the set $\text{dom}(f) = \{x \in \mathcal{X} : f(x) < \infty\}$.

The following result will be used frequently in the development of the triggering mechanism.

Lemma 2.1 (Set-difference lower bound [31])**.** *Let r be a vector in $\mathbb{R}^n$, $\mathcal{B}$ and $\mathcal{C}$ be two compact sets in $\mathbb{R}^n$, and M be a positive definite matrix in $\mathbb{R}^{n \times n}$. Then, $d_M(r + c, \mathcal{B}) \leq d_M(r, \mathcal{B} \ominus \mathcal{C})$, for all $c \in \mathcal{C}$.*

We now revisit some notions from convex analysis (see e.g., [21, Section 2] for a compact exposition of the subject). Given a set $\mathcal{S} \subset \mathbb{R}^n$, the support function of $\mathcal{S}$ evaluated at $\eta \in \mathbb{R}^n$ is $h_{\mathcal{S}}(\eta) := \sup_{s \in \mathcal{S}} \ \langle \eta, s \rangle$. The domain $\mathcal{K}_{\mathcal{S}}$ on which the support function is defined is a convex cone pointed at the origin. If $\mathcal{S}$ is bounded, then $\mathcal{K}_{\mathcal{S}} := \mathbb{R}^n$. Given a matrix $M \in \mathbb{R}^{n \times m}$ and a vector $v \in \mathbb{R}^n$, if $M^\top v \in \mathcal{K}_{\mathcal{S}}$, then $h_{M\mathcal{S}}(v) := h_{\mathcal{S}}(M^\top v)$. Suppose $\mathcal{S} \subset \mathbb{R}^n$ is closed and convex. Then, $\mathcal{S} := \{s \in \mathbb{R}^n : \langle \eta, s \rangle \leq h_{\mathcal{S}}(\eta), \forall \eta \in \mathcal{K}_{\mathcal{S}}\}$, i.e., the intersection of its supporting halfplanes. A set $\mathcal{S} \subset \mathbb{R}^n$ is called a polyhedron, if $\mathcal{S} = \{s \in \mathbb{R}^n : A_{\mathcal{S}} s \leq b_{\mathcal{S}}\}$, $A_{\mathcal{S}} \in \mathbb{R}^{m \times n}$, $b_{\mathcal{S}} \in \mathbb{R}^m$. If the polyhedron $\mathcal{S}$ is bounded, the set is called a *polytope* and its representation given above is known as the *H-representation*. Furthermore, the support function $h_{\mathcal{S}}(\eta)$ of a polytope $\mathcal{S}$ is the solution of the LP, $h_{\mathcal{S}}(\eta) = \max_s \ \langle \eta, s \rangle$ subject to $A_{\mathcal{S}} s \leq b_{\mathcal{S}}$. Given the H-representation of a polytope, we employ the notations $a_{i,\mathcal{S}} \in \mathbb{R}^{1 \times n}$ and $a_{\mathcal{S},j} \in \mathbb{R}^{m \times 1}$ to denote the i-th row and the j-th column of $A_{\mathcal{S}}$, respectively. Moreover, $b_{i,\mathcal{S}}$ is the i-th entry of $b_{\mathcal{S}}$. Given a polyhedron $\mathcal{S} \subset \mathbb{R}^n$ and a set $\mathcal{V} \subset \mathbb{R}^n$, assume that $h_{\mathcal{V}}(a_{i,\mathcal{S}}^\top)$ is well-defined for all $i \in \{1 : m\}$. Then, $\mathcal{S} \ominus \mathcal{V} := \left\{z \in \mathbb{R}^n : \ \langle a_{i,\mathcal{S}}^\top, z \rangle \leq b_{i,\mathcal{S}} - h_{\mathcal{V}}(a_{i,\mathcal{S}}^\top), \forall i \in \{1 : m\}\right\}$. For any vector-pairs $l, u \in \mathbb{R}^n$ such that $l < u$, the full-dimensional convex polytope $\mathcal{B}(l, u) := \{x \in \mathbb{R}^n : \ l \leq x \leq u\} = \{x \in \mathbb{R}^n : \ A_{\mathcal{B}} x \leq b_{\mathcal{B}}\}$ is called a *hyper-rectangle*, where $A_{\mathcal{B}} := [I_n \ \ -I_n]^\top$ and $b_{\mathcal{B}} = [u^\top \ \ -l^\top]^\top$.

3. Robust model Predictive Control Method

In this section, we introduce the class of constrained dynamical systems considered in this paper, followed by the description of the RMPC method. At last, we formally state the problem addressed in this paper.

Consider an LTI system with a bounded additive disturbance given by

$$x^+ = Ax + Bu + w, \tag{1}$$

where x^+ is the successor state and x, u, and w are the current state, input and disturbance, respectively. The current state, input, and disturbance are subject to the hard constraints

$$x \in \mathbb{X} \subset \mathbb{R}^{n_x}, \ u \in \mathbb{U} \subset \mathbb{R}^{n_u}, \ w \in \mathbb{W} \subset \mathbb{R}^{n_x}. \tag{2}$$

A system is called the *nominal* system associated with (1) when $w = 0$. Given a positive integer N, let $\mathscr{U} := \mathbb{U}^N = \prod_{i=0}^{N-1} \mathbb{U}$ ($\mathscr{W} := \mathbb{W}^N$) denote the class of admissible control sequences $\boldsymbol{u} := \{u_i\}_{i \in \{0:N-1\}}$ (admissible disturbance sequences $\boldsymbol{w} := \{w_i\}_{i \in \{0:N-1\}}$). Initiated at state x, the solution to (1) at time i with the control and disturbance sequences $\boldsymbol{u}$ and $\boldsymbol{w}$, respectively, is denoted by $\phi_i^{\boldsymbol{u},\boldsymbol{w}}(x)$. Similarly, we define $\boldsymbol{\phi}^{\boldsymbol{u},\boldsymbol{w}}(x) := \{\phi_i^{\boldsymbol{u},\boldsymbol{w}}(x)\}_{i \in \{0:N\}}$. Moreover, let $\phi_i^{\boldsymbol{u},\boldsymbol{0}}(x)$ denote the nominal solution with the input sequence $\boldsymbol{u}$ initiated at state x. The RMPC method is designed such that the state x and the input u eventually converge to some user-defined *target sets* $\mathbb{T}^{\mathbb{X}} \subset \mathbb{R}^{n_x}$ and $\mathbb{T}^{\mathbb{U}} \subset \mathbb{R}^{n_u}$, respectively, while the constraints (2) are satisfied at all times.

Assumption 3.1 (System & constraint sets)**.** *(i) Nominal controllability: The pair (A, B) is controllable. (ii) Polytopic sets: The sets $\mathbb{X}$, $\mathbb{U}$, $\mathbb{T}^{\mathbb{X}}$, $\mathbb{T}^{\mathbb{U}}$, and $\mathbb{W}$ are all convex, compact polytopes containing their underlying spaces' origin in their interior.*

We start with introducing two types of feedback gains which are used in the RMPC method and are essential for the construction of the triggering mechanism. Let $F \in \mathbb{R}^{n\times m}$ be a given feedback gain that guarantees the stability of the nominal system with $u = Fx$. The *nominal* gain F can be designed so that a satisfactory performance (e.g., in an LQ optimal control sense) is guaranteed for the nominal system.

Let integer $N \geq n_x + 1$ be the horizon length of the RMPC method and integer M be given, where $M \in \{n_x : N-1\}$. Suppose next that a set of feedback gains $\boldsymbol{K} = \{K_i\}_{i\in\{0:N-1\}}$ are given such that $\prod_{i=1}^{M}(A + BK_i) = 0$, i.e., for all $k \geq M$, $\phi_k^{\boldsymbol{u},\boldsymbol{0}}(x) = 0$. We call the set of gains $\boldsymbol{K}$ the *tightening gains* since these gains are employed in the state and input constraint tightening process. We refer the interested reader to [31, Section IV] for a possible approach to construct the gains $\boldsymbol{K}$. The constraint tightening approach is applied to the input, state, input target, and state target sets, that is, for all $i \in \{0 : N-2\}$,

(3a) $$\mathcal{U}_0 = \mathbb{U}, \qquad \mathcal{U}_{i+1} = \mathcal{U}_i \ominus K_i L_i \mathbb{W},$$

(3b) $$\mathcal{X}_0 = \mathbb{X}, \qquad \mathcal{X}_{i+1} = \mathcal{X}_i \ominus L_i \mathbb{W},$$

(3c) $$\mathcal{T}_0^{\mathbb{U}} = \mathbb{T}^{\mathbb{U}}, \quad \mathcal{T}_{i+1}^{\mathbb{U}} = \mathcal{T}_i^{\mathbb{U}} \ominus K_i L_i \mathbb{W},$$

(3d) $$\mathcal{T}_0^{\mathbb{X}} = \mathbb{T}^{\mathbb{X}}, \quad \mathcal{T}_{i+1}^{\mathbb{X}} = \mathcal{T}_i^{\mathbb{X}} \ominus L_i \mathbb{W},$$

where $L_0 = \mathsf{I}_{n_x}$ and $L_{i+1} = (A + BK_i)L_i$ for all $i \in \{0 : N-2\}$. Notice that the M-step nilpotency of the set of gains $\mathbf{K}$ implies that for all $i \in \{M : N-1\}$, $L_i = \mathsf{0}_{n_x}$.

Let the terminal set $\mathcal{X}_f \subset \mathbb{R}^{n_x}$ be a *control invariant set* for the nominal system, i.e., $(A+BF)\xi \in \mathcal{X}_f$ for all $\xi \in \mathcal{X}_f$.

Assumption 3.2 (Terminal set)**.** *For all $\zeta \in \mathcal{X}_f$, the following conditions hold:*

$$\zeta \in \mathcal{X}_{N-1} \cap \mathcal{T}_{N-1}^{\mathbb{X}}, \;\; F\zeta \in \mathcal{U}_{N-1} \cap \mathcal{T}_{N-1}^{\mathbb{U}}.$$

For the sake of notational simplicity, let us define $\mathscr{U}_N := \prod_{i=0}^{N-1} \mathcal{U}_i$ and $\mathscr{X}_N := \prod_{i=0}^{N-1} \mathcal{X}_i \times \mathcal{X}_f$. The *cost function* of the RMPC problem is

(4) $$V_N(x, \boldsymbol{u}) := \sum_{i=0}^{N-1} d_Q(\phi_i^{\boldsymbol{u},\boldsymbol{0}}(x), \mathcal{T}_i^{\mathbb{X}}) + d_R(u_i, \mathcal{T}_i^{\mathbb{U}}) + \delta_{\text{feas}}\big(\boldsymbol{u}, \boldsymbol{\phi}^{\boldsymbol{u},\boldsymbol{0}}(x)\big),$$

where $\delta_{\text{feas}}\big(\boldsymbol{u}, \boldsymbol{\phi}^{\boldsymbol{u},\boldsymbol{0}}(x)\big) = 0$ if $\boldsymbol{u} \in \mathscr{U}_N$ and $\boldsymbol{\phi}^{\boldsymbol{u},\boldsymbol{0}}(x) \in \mathscr{X}_N$, and $= \infty$ otherwise, is the *indicator* function of the set $\mathscr{U}_N \times \mathscr{X}_N$. Notice that the input and state constraints are embedded in the objective function via the indicator function. The optimization problem for a finite horizon N with an initial state x reads as

(5) $$V_N^*(x) := \min_{\boldsymbol{u}} V_N(x, \boldsymbol{u}),$$

with $\boldsymbol{u}^{\text{mpc}}(x) := \text{argmin}_{\boldsymbol{u}} V_N(x, \boldsymbol{u})$ as the optimal input sequence. When it is clear from the context, we may instead use the shorthand notation $\boldsymbol{u}^{\text{mpc}}$. The above sequence of inputs is indeed an optimal solution to a nominal (i.e., $\boldsymbol{w} = \boldsymbol{0}$) finite optimization problem emerging in the context of finite horizon MPC in the rest of the paper. In this light, we denote this nominally optimal controller by a similar label, for which the associated nominal state sequence is $\boldsymbol{\phi}^{\boldsymbol{u}^{\text{mpc}},\boldsymbol{0}}(x)$.

In a standard RMPC setting, the optimal control problem (5) is solved. The first element $u_0^{\text{mpc}}(x)$ of $\boldsymbol{u}^{\text{mpc}}(x)$ is then applied to the plant yielding to the closed-loop dynamics $x^+ = Ax + Bu_0^{\text{mpc}}(x) + w$. In an event-based setting, the triggering mechanism generally exploits the optimal state sequence $\boldsymbol{\phi}^{\boldsymbol{u}^{\text{mpc}},\boldsymbol{0}}(x)$ in order to possibly employ the rest of elements in the nominally optimal input vector $\boldsymbol{u}^{\text{mpc}}(x)$. The challenge in designing the triggering mechanism is then to guarantee robust stability and robust recursive feasibility of the resulting event-triggered, closed-loop dynamics.

Definition 3.3 (Triggering mechanism)**.** *Given an initial state x and a sequence of (possibly) state-dependent, hyper-rectangular sets $\boldsymbol{\mathcal{E}}(x) := \mathcal{E}_0 \cup \{\mathcal{E}_i(x)\}_{i=1}^{N-1} \subset (\mathbb{R}^{n_x})^N$, the triggering instance is defined by*

$$k^{\boldsymbol{w}}_{trig}(x) := \min\big\{j \in \{0 : N-1\} : \phi_j^{\boldsymbol{u}^{mpc},\boldsymbol{w}}(x) - \phi_j^{\boldsymbol{u}^{mpc},\boldsymbol{0}}(x) \notin \mathcal{E}_j(x)\big\}, \tag{6}$$

where $\mathcal{E}_0 := \mathbb{R}^{n_x}$.

The quantity $k^{\boldsymbol{w}}_{\text{trig}}(x)$ is known as the *inter-execution* time in the literature. One can observe that $\phi_0^{\boldsymbol{u}^{\text{mpc}},\boldsymbol{w}}(x) = \phi_0^{\boldsymbol{u}^{\text{mpc}},\boldsymbol{0}}(x) = x$. As a result, $\phi_0^{\boldsymbol{u}^{\text{mpc}},\boldsymbol{w}}(x) - \phi_0^{\boldsymbol{u}^{\text{mpc}},\boldsymbol{0}}(x) = 0 \in \mathbb{R}^{n_x} = \mathcal{E}_0$, and thus $k^{\boldsymbol{w}}_{\text{trig}}(x) \geq 1$. The closed-loop dynamics is then, for all $t \in \mathbb{Z}_{\geq 0}$,

$$\xi_{t+1} = A\xi_t + Bu^{\text{mpc}}_{t-\tau_t}(\xi_{\tau_t}) + w_t, \tag{7a}$$

$$\tau_{t+1} = \begin{cases} \tau_t, & t-\tau_t \leq N-1 \text{ and } \xi_t - \phi_{t-\tau_t}^{\boldsymbol{u}^{\text{mpc}},\boldsymbol{0}}(\xi_{\tau_t}) \in \mathcal{E}_{t-\tau_t}(\xi_{\tau_t}), \\ t, & \text{otherwise,} \end{cases} \tag{7b}$$

given the initial state ξ_0 and the initial triggering instance $\tau_0 = 0$. Here, τ_t denotes the last triggering instance up to time t. Also, notice that a mandatory triggering is put in place at time $\tau_t + N$. The problem addressed in this paper is now introduced.

Problem 3.4. *Consider the closed-loop dynamics* (7) *under Assumptions 3.1-3.2. Devise an approach to construct the sequence of triggering sets $\boldsymbol{\mathcal{E}}(\xi_{\tau_t})$ in* (6) *such that the trajectories of the closed-loop dynamics satisfy:*

- **Recursive feasibility:** *If $V_N^*(\xi_0) < \infty$, then $V_N^*(\xi_t) < \infty$, for all $t \in \mathbb{Z}_{\geq 0}$;*
- **Robust stability:** *The states and inputs of the closed-loop dynamics converge to the target sets $\mathbb{T}^{\mathbb{X}}$ and $\mathbb{T}^{\mathbb{U}}$, respectively ($\lim_{t\to\infty} V_N^*(\xi_t) = 0$).*

Remark 3.5 (Smart actuators and sensors)**.** *The actuator and sensor units are "smart" in the following sense. The actuator (sensor) units can buffer the time-stamped and packetized sequence $\boldsymbol{u}^{mpc}(\xi_{\tau_t})$ ($\{\phi_s^{\boldsymbol{u}^{mpc},\boldsymbol{0}}(\xi_{\tau_t}) \oplus \mathcal{E}_s(\xi_{\tau_t})\}_{s=1}^{N-1}$). The actuator units consecutively apply the input action $u^{mpc}_{s-\tau_t}(\xi_{\tau_t})$ on the plant at each time $s \in \{\tau_t : \tau_{t+1} - 1\}$. The sensor units evaluate the triggering condition*

$$\xi_s \notin \phi_{s-\tau_t}^{\boldsymbol{u}^{mpc},\boldsymbol{0}}(\xi_{\tau_t}) \oplus \mathcal{E}_{s-\tau_t}(\xi_{\tau_t}),$$

at each time $s \in \{\tau_t + 1 : \tau_t + N - 1\}$. When the triggering condition holds at some time s, the sensors send the most recent states ξ_s to the controller and the triggering instance is set to $\tau_{t+1} = s$.

Remark 3.6 (Iteration Complexity)**.** *RMPC problems with linear dynamics, a quadratic cost function, and polytopic constraints are* quadratic programs *for which dedicated solvers provide the complexity per iteration* $\mathcal{O}(N(n_x + n_u)^3)$ [38].

4. Main Results

In this section, we provide several approaches to construct the sequence of sets $\boldsymbol{\mathcal{E}}(x)$ which meets the requirements of Problem 3.4. To this end, we begin with describing a certain type of constrained optimization problem that produces $\boldsymbol{\mathcal{E}}(x)$. Based on these constructed sets, we then state the main theoretical results of this paper.

4.1. Construction of Hyper-Rectangles

Let $j \in \{1 : N-1\}$. The procedure to construct each hyper-rectangle $\mathcal{E}_j(x)$ comprises the parametric representation of $\mathcal{E}_j(x)$, the definition of auxiliary quantities associated with $\mathcal{E}_j(x)$, and finally the optimization problem to find $\mathcal{E}_j(x)$.

Notice that one way to represent a hyper-rectangle $\mathcal{E}_j(x)$ is

$$\mathcal{E}_j(x) := \{\epsilon \in \mathbb{R}^{n_x} : -\underline{e}_j(x) \le \epsilon \le \overline{e}_j(x)\},$$

for some vectors $\underline{e}_j(x), \overline{e}_j(x) \in \mathbb{R}^{n_x}_{\ge 0}$. In other words, each hyper-rectangle $\mathcal{E}_j(x)$ is parameterized by $2n_x$ entries of $\underline{e}_j(x)$ and $\overline{e}_j(x)$.

Let us now introduce the auxiliary quantities involved in the derivation of $\mathcal{E}_j(x)$. Let $A_{\rm cl} := (A + BF)$ be the nominal, closed-loop state matrix. Define the input sequence $\tilde{\boldsymbol{u}}(x; j)$ and the associated state sequence $\boldsymbol{\phi}^{\tilde{\boldsymbol{u}},\boldsymbol{0}}(x; j)$ as

$$\tilde{u}_i(x; j) := \begin{cases} u^{\rm mpc}_{j+i}(x), & i \in \{0 : N - j - 1\}, \\ F A_{\rm cl}^{j+i-N} \phi_N^{\boldsymbol{u}^{\rm mpc},\boldsymbol{0}}(x), & i \in \{N - j : N - 1\}, \end{cases} \tag{8a}$$

$$\phi_i^{\tilde{\boldsymbol{u}},\boldsymbol{0}}(x; j) := \begin{cases} \phi_{j+i}^{\boldsymbol{u}^{\rm mpc},\boldsymbol{0}}(x), & i \in \{0 : N - j\}, \\ A_{\rm cl}^{j+i-N} \phi_N^{\boldsymbol{u}^{\rm mpc},\boldsymbol{0}}(x), & i \in \{N - j + 1 : N\}. \end{cases} \tag{8b}$$

Notice that the above *candidate input sequence* is constructed by concatenating the last $N - j$ elements of $\boldsymbol{u}^{\rm mpc}(x)$ with the nominal feedback F (recursively) applied to the optimal terminal state $\phi_N^{\boldsymbol{u}^{\rm mpc},\boldsymbol{0}}(x)$.

Define $\mathscr{T}_N^{\mathbb{U}} := \prod_{i=0}^{N-1} \mathcal{T}_i^{\mathbb{U}}$ and $\mathscr{T}_N^{\mathbb{X}} := \prod_{i=0}^{N-1} \mathcal{T}_i^{\mathbb{X}}$. Denote now the projections of optimal state and input sequences $\boldsymbol{\phi}^{\boldsymbol{u}^{\rm mpc},\boldsymbol{0}}(x)$ and $\boldsymbol{u}^{\rm mpc}(x)$ onto their corresponding target sets by $\boldsymbol{s}^{\mathbb{X}}(x) \in \mathscr{T}_N^{\mathbb{X}}$ and $\boldsymbol{s}^{\mathbb{U}}(x) \in \mathscr{T}_N^{\mathbb{U}}$, where for all $i \in \{0 : N - 1\}$,

$$s_i^{\mathbb{X}}(x) := \Pi_Q(\phi_i^{\boldsymbol{u}^{\rm mpc},\boldsymbol{0}}(x), \mathcal{T}_i^{\mathbb{X}}),\ s_i^{\mathbb{U}}(x) := \Pi_R(u_i^{\rm mpc}(x), \mathcal{T}_i^{\mathbb{U}}).$$

Based on the above definition, the next two auxiliary quantities are defined as follows. Let $\tilde{\boldsymbol{s}}^{\mathbb{U}}(x; j)$ and $\tilde{\boldsymbol{s}}^{\mathbb{X}}(x; j)$ represent the projection of $\tilde{\boldsymbol{u}}(x; j)$ and $\boldsymbol{\phi}^{\tilde{\boldsymbol{u}},\boldsymbol{0}}(x; j)$ onto $\mathscr{T}_N^{\mathbb{U}}$ and $\mathscr{T}_N^{\mathbb{X}}$, respectively. We have

$$\tilde{s}_i^{\mathbb{U}}(x; j) := \begin{cases} s_{j+i}^{\mathbb{U}}(x), & i \in \{0 : N - j - 1\}, \\ \tilde{u}_{i+j}(x; j), & i \in \{N - j : N - 1\}, \end{cases} \tag{9a}$$

$$\tilde{s}_i^{\mathbb{X}}(x; j) := \begin{cases} s_{j+i}^{\mathbb{X}}(x), & i \in \{0 : N - j\}, \\ \phi_{j+i}^{\tilde{\boldsymbol{u}},\boldsymbol{0}}(x; j), & i \in \{N - j + 1 : N - 1\}. \end{cases} \tag{9b}$$

Let us clarify the conventions used in (9). Notice that the definition of $\tilde{u}_i(x; j)$ in (8a) implies that $\tilde{u}_i(x; j) \in \mathcal{T}_{N-1}^{\mathbb{U}} \subseteq \mathcal{T}_i^{\mathbb{U}}$, for all $i \in \{N - j : N - 1\}$. That is, the distance $d_R(\tilde{u}_i(x; j), \mathcal{T}_i^{\mathbb{U}}) = 0$, and hence, $\Pi_R(\tilde{u}_i(x; j), \mathcal{T}_i^{\mathbb{U}}) = \tilde{u}_i(x; j)$, as given in (9a). A similar line of reasoning has been used in (9b).

We next adopt the feedback gains $\tilde{K}_i$ and the state-transition matrices $\tilde{L}_i$ defined as

$$\tilde{K}_0 = 0_{n_u \times n_x},\ \tilde{K}_{i+1} = K_i,\ \forall i \in \{0 : N - 2\}, \tag{10a}$$

$$\tilde{L}_0 = \mathsf{I}_{n_x},\ \tilde{L}_{i+1} = (A + B\tilde{K}_i)\tilde{L}_i,\ \forall i \in \{0 : N - 1\}. \tag{10b}$$

In the following, we use the matrices (10) to identify certain sets around the optimal state sequence $\boldsymbol{\phi}^{\boldsymbol{u}^{\rm mpc},\boldsymbol{0}}(x)$. These sets in turn will be used to formulate recursive feasibility and robust stability for the event-triggering setting (see the problem (12) and Section 5.1).

Let us now provide two definitions for the volume of $\mathcal{E}_j(x)$, that are

$$\mathrm{vol}_1(\mathcal{E}_j(x)) := \prod_{p \in \{1:n_x\}} \left(\overline{e}_j^p(x) + \underline{e}_j^p(x)\right), \tag{11a}$$

$$\mathrm{vol}_2(\mathcal{E}_j(x)) := \prod_{p \in \{1:n_x\}} \left(\overline{e}_j^p(x) \times \underline{e}_j^p(x)\right), \tag{11b}$$

where $\overline{e}_j^p(x)$ (resp. $\underline{e}_j^p(x)$) denotes the p-th entry of $\overline{e}_j(x)$ (resp. $\underline{e}_j(x)$). Notice that (11a) is the standard definition of volume for $\mathcal{E}_j(x)$ in $\mathbb{R}^{n_x}$. As it will be discussed later on, the application of (11a) to construct $\mathcal{E}_j(x)$ leads to a more asymmetric spread of $\mathcal{E}_j(x)$ around $\phi_j^{\boldsymbol{u}^{\rm mpc},\boldsymbol{0}}(x)$ compared to the application of (11b).

The asymmetry in turn implies that the triggering mechanism has no robustness in certain error directions, see Remark 4.7 for further details. Nonetheless, the definition (11a) leads to the construction of sets that have the maximum possible volume, in particular, higher than the ones constructed based on (11b).

For all $j \in \{1 : N-1\}$, the problem to find each $\mathcal{E}_j(x)$ is

$$\max_{\overline{e}_j(x), \underline{e}_j(x) \geq 0} \ \mathrm{vol}_q(\mathcal{E}_j(x)) \tag{12a}$$

$$\text{s.t.}$$

$$\phi_i^{\tilde{\boldsymbol{u}},\mathbf{0}}(x;j) \in \mathcal{X}_i \ominus \tilde{L}_i \mathcal{E}_j(x), \quad \forall i \in \{0 : N-1\}, \tag{12b}$$

$$\tilde{u}_i(x;j) \in \mathcal{U}_i \ominus \tilde{K}_i \tilde{L}_i \mathcal{E}_j(x), \quad \forall i \in \{0 : N-1\}, \tag{12c}$$

$$\tilde{s}_i^{\mathbb{X}}(x;j) \in \mathcal{T}_i^{\mathbb{X}} \ominus \tilde{L}_i \mathcal{E}_j(x), \quad \forall i \in \{0 : N-1\}, \tag{12d}$$

$$\tilde{s}_i^{\mathbb{U}}(x;j) \in \mathcal{T}_i^{\mathbb{U}} \ominus \tilde{K}_i \tilde{L}_i \mathcal{E}_j(x), \ \forall i \in \{0 : N-1\}, \tag{12e}$$

where $q \in \{1, 2\}$ determines which type of the volume definition in (11) is chosen. Notice that the objective function $\mathrm{vol}_q(\mathcal{E}_j(x))$ is a nonlinear, non-convex function with a decision variable $\mathcal{E}_j(x)$. Hence, the problem (12) is difficult to solve. In the next subsection, we show that this problem remains practically solvable, in particular, the set-based constraints (12b)-(12e) are effectively representable by linear inequalities (i.e., polytopic inequalities) such that (i) the optimization problem (12) has a CP counterpart (in Theorem 4.4), and (ii) the optimization problem (12) admits an LP relaxation (in Theorem 4.5).

4.2. **Event-Based Implementation**

We first show that robust stability of the event-triggered, closed-loop dynamics (7) is guaranteed, which in turn leads to recursive feasibility of the closed-loop system. The triggering mechanism (6) is constructed by the approach proposed in (12). We next establish that the non-convex problem (12) to construct the hyper-rectangles $\boldsymbol{\mathcal{E}}(x)$ has a CP reformulation and an LP relaxation, and therefore can be efficiently solved in practice.

Theorem 4.1 (Robust convergence)**.** *Consider the closed-loop dynamics (7), and suppose that the initial state ξ_0 is feasible (i.e., $V_N^*(\xi_0) < \infty$). For all $s \in \{\tau_t + 1 : \tau_{t+1}\}$, there exists an input sequence $\boldsymbol{u} \in \mathscr{U}_N$ such that*

$$V_N^*(\xi_{\tau_{t+1}}) - V_N^*(\xi_{\tau_t}) \leq V_N\big(\xi_s, \boldsymbol{u}(\xi_s)\big) - V_N^*(\xi_{\tau_t}) \leq -\Big(\sum_{k=0}^{s-\tau_t-1} d_Q(\phi_k^{\boldsymbol{u}^{mpc},\mathbf{0}}(\xi_{\tau_t}), \mathcal{T}_k^{\mathbb{X}}) + d_R(u_k^{mpc}(\xi_{\tau_t}), \mathcal{T}_k^{\mathbb{U}})\Big). \tag{13}$$

In particular, the closed-loop dynamics (7) is asymptotically stable, i.e., $\lim_{t\to\infty} V_N^*(\xi_t) = 0$.

Remark 4.2 (Recursive feasibility)**.** *Notice that the second inequality in (13) implies that $V_N\big(\xi_s, \boldsymbol{u}(\xi_s)\big) < \infty$, for all $s \in \{\tau_t + 1 : \tau_{t+1}\}$. In other words, the optimization problem (5) remains feasible for all time $t \in \mathbb{Z}_{>0}$.*

Remark 4.3 (Transmission protocol)**.** *We assume that all sensor and actuator units are clock-synchronized. When the problem (5) is solved, the controller node sends: (i) $\boldsymbol{u}^{mpc}(\xi_{\tau_t})$ to the actuator nodes and (ii) each entry of $\phi_j^{\boldsymbol{u}^{mpc},\mathbf{0}}(\xi_{\tau_t}) - \underline{e}_j(\xi_{\tau_t})$ and $\phi_j^{\boldsymbol{u}^{mpc},\mathbf{0}}(\xi_{\tau_t}) + \overline{e}_j(\xi_{\tau_t})$ to the corresponding sensory nodes, for all $j \in \{1 : N-1\}$. Moreover, the n_x sensor units declare a triggering instance to each other, through a cost-efficient short-range transmission. Then, all sensors declare their time-stamped, observed states to the controller.*

The successful usage of the above results is conditioned upon the premise that there exist computationally tractable methods to construct the sets $\boldsymbol{\mathcal{E}}(x)$. We now revisit problem (12) to show that such a premise is valid by providing two frameworks: one in a CP form and another one in an LP form. In these frameworks, the parametric-in-set constraints (12b)-(12e) can be reformulated into a new set of linear inequalities in terms of the vertices of each set $\mathcal{E}_j(x)$. We shall call the polytope represented by the derived linear inequalities, the *principal* polytope $\bar{\mathcal{S}}$. Both frameworks try to find a maximum-volume hyper-rectangle $\mathcal{E}_j(x)$ inscribed (or contained) in the principal polytope such that $0 \in \mathcal{E}_j(x)$. In the LP framework, we partly employ some results from [5], see Section 5.2 and avoid reiterating the proofs of borrowed material. For notational convenience, let $\xi \in \mathcal{S} \ominus M\mathcal{B}(l,u)$ represent a concatenated version of the constraint (12b)-(12e) where, in particular, $\mathcal{B}(l,u) := \mathcal{E}_j(x)$. Hereafter, when we take volume (of a hyper-rectangle) as defined in (11) with index $q = 1$ and $q = 2$ referring to (11a) and (11b), respectively.

Theorem 4.4 (Volume maximization - CP reformulation)**.** *Consider a vector $\xi \in \mathbb{R}^p$, a matrix $M \in \mathbb{R}^{p\times k}$, and a polytope $\mathcal{S} = \{s \in \mathbb{R}^p : \; A_{\mathcal{S}} s \le b_{\mathcal{S}}\}$ containing the origin where $A_{\mathcal{S}} \in \mathbb{R}^{m\times p}$ and $b_{\mathcal{S}} \in \mathbb{R}^m$. The maximum volume hyper-rectangle $\mathcal{B}(l,u) \subset \mathbb{R}^k$ that contains the origin and satisfies $\xi \in \mathcal{S} \ominus M\mathcal{B}(l,u)$ is $\mathcal{B}(-\underline{v}^*, \overline{v}^*)$ where $\underline{v}^*$ and $\overline{v}^*$ are the optimal solutions of the problem*

$$\begin{aligned} \min_{\underline{v},\overline{v}} \quad & f_q(\overline{v}, \underline{v}) \\ \text{s.t.} \quad & \langle w^i, [\overline{v}^\top \; \underline{v}^\top]^\top \rangle \le b_{i,\mathcal{S}} - a_{i,\mathcal{S}}\xi, \forall i \in \{1:m\}, \\ & \overline{v} \ge 0, \; \underline{v} \ge 0, \end{aligned} \tag{14}$$

where for $q \in \{1,2\}$

$$f_1(\overline{v}, \underline{v}) := - \sum_{j\in\{1:k\}} \log(\overline{v}_j + \underline{v}_j), \tag{15a}$$

$$f_2(\overline{v}, \underline{v}) := - \sum_{j\in\{1:k\}} \log(\overline{v}_j) + \log(\underline{v}_j), \tag{15b}$$

and for all $j \in \{1:k\}$

$$w^i_j = \begin{cases} (M^\top a^\top_{i,\mathcal{S}})_j, & \text{if } \hat{w}^i_j = 1, \\ 0, & \text{otherwise,} \end{cases} \tag{16a}$$

$$w^i_{k+j} = \begin{cases} -(M^\top a^\top_{i,\mathcal{S}})_j, & \text{if } \hat{w}^i_j = -1, \\ 0, & \text{otherwise,} \end{cases} \tag{16b}$$

with $\hat{w}^i := \operatorname{sign}(M^\top a^\top_{i,\mathcal{S}})$, for all $i \in \{1:m\}$.

Theorem 4.5 (Volume maximization - LP relaxation)**.** *Suppose the hypotheses in Theorem 4.4 hold.*

- **($\mathbf{q = 1}$)** *The maximum volume r-constrained hyper-rectangle $\mathcal{B}(l,u) \subset \mathbb{R}^k$ that contains the origin and satisfies $\xi \in \mathcal{S} \ominus M\mathcal{B}(l,u)$ is $\mathcal{B}(z^*, z^* + \lambda^* r)$ for which $z^* \in \mathbb{R}^k$ and $\lambda^* \in \mathbb{R}$ are the optimal solution of the problem*

$$\begin{aligned} \max_{z,\lambda} \quad & \lambda \\ \text{s.t.} \quad & A_{\mathcal{S}}Mz + (A_{\mathcal{S}}M)^+ r\lambda \le b_{\mathcal{S}} - A_{\mathcal{S}}\xi \\ & z + \lambda r \ge 0, \; z \le 0, \end{aligned} \tag{17a}$$

  where the j-th entry of r, $j \in \{1:k\}$, is defined as

$$\begin{aligned} r_j(\bar{\mathcal{S}}) := \max_{z,\omega} \quad & \omega \\ \text{s.t.} \quad & A_{\mathcal{S}}Mz \le b_{\mathcal{S}} - A_{\mathcal{S}}\xi \\ & A_{\mathcal{S}}M(z + \omega e_j) \le b_{\mathcal{S}} - A_{\mathcal{S}}\xi \\ & z + \omega e_j \ge 0, \; z \le 0, \end{aligned} \tag{17b}$$

  where $e_j \in \mathbb{R}^k$ is the unit vector in the j-th direction and the polytope $\bar{\mathcal{S}}$ is

$$\bar{\mathcal{S}} := \{z \in \mathbb{R}^k : \; A_{\mathcal{S}}Mz \le b_{\mathcal{S}} - A_{\mathcal{S}}\xi\}.$$

- $(\mathbf{q=2})$ *The maximum volume r-constrained hyper-rectangle $\mathcal{B}(l,u) \subset \mathbb{R}^k$ that contains the origin and satisfies $\xi \in \mathcal{S} \ominus M\mathcal{B}(l,u)$ is $\mathcal{B}(-\lambda^* r_1, \lambda^* r_2)$ for which $\lambda^* \in \mathbb{R}$ is the optimal solution of the problem*

$$\begin{aligned} &\max_{\lambda} \quad \lambda \\ &\text{s.t.} \quad (W)^{+} r\lambda \le B, \end{aligned} \tag{18a}$$

where $r = \left(r_2^\top, r_1^\top\right)^\top$ and the j-th entry of r, $j \in \{1:2k\}$, is defined as

$$\begin{aligned} r_j := \ &\max_{\omega} \quad \omega \\ &\text{s.t.} \quad W'(\omega e_j) \le B', \end{aligned} \tag{18b}$$

where $e_j \in \mathbb{R}^{2k}$ is the unit vector in the j-th direction,

$$W = \left(w^1, \cdots, w^m\right)^\top, \qquad W' = \begin{pmatrix} & W & \\ -\mathsf{I}_k & & 0_{k\times 1} \\ 0_{k\times 1} & & -\mathsf{I}_k \end{pmatrix},$$

$$B = b_{\mathcal{S}} - A_{\mathcal{S}}\xi, \qquad B' = \left(B^\top, 0_{1\times 2k}\right)^\top,$$

and for all $i \in \{1:m\}$, w^i are defined in (16).

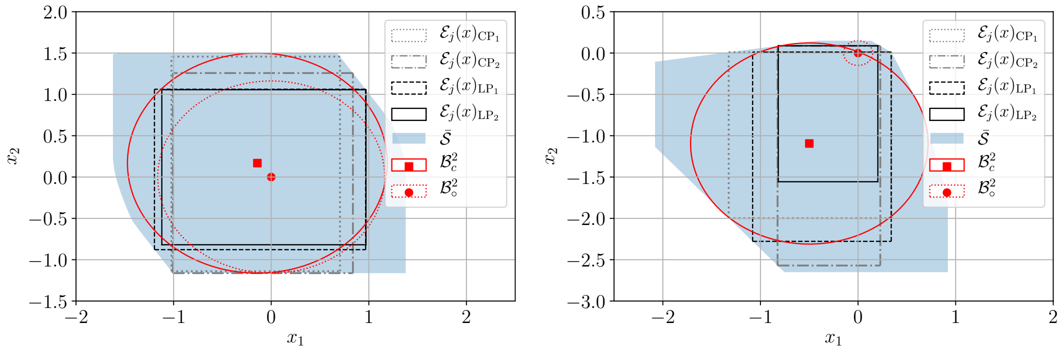
We should emphasize that although Theorems 4.4 & 4.5 provide a way to construct $\mathcal{E}_j(x)$ with a maximal volume, the derived set is not unique (the corresponding cost functions of these approaches are not strictly convex to guarantee the uniqueness of solution). In the remainder of the paper, we denote the construction approach based on the CP (14) with $q = 1$ and $q = 2$ by CP_1 and CP_2, respectively. Furthermore, LP_1 represents the LP relaxation (17) of CP_1 and LP_2 denotes the LP relaxation (18) of CP_2.

4.3. Further Comments on Complexity and Sensitivity

In the rest of this section, we allude briefly to two important practical aspects of the proposed construction approaches and possible directions to improve them. First, since these approaches are implemented online, they require an extra computation step besides the computation of the optimal input sequence. Notice that fixed-thresholding approaches in the literature, for example [23], avoid this extra step by considering pre-defined triggering sets. We provide the arithmetic complexity of the proposed approaches to quantify the extra computational burden. To this end, we adopt the following notion of an oracle to represent the optimization problems in this paper. Let $A \in \mathbb{R}^{n_c \times n_d}$, $b \in \mathbb{R}^{n_c}$, $c \in \mathbb{R}^{n_d}$, and $f : \mathbb{R}^{n_d} \to \mathbb{R}$ be a concave function. Also, let $\mathbf{lp}(n_c, n_d)$ denote the oracle complexity for solving $\max_\eta\{c^\top \eta : A\eta \le b\}$, and $\mathbf{cp}(n_c, n_d)$ denote the oracle complexity for solving $\max_\eta\{f(\eta) : A\eta \le b\}$.

Remark 4.6 (Computational complexity)**.** *The oracle complexity of the CP reformulations* (14) *in Theorem 4.4 is* $\mathbf{cp}(m+2k, 2k)$ *and of the LP reformulations* (17) *and* (18) *in Theorem 4.5 are* $\mathbf{lp}(m+2k, k+1) + k \times \mathbf{lp}(2m+2k, k+1)$ *and* $\mathbf{lp}(m,1) + 2k \times \mathbf{lp}(m+2k, 1)$*, respectively. A possible remedy to circumvent these computations is to introduce a* state-independent *triggering law, as opposed to the current state-dependent law* (6)*. This extension would allow to compute the desired sets offline and only once.*

The other issue regarding the proposed approaches is the asymmetry of the triggering sets with respect to the optimal state sequence. Let polytope $\overline{\mathcal{S}} \subset \mathbb{R}^{n_x}$ represent the constraints (12b)-(12e) that the triggering set $\mathcal{E}_j(x)$ satisfies. In other words, $\mathcal{E}_j(x)$ is constructed inside $\overline{\mathcal{S}}$. Recall that $\mathcal{E}_j(x)$ represents the "allowable" prediction error so that the triggering mechanism is not activated. Qualitatively speaking, for a "better" directional resilience against prediction errors, one would prefer symmetry in the constructed $\mathcal{E}_j(x)$. The above statements are schematically depicted in Figure 1. When $\overline{\mathcal{S}}$ is well-shaped as in Figure 1(a), the approaches in Theorems 4.4 & 4.5 lead to a relatively symmetric set $\mathcal{E}_j(x)$ with respect to the origin. When $\overline{\mathcal{S}}$ is ill-shaped as in Figure 1(b), the constructed set $\mathcal{E}_j(x)$ is however extremely asymmetric with respect to the origin along some coordinates. This difference is well-captured by the geometric measure $\frac{r_c}{r_\circ}$ of $\overline{\mathcal{S}}$, where r_c is



(a) Well-shaped polytope example, $\frac{r_c}{r_\circ} = 1.0861$. (b) Ill-shaped polytope example, $\frac{r_c}{r_\circ} = 8.0669$.

FIGURE 1. Comparison of the CP and LP approaches to construct $\mathcal{E}_j(x) \subseteq \overline{\mathcal{S}}$. **(a)** $\overline{\mathcal{S}}$ is distributed in a fairly uniform manner around the origin. All the approaches provide close behaviors. **(b)** $\overline{\mathcal{S}}$ is distributed in a relatively uneven manner around the origin. The approaches CP_2 and LP_2 promote more symmetric constructions compared to the approaches CP_1 and LP_1.

the radius of the maximal 2-norm ball inside $\overline{\mathcal{S}}$, and $r_\circ$ is the radius of the maximal 2-norm ball, centered at the origin and inside $\overline{\mathcal{S}}$. By definition, we have $\frac{r_c}{r_\circ} \geq 1$. Observe that in well-shaped cases $r_c/r_\circ \approx 1$ and in ill-shaped cases $r_c/r_\circ \gg 1$.

Remark 4.7 (Directional sensitivity to prediction errors)**.** *The directional sensitivity issue is the main reason for introducing the second definition* (11b) *of the volume. To see this, assume first that our goal is to maximize the* log *value of the volume of $\mathcal{E}_j(x)$. Notice that the first definition* (11a) *solely aims for maximizing the width of $\mathcal{E}_j(x)$ within $\overline{\mathcal{S}}$ along each coordinate. On the other hand, the second definition* (11b) *maximizes the width of $\mathcal{E}_j(x)$ in both positive and negative directions along each coordinate. As shown in Figure 1, in both cases the set $\mathcal{E}_j(x)$ constructed by the approaches CP_2 and LP_2 is typically more symmetric compared to those constructed by the approaches CP_1 and LP_1. An interesting research direction to alleviate this sensitivity issue is to investigate the impact of the MPC design parameters (e.g., the tightening gains $\boldsymbol{K}$ or the target sets $\mathbb{T}^{\mathbb{X}}$ and $\mathbb{T}^{\mathbb{U}}$).*

5. Technical Proofs

5.1. Proof of Theorem 4.1

The proof consists of five main steps. Each step is labeled by the guaranteed property. Let $x := \xi_{\tau_t}$ be the state at the last triggering instance. Define the prediction error

$$e_j^{\boldsymbol{w}}(x) = \phi_j^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{w}}(x) - \phi_j^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{0}}(x), \tag{19}$$

indicating the mismatch between the perturbed system and the nominal one. For some integer $j \in \{0 : N-1\}$, suppose that the mechanism is enabled at time $j+1$, that is, either (1) $j < N-1$ so that for all $i \in \{0 : j\}$, $e_i^{\boldsymbol{w}}(x) \in \mathcal{E}_i(x)$ and $e_{j+1}^{\boldsymbol{w}}(x) \notin \mathcal{E}_{j+1}(x)$, or (2) $j = N-1$ (see equation (7b)). We omit the arguments of variables for convenience when it is clear from the context (unless mentioned otherwise). In what follows, we also use the notation $\ell(x_i, u_i)$ for $d_Q(x_i, \mathcal{T}_i^{\mathbb{X}}) + d_R(u_i, \mathcal{T}_i^{\mathbb{U}})$ for notational simplicity.

1) Inter-event recursive feasibility: Define the *candidate* input sequence $\boldsymbol{u}^{\mathrm{c}}(x; j)$ such that for all $i \in \{0 : N-1\}$,

$$u_i^{\mathrm{c}} := \tilde{u}_i + \tilde{K}_i \tilde{L}_i e_j^{\boldsymbol{w}}, \tag{20a}$$

and its associated *candidate* state sequence $\boldsymbol{\phi}^{\boldsymbol{u}^{\mathrm{c}},\mathbf{0}}(x;j)$, where for all $i \in \{0:N\}$,

$$\phi_i^{\boldsymbol{u}^{\mathrm{c}},\mathbf{0}} := \phi_i^{\tilde{\boldsymbol{u}},\mathbf{0}} + \tilde{L}_i e_j^{\boldsymbol{w}}. \tag{20b}$$

Note that $\phi_0^{\boldsymbol{u}^{\mathrm{c}},\mathbf{0}} = \phi_j^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{w}}$ and $u_0^{\mathrm{c}} = u_j^{\mathrm{mpc}}$. We now establish that the sequences $\boldsymbol{u}^{\mathrm{c}}$ and $\boldsymbol{\phi}^{\boldsymbol{u}^{\mathrm{c}},\mathbf{0}}$ satisfy $\boldsymbol{u}^{\mathrm{c}} \in \mathscr{U}_N$ and $\boldsymbol{\phi}^{\boldsymbol{u}^{\mathrm{c}},\mathbf{0}} \in \mathscr{X}_N$, i.e., $V_N(\phi_j^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{w}}, \boldsymbol{u}^{\mathrm{c}}) < \infty$. By assumption, $e_j^{\boldsymbol{w}} \in \mathcal{E}_j$. Moreover, $\mathcal{E}_j$ satisfies (12b)-(12c). From the definition of the Pontryagin difference, it follows that $u_i^{\mathrm{c}} \in \mathcal{U}_i$ and $\phi_i^{\boldsymbol{u}^{\mathrm{c}},\mathbf{0}} \in \mathcal{X}_i$, for all $i \in \{0:N-1\}$. Recall that $L_{N-1} = 0$. Hence, $\tilde{L}_N = 0$ and $\phi_N^{\boldsymbol{u}^{\mathrm{c}},\mathbf{0}} = \phi_N^{\tilde{\boldsymbol{u}},\mathbf{0}}$. From (8b), we have $\phi_N^{\tilde{\boldsymbol{u}},\mathbf{0}} = A_{\mathrm{cl}}^j \phi_N^{\boldsymbol{u}^{\mathrm{mpc}},\mathbf{0}}(x)$. Since $\mathcal{X}_f$ is a control invariant set, $\phi_N^{\boldsymbol{u}^{\mathrm{c}},\mathbf{0}} \in \mathcal{X}_f$. We conclude that $V_N(\phi_j^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{w}}, \boldsymbol{u}^{\mathrm{c}}) < \infty$.

2) Inter-event cost function decay: Observe that

$$d_Q(\phi_i^{\boldsymbol{u}^{\mathrm{c}},\mathbf{0}}, \mathcal{T}_i^{\mathbb{X}}) = d_Q(\phi_i^{\tilde{\boldsymbol{u}},\mathbf{0}} + \tilde{L}_i e_j^{\boldsymbol{w}}, \mathcal{T}_i^{\mathbb{X}}) \leq d_Q(\phi_i^{\tilde{\boldsymbol{u}},\mathbf{0}}, \mathcal{T}_i^{\mathbb{X}} \ominus \tilde{L}_i \mathcal{E}_j),$$

where we made use of the definition (20b) and Lemma 2.1, respectively. Recall from (12d) that $\tilde{s}_i^{\mathbb{X}} \in \mathcal{T}_i^{\mathbb{X}} \ominus \tilde{L}_i \mathcal{E}_j$. Hence,

$$0 \leq d_Q(\phi_i^{\boldsymbol{u}^{\mathrm{c}},\mathbf{0}}, \mathcal{T}_i^{\mathbb{X}}) \leq d_Q(\phi_i^{\tilde{\boldsymbol{u}},\mathbf{0}}, \tilde{s}_i^{\mathbb{X}}). \tag{21a}$$

Similarly, one can arrive at

$$0 \leq d_R(u_i^{\mathrm{c}}, \mathcal{T}_i^{\mathbb{U}}) \leq d_R(\tilde{u}_i, \tilde{s}_i^{\mathbb{U}}). \tag{21b}$$

Consider $i \in \{0 : N-j-1\}$. In light of the definitions (8b) and (9b), we have $d_Q(\phi_i^{\tilde{\boldsymbol{u}},\mathbf{0}}, \tilde{s}_i^{\mathbb{X}}) = d_Q(\phi_{j+i}^{\boldsymbol{u}^{\mathrm{mpc}},\mathbf{0}}, \mathcal{T}_{j+i}^{\mathbb{X}})$. Thus,

$$d_Q(\phi_i^{\boldsymbol{u}^{\mathrm{c}},\mathbf{0}}, \mathcal{T}_i^{\mathbb{X}}) \leq d_Q(\phi_{j+i}^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{w}}, \mathcal{T}_{j+i}^{\mathbb{X}}). \tag{22a}$$

In a similar fashion, we can show

$$d_R(u_i^{\mathrm{c}}, \mathcal{T}_i^{\mathbb{U}}) \leq d_Q(u_{j+i}^{\mathrm{mpc}}, \mathcal{T}_{j+i}^{\mathbb{U}}). \tag{22b}$$

From (22), it is then straightforward that

$$\sum_{i=0}^{N-j-1} \ell(\phi_i^{\boldsymbol{u}^{\mathrm{c}},\mathbf{0}}, u_i^{\mathrm{c}}) \leq \sum_{i=0}^{N-j-1} \ell(\phi_{j+i}^{\boldsymbol{u}^{\mathrm{mpc}},\mathbf{0}}, u_{j+i}^{\mathrm{mpc}}). \tag{23a}$$

Now, let $i \in \{N-j : N-1\}$ and consider the definition (9). Then, $d_Q(\phi_i^{\tilde{\boldsymbol{u}},\mathbf{0}}, \tilde{s}_i^{\mathbb{X}}) = d_R(\tilde{u}_i, \tilde{s}_i^{\mathbb{U}}) = 0$. These equality relations coupled with (21) give rise to

$$\sum_{i=N-j}^{N-1} \ell(\phi_i^{\boldsymbol{u}^{\mathrm{c}},\mathbf{0}}, u_i^{\mathrm{c}}) = 0. \tag{23b}$$

From (23), we finally infer that if $e_j^{\boldsymbol{w}} \in \mathcal{E}_j$, then

$$V_N(\phi_j^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{w}}, \boldsymbol{u}^{\mathrm{c}}) = V_N(\phi_0^{\boldsymbol{u}^{\mathrm{c}},\mathbf{0}}, \boldsymbol{u}^{\mathrm{c}}) \leq V_N^*(x) - \sum_{i=0}^{j-1} \ell(\phi_i^{\boldsymbol{u}^{\mathrm{mpc}},\mathbf{0}}, u_i^{\mathrm{mpc}}). \tag{24}$$

3) At-event recursive feasibility: Consider now the *new candidate* input sequence $\hat{\boldsymbol{u}}^{\mathrm{c}}(x;j+1)$ where

$$\hat{u}_i^{\mathrm{c}} := \begin{cases} u_{i+1}^{\mathrm{c}} + K_i L_i w_j, & \forall i \in \{0 : N-2\}, \\ F\phi_N^{\boldsymbol{u}^{\mathrm{c}},\mathbf{0}} + FL_{N-1} w_j, & i = N-1, \end{cases} \tag{25a}$$

and its associated *candidate* state sequence $\hat{\boldsymbol{\phi}}^{\hat{\boldsymbol{u}}^{\mathrm{c}},\mathbf{0}}(x;j+1)$ such that

$$\hat{\phi}_i^{\hat{\boldsymbol{u}}^{\mathrm{c}},\mathbf{0}} := \begin{cases} \phi_{i+1}^{\boldsymbol{u}^{\mathrm{c}},\mathbf{0}} + L_i w_j, & \forall i \in \{0 : N-1\}, \\ A_{\mathrm{cl}} \hat{\phi}_{N-1}^{\hat{\boldsymbol{u}}^{\mathrm{c}},\mathbf{0}}, & i = N, \end{cases} \tag{25b}$$

where $w_j \in \mathbb{W}$. Observe that

$$\hat{\phi}_0^{\hat{\boldsymbol{u}}^{\mathrm{c}},\mathbf{0}} = \phi_1^{\boldsymbol{u}^{\mathrm{c}},\mathbf{0}} + w_j = A\phi_0^{\boldsymbol{u}^{\mathrm{c}},\mathbf{0}} + Bu_0^{\mathrm{c}} + w_j$$

$$= A\phi_j^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{w}} + Bu_j^{\mathrm{mpc}} + w_j = \phi_{j+1}^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{w}}.$$

We now show that $\hat{\boldsymbol{u}}^{\mathrm{c}} \in \mathscr{U}_N$ and $\hat{\boldsymbol{\phi}}^{\hat{\boldsymbol{u}}^{\mathrm{c}},\boldsymbol{0}} \in \mathscr{X}_N$, i.e., $V_N(\phi_{j+1}^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{w}}, \hat{\boldsymbol{u}}^{\mathrm{c}}) < \infty$. Observe that $u_{i+1}^{\mathrm{c}} \in \mathcal{U}_{i+1}$ and $w_j \in \mathbb{W}$. Hence, $u_{i+1}^{\mathrm{c}} \in \mathcal{U}_{i+1} \oplus K_i L_i \mathbb{W}$ for all $i \in \{0 : N-2\}$. Since $\mathcal{U}_{i+1} = \mathcal{U}_i \ominus K_i L_i \mathbb{W}$, we have $\hat{u}_i^{\mathrm{c}} \in \mathcal{U}_i$ for all $i \in \{0 : N-2\}$. Recall now $\phi_N^{\boldsymbol{u}^{\mathrm{c}},\boldsymbol{0}} \in \mathcal{X}_f$ (from Step 1). Assumption 3.2 along with $L_{N-1} = 0$ imply that $\hat{u}_{N-1}^{\mathrm{c}} \in \mathcal{U}_{N-1}$. We have $\phi_{i+1}^{\boldsymbol{u}^{\mathrm{c}},\boldsymbol{0}} \in \mathcal{X}_{i+1}$ for all $i \in \{0 : N-2\}$. Then, $\hat{\phi}_i^{\hat{\boldsymbol{u}}^{\mathrm{c}},\boldsymbol{0}} \in \mathcal{X}_{i+1} \oplus L_i \mathbb{W}$. For all $i \in \{0 : N-2\}$, it follows from $\mathcal{X}_{i+1} = \mathcal{X}_i \ominus L_i \mathbb{W}$ that $\hat{\phi}_i^{\hat{\boldsymbol{u}}^{\mathrm{c}},\boldsymbol{0}} \in \mathcal{X}_i$. Recall that $\phi_N^{\boldsymbol{u}^{\mathrm{c}},\boldsymbol{0}} \in \mathcal{X}_f$ and $L_{N-1} = 0$. Hence, we arrive at $\hat{\phi}_{N-1}^{\hat{\boldsymbol{u}}^{\mathrm{c}},\boldsymbol{0}} \in \mathcal{X}_f$ and as a result $\hat{\phi}_N^{\hat{\boldsymbol{u}}^{\mathrm{c}},\boldsymbol{0}} \in \mathcal{X}_f$. We thus have $\hat{\boldsymbol{u}}^{\mathrm{c}} \in \mathscr{U}_N$ and $\hat{\boldsymbol{\phi}}^{\hat{\boldsymbol{u}}^{\mathrm{c}},\boldsymbol{0}} \in \mathscr{X}_N$, i.e., $V_N(\phi_{j+1}^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{w}}, \hat{\boldsymbol{u}}^{\mathrm{c}}) < \infty$.

4) At-event value function decay: Consider now $\hat{\boldsymbol{u}}^{\mathrm{c}}$ and $\hat{\boldsymbol{\phi}}^{\hat{\boldsymbol{u}}^{\mathrm{c}},\boldsymbol{0}}$ as the candidate input and state sequences at time $j+1$, respectively. For all $i \in \{0 : N-2\}$ and for all $w_j \in \mathbb{W}$,

$$\begin{aligned} d_Q(\hat{\phi}_i^{\hat{\boldsymbol{u}}^{\mathrm{c}},\boldsymbol{0}}, \mathcal{T}_i^{\mathbb{X}}) &= d_Q(\phi_{i+1}^{\boldsymbol{u}^{\mathrm{c}},\boldsymbol{0}} + L_i w_j, \mathcal{T}_i^{\mathbb{X}}) \\ &\leq d_Q(\phi_{i+1}^{\boldsymbol{u}^{\mathrm{c}},\boldsymbol{0}}, \mathcal{T}_i^{\mathbb{X}} \ominus \tilde{L}_i \mathcal{E}_j) = d_Q(\phi_{i+1}^{\boldsymbol{u}^{\mathrm{c}},\boldsymbol{0}}, \mathcal{T}_{i+1}^{\mathbb{X}}), \end{aligned} \tag{26a}$$

where the first inequality follows from (25b), the inequality is implied by Lemma 2.1, and the second equality is derived from (3b). Following a similar argument, we arrive at

$$d_R(\hat{u}_i^{\mathrm{c}}, \mathcal{T}_i^{\mathbb{U}}) \leq d_R(u_{i+1}^{\mathrm{c}}, \mathcal{T}_{i+1}^{\mathbb{U}}). \tag{26b}$$

Since $L_{N-1} = 0$, $\hat{\phi}_{N-1}^{\hat{\boldsymbol{u}}^{\mathrm{c}},\boldsymbol{0}} = \phi_N^{\boldsymbol{u}^{\mathrm{c}},\boldsymbol{0}}$ and $\hat{u}_{N-1}^{\mathrm{c}} = F\phi_N^{\boldsymbol{u}^{\mathrm{c}},\boldsymbol{0}}$. In Step 3, it is shown that $\hat{\phi}_{N-1}^{\hat{\boldsymbol{u}}^{\mathrm{c}},\boldsymbol{0}} \in \mathcal{X}_f$. Then Asuumption 3.2 implies that $\hat{\phi}_{N-1}^{\hat{\boldsymbol{u}}^{\mathrm{c}},\boldsymbol{0}} \in \mathcal{T}_{N-1}^{\mathbb{X}}$ and $\hat{u}_{N-1}^{\mathrm{c}} \in \mathcal{T}_{N-1}^{\mathbb{U}}$. Hence, $\ell(\hat{\phi}_{N-1}^{\hat{\boldsymbol{u}}^{\mathrm{c}},\boldsymbol{0}}, \hat{u}_{N-1}^{\mathrm{c}}) = 0$. By virtue of the inequalities in (26), we then arrive at

$$\begin{aligned} V_N(\phi_{j+1}^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{w}}, \hat{\boldsymbol{u}}^{\mathrm{c}}) = V_N(\hat{\phi}_0^{\hat{\boldsymbol{u}}^{\mathrm{c}},\boldsymbol{0}}, \hat{\boldsymbol{u}}^{\mathrm{c}}) &\leq \sum_{i=1}^{N-1} \ell(\phi_i^{\boldsymbol{u}^{\mathrm{c}},\boldsymbol{0}}, u_i^{\mathrm{c}}) \\ &= V_N(\phi_0^{\boldsymbol{u}^{\mathrm{c}},\boldsymbol{0}}, \boldsymbol{u}^{\mathrm{c}}) - \ell(\phi_0^{\boldsymbol{u}^{\mathrm{c}},\boldsymbol{0}}, u_0^{\mathrm{c}}) \\ &= V_N(\phi_0^{\boldsymbol{u}^{\mathrm{c}},\boldsymbol{0}}, \boldsymbol{u}^{\mathrm{c}}) - \ell(\phi_j^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{0}}, u_j^{\mathrm{mpc}}) \\ &\leq V_N^*(x) - \sum_{i=0}^{j} \ell(\phi_i^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{0}}, u_i^{\mathrm{mpc}}). \end{aligned} \tag{27}$$

It follows from the optimality principle that $V_N^*(\phi_{j+1}^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{w}}) \leq V_N(\phi_{j+1}^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{w}}, \hat{\boldsymbol{u}}^{\mathrm{c}})$. This inequality along with (27) in turn implies that

$$V_N^*(\phi_{j+1}^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{w}}) \leq V_N^*(x) - \sum_{i=0}^{j} \ell(\phi_i^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{0}}, u_i^{\mathrm{mpc}}). \tag{28}$$

5) Robust convergence: First, observe that (13) is an immediate consequence of (24) and (28). Let us now recall that $x = \xi_{\tau_t}$ and $\phi_{j+1}^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{w}}(x) = \xi_{\tau_{t+1}}$. Then, one can rewrite (28) as follows:

$$V_N^*(\xi_{\tau_{t+1}}) - V_N^*(\xi_{\tau_t}) \leq -\sum_{i=0}^{\tau_{t+1}-\tau_t-1} \ell\big(\phi_i^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{0}}(\xi_\tau), u_i^{\mathrm{mpc}}(\xi_\tau)\big).$$

Notice that the right-hand side of the above inequality is strictly negative unless when $\phi_i^{\boldsymbol{u}^{\mathrm{mpc}},\boldsymbol{0}}(\xi_\tau) \in \mathcal{T}_i^{\mathbb{X}}$ and $u_i^{\mathrm{mpc}}(\xi_\tau) \in \mathcal{T}_i^{\mathbb{U}}$ for all $i \in \{0 : \tau_{t+1} - \tau_t - 1\}$. Since $V_N^*(\xi_{\tau_{t+1}})$ is a non-negative value, it is straightforward to observe that the states and inputs of the closed-loop dynamics (7) converge to their corresponding target sets. This concludes the proof.

5.2. **Proof of Theorems 4.4 & 4.5**

We first begin with a preliminary argument that is shared between both theorems. We then carry on with the proof of each case in an orderly fashion. Notice that $\xi \in \mathcal{S} \ominus M\mathcal{B}(l,u)$ and $\mathcal{S}$ is a polytope by the theorems' hypothesis. Let $h_{M\mathcal{B}}$ be the support function of $M\mathcal{B}$. One can infer that

$$\langle a_{i,\mathcal{S}}^\top, \xi\rangle \le b_{i,\mathcal{S}} - h_{M\mathcal{B}}(a_{i,\mathcal{S}}^\top), \forall i \in \{1:m\}.$$

Next, observe that $\mathcal{B}(l,u) \subset \mathbb{R}^k$ is a polytope (and as a result bounded), and the domain $\mathcal{K}_\mathcal{B}$ on which the support function $h_\mathcal{B}$ is defined is the whole space, i.e., $\mathcal{K}_\mathcal{B} = \mathbb{R}^k$. Hence, $h_{M\mathcal{B}}(a_{i,\mathcal{S}}^\top) = h_\mathcal{B}(M^\top a_{i,\mathcal{S}}^\top)$, and as a consequence

$$\langle a_{i,\mathcal{S}}^\top, \xi\rangle \le b_{i,\mathcal{S}} - h_{\mathcal{B}}(M^\top a_{i,\mathcal{S}}^\top), \forall i \in \{1:m\}.$$

Rearranging the above inequality, we arrive at

$$h_{\mathcal{B}}(M^\top a_{i,\mathcal{S}}^\top) \le b_{i,\mathcal{S}} - \langle a_{i,\mathcal{S}}^\top, \xi\rangle, \forall i \in \{1:m\},$$

where the only unknown entity is $h_\mathcal{B}(M^\top a_{i,\mathcal{S}}^\top)$ with $M^\top a_{i,\mathcal{S}}^\top \in \mathbb{R}^k$. It follows from the definition of the support function that $\langle M^\top a_{i,\mathcal{S}}^\top, z\rangle \le h_\mathcal{B}(M^\top a_{i,\mathcal{S}}^\top)$ for all $z \in \mathbb{R}^k$. Thus,

$$\langle M^\top a_{i,\mathcal{S}}^\top, z\rangle \le b_{i,\mathcal{S}} - \langle a_{i,\mathcal{S}}^\top, \xi\rangle, \forall i \in \{1:m\}, \forall z \in \mathcal{B}. \tag{29}$$

Let us now define for all $i \in \{1:m\}$, $a_{i,\bar{\mathcal{S}}}^\top := M^\top a_{i,\mathcal{S}}^\top$, $b_{i,\bar{\mathcal{S}}} := b_{i,\mathcal{S}} - \langle a_{i,\mathcal{S}}^\top, \xi\rangle$, and the convex polytope (which we referred to as the *principal* polytope in the paragraph before Theorem 4.4)

$$\begin{aligned}\bar{\mathcal{S}} &:= \{s \in \mathbb{R}^k :\ \langle a_{i,\bar{\mathcal{S}}}^\top, s\rangle \le b_{i,\bar{\mathcal{S}}}, \forall i \in \{1:m\}\}\\ &= \{s \in \mathbb{R}^k :\ A_{\bar{\mathcal{S}}} s \le b_{\bar{\mathcal{S}}}\},\end{aligned} \tag{30}$$

where $A_{\bar{\mathcal{S}}} := [a_{1,\bar{\mathcal{S}}}^\top, \cdots, a_{m,\bar{\mathcal{S}}}^\top]^\top = (M^\top A_\mathcal{S}^\top)^\top = A_\mathcal{S} M$ and $b_{\bar{\mathcal{S}}} := [b_{1,\bar{\mathcal{S}}}, \cdots, b_{m,\bar{\mathcal{S}}}]^\top = b_\mathcal{S} - A_\mathcal{S}\xi$. Now, one can deduce from the inequalities (29) and the definition (30) that the convex polytope $\bar{\mathcal{S}}$ contains the hyper-rectangle $\mathcal{B}(l,u)$, i.e., $\mathcal{B}(l,u) \subseteq \bar{\mathcal{S}}$. Notice that $\mathcal{B}(l,u)$ is parametric in the variables l and u.

Theorem 4.4: In the CP framework, we propose a convex nonlinear program to compute the hyper-rectangle $\mathcal{B}(l,u) \subseteq \bar{\mathcal{S}}$ such that its volume is maximized. Suppose $\mathcal{B}(l,u)$ is parameterized as $l := -\underline{v} = [-\underline{v}_1, \cdots, -\underline{v}_k]^\top$ and $u := \overline{v} = [\overline{v}_1, \cdots, \overline{v}_k]^\top$ such that for all $i \in \{1:k\}$, $\underline{v}_i$ and $\overline{v}_i$ are positive scalars (this condition has to do with the fact that the resulting hyper-rectangle should contain the origin). Recall the inequality (29), that is $\langle M^\top a_{i,\mathcal{S}}^\top, z\rangle \le b_{i,\mathcal{S}} - a_{i,\mathcal{S}}\xi$, for all $i \in \{1:m\}$ and for all $z \in \mathcal{B}$. In what follows, we show that although the hyper-rectangle $\mathcal{B}(l,u) = \mathcal{B}(-\underline{v}, \overline{v})$ is parametric, one can provide a closed form for its support function evaluated at $M^\top a_{i,\mathcal{S}}^\top$. By definition of a support function,

$$\begin{aligned} h_\mathcal{B}(M^\top a_{i,\mathcal{S}}^\top) = \max_z\ & \langle M^\top a_{i,\mathcal{S}}^\top, z\rangle \\ \text{s.t. } & A_\mathcal{B} z \le b_\mathcal{B}, \end{aligned} \tag{31}$$

where $A_\mathcal{B} = [\mathsf{I}_k\ \ -\mathsf{I}_k]^\top$ and $b_\mathcal{B} = [\overline{v}^\top\ \ \underline{v}^\top]^\top$. The above problem is an LP with a bounded feasible set. Thus, the optimal solution lies on the boundary of the hyper-rectangle towards which the normal $M^\top a_{i,\mathcal{S}}^\top$ points. Let us define, for all $i \in \{1:m\}$, $\hat{w}^i := \text{sign}(M^\top a_{i,\mathcal{S}}^\top) \in \mathbb{R}^k$, where the sign operator is applied entry-wise. (Notice that this vector simply indicates the orthant(s) that the vector $M^\top a_{i,\mathcal{S}}^\top$ points to.) It then becomes clear that the vectors $w^i \in \mathbb{R}^{2k}$, as defined in (16), enable us to express the optimal solution of (31) in terms of a linear combination of the vertices of $\mathcal{B}$, i.e.,

$$h_\mathcal{B}\big(M^\top a_{i,\mathcal{S}}^\top\big) = \langle w^i, [\overline{v}^\top\ \ \underline{v}^\top]^\top\rangle, \forall i \in \{1:m\}.$$

Based on the above relation, the inequality (29) simplifies to

$$\langle w^i, [\overline{v}^\top\ \ \underline{v}^\top]^\top\rangle \le b_{i,\mathcal{S}} - a_{i,\mathcal{S}}\xi, \forall i \in \{1:m\},$$

in which the vectors $\underline{v}, \overline{v} \in \mathbb{R}^k$ are the decision variables. Intuitively, the above inequalities represents the linear constraints that the vertices of the hyper-rectangle $\mathcal{B}(-\underline{v}, \overline{v})$ should satisfy in order to guarantee $\xi \in \mathcal{S} \ominus M\mathcal{B}(-\underline{v}, \overline{v})$.

Based on the chosen definition of volume for $\mathcal{B}(-\underline{v}, \overline{v})$ in (11), we intend to find a hyper-rectangle $\mathcal{B}(-\underline{v}, \overline{v})$ that possesses the maximal volume. Unfortunately, regardless of the definition choice for the volume, the resulting objective function is non-convex and becomes unsuitable for optimization. Interestingly enough, one can simply use the logarithmic mapping for the volume definitions in (11) to obtain the objective functions suggested in (15), that are monotonic nonlinear concave functions. Then, it follows that a maximum hyper-rectangle $\mathcal{B}$ that contains the origin and satisfies $\xi \in \mathcal{S} \ominus M\mathcal{B}$ is the solution of the CP (14).

Theorem 4.5: In the LP framework, we follow the procedure proposed in [5] with which one is able to cast the problem as a linear program. We first provide the proof for the LP relaxation of the problem (14) with $q = 1$. Let us denote the maximum length of a line segment containing the origin, parallel to the j-th coordinate axis, and contained in $\bar{\mathcal{S}}$ by r_j. It follows from [5, Proposition3] that one can use (17b) to find r_j, for all $j \in \{1 : k\}$. It is worth nothing that in the LP (17b), the constraints $z \leq 0$ and $z + \omega e_j \geq 0$ are two extra regularity conditions that we placed on the line segment compared to [5, Proposition3]. These conditions ensure that the origin lies inside this line segment. Now, define the strictly positive vector $r \in \mathbb{R}^k$ by $r_j = \omega_j$ for all $j \in \{1 : k\}$. Then, it follows from [5, Proposition2] that a maximum r-constrained inner hyper-rectangular $\mathcal{B}$ of $\bar{\mathcal{S}}$ that contains the origin is given by $\mathcal{B}(z^*, z^* + \lambda^* r)$ where z^* and λ^* are the optimal solutions of (17a). Here, we also emphasize the fact that we have introduced the extra constraints $z \leq 0$ and $z + \lambda r \geq 0$ with respect to [5, Proposition2]. By doing so, the LP (17a) is forced to find a hyper-rectangular $\mathcal{B}$ such that it contains the origin. Then, the claim for the LP case follows.

We now present a sketch of proof for the LP relaxation of the problem (14) with $q = 2$. Observe that the polytope $\bar{\mathcal{S}}' := \{s \in \mathbb{R}^{2k} : W's \leq B'\}$ is the inequality representation of the constraints in the CP (14), where W' and B' are defined in Theorem 4.5. We seek to find a hyper-rectangle that fits inside this *lifted* polytope as follows. In the first step, we place a vertex of the hyper-rectangle at the origin. We then find the width of the line segment along each coordinate that is inside the lifted polytope and contains the origin using (18b). In the second step, we use (18a) to find a scaling factor λ such that the λ-scaled hyper-rectangle constructed based on the first step fits inside the polytope $\bar{\mathcal{S}}'$. This concludes the proof.

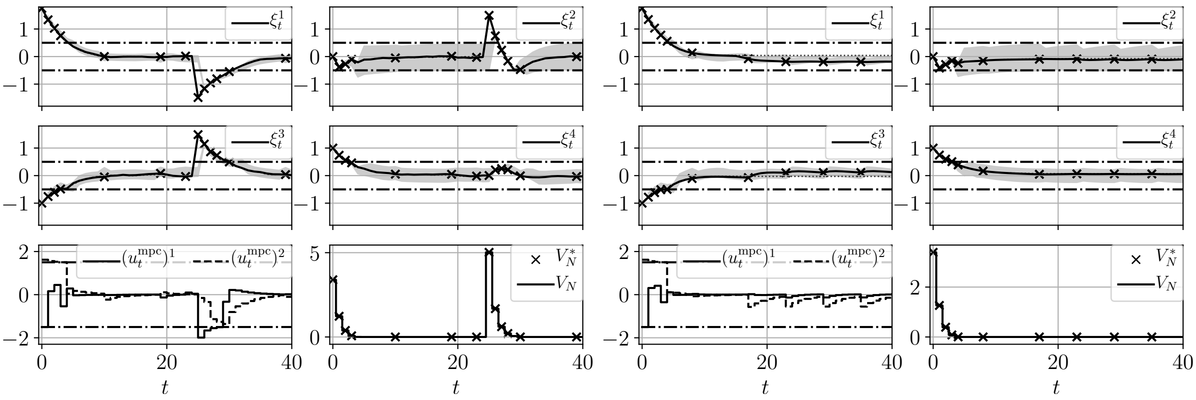
6. Numerical Examples

In this section, we provide a numerical example to study the results presented in Section 4. For the numerical simulations, we use CVXOPT [1] and (py)cddlib [14]. The system is an unstable batch reactor borrowed from [33, Page 213]. We discretized the model using the zero-order-hold method with step-size 0.05, that is,

$$x^+ = \begin{pmatrix} 1.08 & -0.05 & 0.29 & -0.24 \\ -0.03 & 0.81 & 0.00 & 0.03 \\ 0.04 & 0.19 & 0.73 & 0.24 \\ 0.00 & 0.19 & 0.05 & 0.91 \end{pmatrix} x + \begin{pmatrix} 0.00 & -0.02 \\ 0.26 & 0.00 \\ 0.08 & -0.13 \\ 0.08 & -0.00 \end{pmatrix} u + w$$

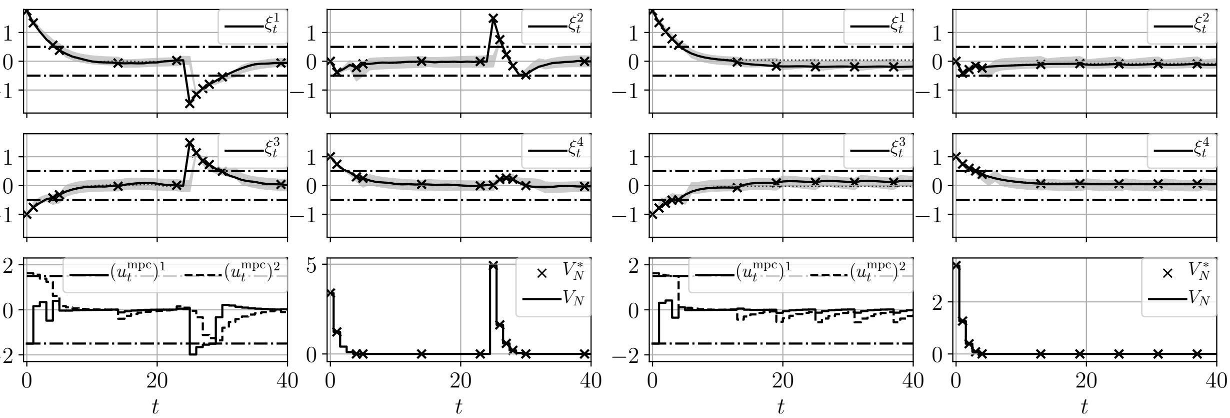
where the state and input constraint sets are $\mathbb{X} = \{x \in \mathbb{R}^4 : \|x\|_\infty \leq 2\}$ and $\mathbb{U} = \{u \in \mathbb{R}^2 : \|u\|_\infty \leq 2\}$, respectively. The disturbance set is defined as $\mathbb{W} = \{w \in \mathbb{R}^4 : \|w\|_\infty \leq 0.02\}$. The state and input target sets are $\mathbb{T}_x = \{x \in \mathbb{R}^4 : \|x\|_\infty \leq 0.5\}$ and $\mathbb{T}_u = \{u \in \mathbb{R}^2 : \|u\|_\infty \leq 1.5\}$, respectively. The horizon length N is set to 10. The weight matrices in the cost function (4) are $Q = 2 \times \mathsf{I}_4$ and $r = \mathsf{I}_2$. Finally, the terminal set is $\mathcal{X}_f = \{x \in \mathbb{R}^4 : \|x\|_\infty \leq 0.2\}$.

In what follows, we employ the triggering set construction approaches of Theorems 4.4 & 4.5 for $q = 1$. Two types of disturbance realizations are considered: (1) a uniform distribution with the bounded support $\mathbb{W}$, and (2) a worst case disturbance $w_t = \operatorname{argmax}_{w \in \mathbb{W}} \xi_t^\top w$ at each time t. In the case of uniform disturbance, we also manually applied an impulse-type disturbance to the closed-loop dynamics by resetting the second state ξ_{25}^2 to 1.7. This disturbance does not belong to the admissible disturbance set $\mathbb{W}$.



(a) Uniform case disturbance. (b) Worst case disturbance.

FIGURE 2. Comparison of the event-based implementation using the construction approach CP_1 with the standard implementation. (Top four) The Solid lines are the evolution of states. The crosses are the states at triggering instances. The (gray) shaded areas are the projection of constructed hyper-rectangles $\boldsymbol{\mathcal{E}}$ on the corresponding state's coordinate axis. (Bottom left) The lines are the input of the closed-loop system. (Bottom right) The crosses are the value function V_N^* at triggering instances. The solid line is the inter-event cost function V_N computed using Theorem 4.1.



(a) Uniform case disturbance. (b) Worst case disturbance.

FIGURE 3. Comparison of the event-based implementation using the construction approach LP_1 with the standard implementation. (Top four) The Solid lines are the evolution of states. The crosses are the states at triggering instances. The (gray) shaded areas are the projection of constructed hyper-rectangles $\boldsymbol{\mathcal{E}}$ on the corresponding state's coordinate axis. (Bottom left) The lines are the input of the closed-loop system. (Bottom right) The crosses are the value function V_N^* at triggering instances. The solid line is the inter-event cost function V_N computed using Theorem 4.1.

Figures 2 and 3 show the behavior of the event-based implementation of the MPC method. (Notice that the behavior of the standard MPC was almost identical, we did not include the results of the standard MPC for the sake clarity.)

We begin with pointing out the shared properties of the approaches CP_1 and LP_1. First of all, it is evident that the number of instances that the optimization problem (5) is solved has effectively reduced in all considered cases compared to standard periodic implementation. Observe that the inputs and states of the closed-loop dynamics (7) do not violate the constraint sets $\mathbb{X}$ and $\mathbb{U}$, respectively, in all considered cases. Moreover, the closed-loop states ξ_t and the inputs u_t converge to the target sets $\mathbb{T}_x$ and $\mathbb{T}_u$, respectively. Finally, both of the approaches CP_1 and LP_1 can effectively recover from the impulse-type disturbance applied on time $t = 25$. We also note that the event-based implementations exhibit an almost limit-cyclic behavior inside the target set $\mathbb{T}_x$ in the worst case disturbance realizations.

Let us now highlight the difference between the construction approaches CP_1 and LP_1. As depicted in the top right plots of Figures 2(a) and 3(a), the construction method LP_1 is more conservative in comparison with the construction method CP_1. The width of the shaded areas represents the projection of the triggering sets $\boldsymbol{\mathcal{E}}(x)$. In Figure 2(a), one can also observe in the top right plot that the triggering intervals are tight with respect to the target sets, as well.

7. Future Directions

In this paper, an event-triggering approach was proposed to implement an RMPC method to constrained, perturbed LTI systems. The procedure to design the triggering mechanism is online, and is decoupled from the controller design. Specifically, we introduced two theoretical frameworks to construct the triggering mechanism as a volume maximization problem. There are multiple directions that one can pursue to extend the results in this paper. First, it is interesting to investigate the possibility of extending the results of this paper to a nonlinear MPC case. In qualitative manner, we have observed that the choice of tightening gains $\boldsymbol{K}$ directly impacts the constructed triggering sets $\boldsymbol{\mathcal{E}}$. Hence, another possible direction is to explore the possibility of characterizing this unknown dependency in a more quantitative manner. Lastly, the triggering approach proposed in this paper is online (and in fact state-dependent). It is thus valuable to investigate whether it is possible to make the triggering set design offline.

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