Outline

1. Course overview
2. History
3. Principles of light propagation
4. Adaptive Optics system components
5. Applications for AO
Course overview — Teaching staff

**Rufus Fraanje**
TU Delft, DCSC, 3ME, room 8c2-19,
Email: p.r.fraanje@tudelft.nl
Research: efficient control for large scale adaptive optics systems, adaptive control algorithms, distributed system identification
Course overview — Teaching staff

**Rufus Fraanje**
TU Delft, DCSC, 3ME, room 8c2-19,
Email: p.r.fraanje@tudelft.nl
Research: efficient control for large scale adaptive optics systems, adaptive control algorithms, distributed system identification

**Silvania Pereira**
TU Delft, Optics Research Group, TNW, room E018,
Email: s.f.pereira@tudelft.nl
Research: high resolution optical recording, microscopy and optical aperture synthesis
Course overview — Teaching staff

**Rufus Fraanje**
TU Delft, DCSC, 3ME, room 8c2-19,
Email: p.r.fraanje@tudelft.nl
Research: efficient control for large scale adaptive optics systems, adaptive control algorithms, distributed system identification

**Silvania Pereira**
TU Delft, Optics Research Group, TNW, room E018,
Email: s.f.pereira@tudelft.nl
Research: high resolution optical recording, microscopy and optical aperture synthesis

**Matthew Kenworthy**
Leiden Univ., Leiden Observatory,
Email: kenaryworthy@strw.leidenuniv.nl
Research: searches for extrasolar planets and techniques to find them, e.g., coronagraphy and point spread function reconstruction
Course overview — Motivation

**Smart Optics Systems**, STW Perspectief programme, 20 PhD/Postdocs

**Program leader:** Prof. dr. ir. Michel Verhaegen (TU Delft)

**Goal:** Integration of optical, mechanical, electronic, and control systems for increasing image resolution taking cost and complexity of the design into account.

**Application fields:** Astronomy, Lithography, Microscope, Camera systems, LASER, ...
Course overview — Motivation

- **Center for Adaptive Optics (CfAO)**, large group of researchers in California
  Yearly AO Summerschool
  [http://cfao.ucolick.org](http://cfao.ucolick.org)
Course overview — Motivation

- **Center for Adaptive Optics (CfAO)**, large group of researchers in California Yearly AO Summerschool
  http://cfao.ucolick.org

- **AO enabling technology for European Extremely Large Telescope**, Dutch consortium for E-ELT instrumentation
  http://esfri.strw.leidenuniv.nl
Course overview — Motivation

- **Center for Adaptive Optics (CfAO),** large group of researchers in California
  Yearly AO Summerschool
  [http://cfao.ucolick.org](http://cfao.ucolick.org)

- **AO enabling technology for European Extremely Large Telescope,**
  Dutch consortium for E-ELT instrumentation
  [http://esfri.strw.leidenuniv.nl](http://esfri.strw.leidenuniv.nl)

- **Conferences:** SPIE, OSA, AO4ELT (Paris, 2009), (Victoria, 2011), ...
Course overview — Motivation

- **Center for Adaptive Optics (CfAO)**, large group of researchers in California
  Yearly AO Summerschool
  http://cfao.ucolick.org

- **AO enabling technology for European Extremely Large Telescope**, Dutch consortium for E-ELT instrumentation
  http://esfri.strw.leidenuniv.nl

- **Conferences**: SPIE, OSA, AO4ELT (Paris, 2009), (Victoria, 2011), ...

- Special issues in journals:
  - European Journal of Control, appearing
Course overview — Motivation

- **Center for Adaptive Optics (CfAO)**, large group of researchers in California
  Yearly AO Summerschool
  http://cfao.ucolick.org

- **AO enabling technology for European Extremely Large Telescope**, Dutch consortium for E-ELT instrumentation
  http://esfri.strw.leidenuniv.nl

- **Conferences**: SPIE, OSA, AO4ELT (Paris, 2009), (Victoria, 2011), …

- Special issues in journals:
  - European Journal of Control, appearing

- In the news: BBC Feb. 18, 2011, ’Adaptive optics’ come into focus’
Course overview — Teaching objectives

After successful completion of the course you will be able to:

- Design simple adaptive optics systems;
Course overview — Teaching objectives

After successful completion of the course you will be able to:

- Design simple adaptive optics systems;
- Understand the working and main specifications of individual components:
  - Imaging system (e.g., telescope, microscope, ...)
  - Deformable mirror;
  - Wavefront sensor;
  - Controller.
Course overview — Teaching objectives

After successful completion of the course you will be able to:

- Design simple adaptive optics systems;
- Understand the working and main specifications of individual components:
  - Imaging system (e.g., telescope, microscope, ...)
  - Deformable mirror;
  - Wavefront sensor;
  - Controller.
- Model dynamic wavefront distortions due to turbulence;
Course overview — Teaching objectives

After successful completion of the course you will be able to:

- Design simple adaptive optics systems;
- Understand the working and main specifications of individual components:
  - Imaging system (e.g., telescope, microscope, ...)
  - Deformable mirror;
  - Wavefront sensor;
  - Controller.
- Model dynamic wavefront distortions due to turbulence;
- Evaluate various algorithms for wavefront reconstruction;
Course overview — Teaching objectives

After successful completion of the course you will be able to:

- Design simple adaptive optics systems;
- Understand the working and main specifications of individual components:
  - Imaging system (e.g., telescope, microscope, ...)
  - Deformable mirror;
  - Wavefront sensor;
  - Controller.
- Model dynamic wavefront distortions due to turbulence;
- Evaluate various algorithms for wavefront reconstruction;
- Design controllers for adaptive optics systems;
Course overview — Teaching objectives

After successful completion of the course you will be able to:

- Design simple adaptive optics systems;
- Understand the working and main specifications of individual components:
  - Imaging system (e.g., telescope, microscope, ...)
  - Deformable mirror;
  - Wavefront sensor;
  - Controller.
- Model dynamic wavefront distortions due to turbulence;
- Evaluate various algorithms for wavefront reconstruction;
- Design controllers for adaptive optics systems;
- Understand the role of AO in various applications, especially in astronomical observation.
Course overview — Teaching objectives

After successful completion of the course you will be able to:

- Design simple adaptive optics systems;
- Understand the working and main specifications of individual components:
  - Imaging system (e.g., telescope, microscope, ...)
  - Deformable mirror;
  - Wavefront sensor;
  - Controller.
- Model dynamic wavefront distortions due to turbulence;
- Evaluate various algorithms for wavefront reconstruction;
- Design controllers for adaptive optics systems;
- Understand the role of AO in various applications, especially in astronomical observation.
- Understand current and future research in AO.
Course overview — Teaching objectives

After successful completion of the course you will be able to:

- Design simple adaptive optics systems;
- Understand the working and main specifications of individual components:
  - Imaging system (e.g., telescope, microscope, ...)
  - Deformable mirror;
  - Wavefront sensor;
  - Controller.
- Model dynamic wavefront distortions due to turbulence;
- Evaluate various algorithms for wavefront reconstruction;
- Design controllers for adaptive optics systems;
- Understand the role of AO in various applications, especially in astronomical observation.
- Understand current and future research in AO.

Examination:

- Short paper (4 to 6 pages) on one topic (50%);
- Oral exam on paper and all topics (50%).
Course overview — System design

Integrated systems design course:

- Optics
- Photonics
- Image Processing
- Electronics
- Mechanics
- Control
- Smart Optics Systems

- Multidisciplinary approach, focus on interrelations;
- Details so far relevant for understanding system;
- Follow other courses for understanding disciplines, e.g.:
  - Geometrical optics (AP3391, TU Delft, TNW)
  - Imaging Systems (AP3121, TU Delft, TNW)
  - Systems and Control Engineering (WB2207, TU Delft, DCSC)
  - Filtering and Identification (SC4040, TU Delft, DCSC)
  - Detection of Light (Leiden);
  - Mechanical design in mechatronics (WB2428-03, TU Delft, PME);
  - ...
Course overview — Study material

- Main study material:
  - Lecture slides
Course overview — Study material

- **Main study material:**
  - Lecture slides

- **Helpful study material on optics:**
  - Born & Wolf, Principles of optics, Cambridge, 2009
  - Chartier, Introduction to Optics, Springer, 2005
  - Hecht, Optics, Addison Wesley, 2002
Course overview — Study material

- **Main study material:**
  - Lecture slides

- **Helpful study material on optics:**
  - Born & Wolf, Principles of optics, Cambridge, 2009
  - Chartier, Introduction to Optics, Springer, 2005
  - Hecht, Optics, Addison Wesley, 2002

- **Helpful study material on adaptive optics:**
  - Roddier, Adaptive optics in astronomy, Cambridge, 1999
  - Tyson, Adaptive optics engineering handbook, Dekker, 2000
  - Porter, Adaptive optics for vision science, Wiley, 2006
Helpful study material on control and identification:

- Franklin et al., Feedback Control of Dynamic Systems, Pearson, 2008
Course overview — Study material (Cont.)

- Helpful study material on control and identification:
  - Franklin et al., Feedback Control of Dynamic Systems, Pearson, 2008

- Online texts:
  - Tokovins Adaptive optics tutorial at CTIO: http://www.ctio.noao.edu/~atokovin/tutorial/intro.html
  - Claire Max’s Introduction to Adaptive Optics and its History: http://www.ucolick.org/~max/max-web/History_AO_Max.htm
  - also see her AO course http://www.ucolick.org/~max/289C/
Helpful study material on control and identification:
- Franklin et al., Feedback Control of Dynamic Systems, Pearson, 2008

Online texts:
- Tokovins Adaptive optics tutorial at CTIO:
  http://www.ctio.noao.edu/~atokovin/tutorial/intro.html
- Adaptive optics at ESO:
  http://www.eso.org/sci/facilities/develop/ao/tecno/index.html
- Claire Max’s Introduction to Adaptive Optics and its History:
  http://www.ucolick.org/~max/max-web/History_AO_Max.htm
  also see her AO course http://www.ucolick.org/~max/289C/

References given at lectures.
Course overview — Schedule

Schedule:

<table>
<thead>
<tr>
<th>DATE</th>
<th>TOPIC</th>
<th>LECTURER</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 Apr</td>
<td>Introduction to AO systems</td>
<td>Fraanje</td>
</tr>
<tr>
<td>2 May</td>
<td>Optical image formation</td>
<td>Pereira</td>
</tr>
<tr>
<td>9 May</td>
<td>Optical wavefront sensors</td>
<td>Pereira</td>
</tr>
<tr>
<td>16 May</td>
<td>Turbulence modeling</td>
<td>Fraanje</td>
</tr>
<tr>
<td>30 May</td>
<td>Reconstruction and prediction</td>
<td>Fraanje</td>
</tr>
<tr>
<td>6 Jun</td>
<td>Astronomical observation</td>
<td>Kenworthy</td>
</tr>
<tr>
<td>13 Jun</td>
<td>Deformable mirror control</td>
<td>Fraanje</td>
</tr>
</tbody>
</table>

- Time: 10.45 - 12.30h
- Location: TU Delft, API-CZ Rudolf Diesel (building 46)
History

Newton:

*If the Theory of making Telescopes could at length be fully brought into Practice, yet there would be certain Bounds beyond which Telescopes could not perform. For the Air through which we look upon the Stars, is in a perpetual Tremor.*
Newton:

*If the Theory of making Telescopes could at length be fully brought into Practice, yet there would be certain Bounds beyond which Telescopes could not perform. For the Air through which we look upon the Stars, is in a perpetual Tremor.*

Until Babcock (1953):

**PUBLICATIONS OF THE**

**ASTRONOMICAL SOCIETY OF THE PACIFIC**

<table>
<thead>
<tr>
<th>Vol. 65</th>
<th>October 1953</th>
<th>No. 386</th>
</tr>
</thead>
</table>

**THE POSSIBILITY OF COMPENSATING**

**ASTRONOMICAL SEEING**

**H. W. Babcock**

Mount Wilson and Palomer Observatories
Carnegie Institution of Washington
California Institute of Technology

The severe limitations imposed upon nearly all astronomical observations by "seeing"—the effects resulting from passage of light rays through the turbulent atmosphere of the earth—are familiar to every observer. With a small instrument the effect may appear largely as a continual shifting and scintillation of the image of a star, but with a large telescope poor seeing usually manifests itself mainly as an enlargement and blurring of the image. The reason for this difference becomes apparent when one makes a knife-edge test with a large telescope on a bright star. The turbulent elements of the atmosphere appear as shifting dark and bright areas on the image of the objective. In general
Fig. 1.—Schematic diagram of seeing compensator. A blurred image of the telescope objective, formed on the image orthicon, is transferred electronically in the form of a modulated electric charge to the surface of the oil film covering the Eidophor mirror. This feed-back process results in the correction of ray deviations due either to atmospheric turbulence or to optical imperfections. F is a field lens and C is a fast guide for centering the control star on the knife-edge, K. The two mirrors are off-axis paraboloids.
History — Adaptive Optics

Without adaptive optics

Atmospheric Turbulence

Beam Splitter

DM

WFS

Detector
History — AO @ Keck
10m Keck telescope @ Mauna Kea, Hawai
History — AO @ Keck
Feb. 5, 1999: First AO observation, V=5.6 A0 star

http://www2.keck.hawaii.edu/ao/aolight.html
History — AO @ Keck

May 26, 1999: Mauna Kea, Hawai
10mø Keck 2 pointed at galactic center with and without AO

http://cfao.ucolick.org/pgallery/gc.php
History — AO @ Keck


Fig. 1.— A comparison of the first LGS-AO image (left) and the best NGS-AO image (right) taken with the W. M. Keck II 10 m telescope (2002-2004) in the L' (3.8 μm) photometric bandpass of the central 7″ × 7″5 of our Galaxy. In both images, the cross denotes the location of the central supermassive black hole and the orientation is North up and East to the left. The LGS-AO image has a Strehl ratio that is a factor of two higher than that obtained in the NGS-AO image; furthermore, the LGS-AO image resulted from an exposure time of only 8 min, a factor of ~20 less than the comparison NGS-AO image. The LGS-AO system has therefore dramatically improved the image quality that can be obtained on the Galactic center with the Keck telescope.
History of largest optical telescopes

<table>
<thead>
<tr>
<th>Year</th>
<th>Diameter [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1600</td>
<td>10^{-2}</td>
</tr>
<tr>
<td>1700</td>
<td>10^{-1}</td>
</tr>
<tr>
<td>1800</td>
<td>10^{0}</td>
</tr>
<tr>
<td>1900</td>
<td>10^{1}</td>
</tr>
<tr>
<td>2000</td>
<td>10^{2}</td>
</tr>
</tbody>
</table>

- **Galileo**
- **Huygens**
- **Newton**
- **Herschel**
- **Hale**
- **Keck**
- **VLT**
- **Keck**
- **VLT**
- **E-ELT**
- **TMT**

Lenses: ○
Mirrors: +
Achromats: ○
Shortcomings: ×
Segmented mirrors: +

### History — Future E-ELT

The European Extremely Large Telescope

**Nasmyth configuration**

- **Aperture:** 38m Ø
- **Optical design:** 5 mirrors, M1&M3 for AO
- **M1:** 984(?) segments, 38m Ø
- **M3:** 8000 actuators, 2.5m Ø
- **Field of view:** 10 arcminute
- **Location:** Cerro Armazones, Chile
- **First light:** 2020
History — Future E-ELT

The European Extremely Large Telescope (E-ELT)

- Diffraction limit (Rayleigh criterion): 

\[ \sin(\theta) = 1.22 \frac{\lambda}{D} \]

\[ \theta < 10\text{mas in near-infrared} \]

\(1\text{mas} \approx 4.8 \cdot 10^{-9}\text{rad}\)
History — Future E-ELT

The European Extremely Large Telescope (E-ELT)

- Diffraction limit (Rayleigh criterion):
  \[
  \sin(\theta) = 1.22 \frac{\lambda}{D}
  \]

\[\theta < 10\text{mas in near-infrared}
(1\text{mas} \approx 4.8 \cdot 10^{-9}\text{rad})\]

Seeing conditions:

- Coherence length (Fried parameter): \(r_0 = 20\text{cm}\)
  Effective resolution without AO: \(1.22\lambda/r_0 \approx 1800\text{mas}\)

Adaptive Optics:

\[\rightarrow (D/r_0 = 200)^2 = 40.000\text{ actuators/sensors}\]
History — Future E-ELT

Development of computing power

Instructions/flops per second

Clock Frequency [Hz]


Intel 8080
Motorola 6800
Intel 286
Intel 386DX
Intel 486DX
DEC Alpha
Intel Pentium Pro
AMD Athlon
Xbox360 (3 cores)
Intel I7 (6 cores)
AMD Phenom (6 cores)
E-ELT AO

Year

Principles of light propagation

Literature: G. Chartier, *Introduction to Optics*, Springer, 2005 [online at TU Library], Sec. 1.2, 1.3, parts Ch. 2, and Sec. 3.1, 3.2

Light acts as a particle and a wave

Photons characterized by:
- individual energy $W$;
- momentum $p$.

**Planck relationship:** $W = h\nu$

Waves characterized by:
- frequency $\nu$;
- wave vector $k$.

**De Broglie relationship:** $p = \frac{h}{2\pi} k$

($h = 6.626 \cdot 10^{-34}$ Js, Planck’s constant)
Principles of light propagation

4.1G 41M 0.41M 4.1K 41 0.41 4.1m 41u 0.41u 4.1p 0.41p 4.1f W (eV)

Increasing Energy (W)

← Increasing Frequency (v)

γ rays X rays UV IR Microwave FM Radio waves AM Long radio waves

10^-16 10^-14 10^-12 10^-10 10^-8 10^-6 10^-4 10^-2 10^0 10^2 10^4 10^6 10^8 λ (m)

Increasing Wavelength (λ) →

Visible spectrum

3.1eV 1.8eV Energy

7.10^14Hz 4.10^14Hz Frequency

400nm 700nm Wavelength (in vacuum)

(source: Wikipedia)
Principles of light propagation

Waves propagate, mathematical formulation:

\[ g(t, x) = f(t \pm x/V) \]

\( V \) is velocity of wave
Principles of light propagation

Waves propagate, mathematical formulation:

\[ g(t, x) = f(t \pm x / V) \]

\( V \) is velocity of wave

Planar wave:

\[ g(t, x) = f(t - u \cdot x / V), \quad u, \text{ direction of wave} \]
Principles of light propagation

Waves propagate, mathematical formulation:

\[ g(t, x) = f(t \pm x/V) \]

\( V \) is velocity of wave

Planar wave:

\[ g(t, x) = f(t - u \cdot x/V), \quad u, \text{ direction of wave} \]

Amplitude variation due to absorption:

\[ g(t, x) = e^{-\alpha u \cdot x} f(t - u \cdot x/V) \]
Principles of light propagation

Waves propagate, mathematical formulation:

\[ g(t, x) = f(t \pm x/V) \]

\( V \) is velocity of wave

Planar wave:

\[ g(t, x) = f(t - u \cdot x/V), \quad u, \text{ direction of wave} \]

Amplitude variation due to absorption:

\[ g(t, x) = e^{-\alpha u \cdot x} f(t - u \cdot x/V) \]

Spherical waves:

\[ g(t, x) = \frac{e^{-\alpha r}}{r} f(t \pm r/V), \quad r = \sqrt{x^T x} \]
Principles of light propagation

Harmonic planar waves:

\[ g(t, x) = A \cos(\omega t - k \cdot x) \]

- **Frequency**: \( \nu \).
Principles of light propagation

Harmonic planar waves:

\[ g(t, x) = A \cos(\omega t - k \cdot x) \]

- **Frequency:** \( \nu \).
- **Pulsation:** \( \omega \) (also angular frequency), \( \omega = 2\pi \nu \).
Principles of light propagation

Harmonic planar waves:

\[ g(t, x) = A \cos(\omega t - k \cdot x) \]

- **Frequency:** \( \nu \).
- **Pulsation:** \( \omega \) (also angular frequency), \( \omega = 2\pi \nu \).
- **Period:** \( T = \frac{1}{\nu} = \frac{2\pi}{\omega} \).
Principles of light propagation

Harmonic planar waves:

\[ g(t, \mathbf{x}) = A \cos(\omega t - \mathbf{k} \cdot \mathbf{x}) \]

- **Frequency:** \( \nu \).
- **Pulsation:** \( \omega \) (also angular frequency), \( \omega = 2\pi \nu \).
- **Period:** \( T = \frac{1}{\nu} = \frac{2\pi}{\omega} \).

**Speed of propagation:** \( \mathbf{V} = (V_x, V_y, V_z) \), \( \mathbf{V} = \| \mathbf{V} \|_2 = \sqrt{V_x^2 + V_y^2 + V_z^2} \).
Principles of light propagation

Harmonic planar waves:

\[ g(t, x) = A \cos(\omega t - k \cdot x) \]

- **Frequency:** \( \nu \).
- **Pulsation:** \( \omega \) (also angular frequency), \( \omega = 2\pi\nu \).
- **Period:** \( T = \frac{1}{\nu} = \frac{2\pi}{\omega} \).
- **Speed of propagation:** \( V = (V_x, V_y, V_z) \), \( V = \|V\|_2 = \sqrt{V_x^2 + V_y^2 + V_z^2} \).
- **Wavelength:** (space period), \( \lambda = VT = \frac{V}{\nu} = \frac{2\pi}{k} \).
Principles of light propagation

Harmonic planar waves:

\[ g(t, x) = A \cos(\omega t - k \cdot x) \]

- **Frequency:** \( \nu \).
- **Pulsation:** \( \omega \) (also angular frequency), \( \omega = 2\pi \nu \).
- **Period:** \( T = \frac{1}{\nu} = \frac{2\pi}{\omega} \).
- **Speed of propagation:** \( V = (V_x, V_y, V_z) \),
  \( V = \|V\|_2 = \sqrt{V_x^2 + V_y^2 + V_z^2} \).
- **Wavelength:** (space period), \( \lambda = VT = \frac{V}{\nu} = \frac{2\pi}{k} \).
- **Wave vector:** (spatial frequency) \( k = (k_x, k_y, k_z) \),
  \( k = \|k\|_2 = \frac{2\pi}{\lambda} = \frac{\omega}{V} \).
Principles of light propagation

Harmonic planar waves:

\[ g(t, \mathbf{x}) = A \cos(\omega t - \mathbf{k} \cdot \mathbf{x}) \]

- **Frequency:** \( \nu \).
- **Pulsation:** \( \omega \) (also angular frequency), \( \omega = 2\pi \nu \).
- **Period:** \( T = \frac{1}{\nu} = \frac{2\pi}{\omega} \).

**Speed of propagation:** \( \mathbf{V} = (V_x, V_y, V_z) \),
\[
\mathbf{V} = \|\mathbf{V}\|_2 = \sqrt{V_x^2 + V_y^2 + V_z^2}.
\]

**Wavelength:** (space period), \( \lambda = \mathbf{V}T = \frac{V}{\nu} = \frac{2\pi}{k} \).

**Wave vector:** (spatial frequency) \( \mathbf{k} = (k_x, k_y, k_z) \),
\[
k = \|\mathbf{k}\|_2 = \frac{2\pi}{\lambda} = \frac{\omega}{V}.
\]

**Direction of propagation:** \( \mathbf{u} = \mathbf{V}/V = \mathbf{k}/k \),
\[
k = k\mathbf{u} = k\mathbf{V}/V = \frac{\omega}{V}\mathbf{u}.
\]
Principles of light propagation

Propagation of light through media described by Maxwell Equations

Vector variables:

- \( \mathbf{E}(t, \mathbf{x}) = (E_x, E_y, E_z)(t, \mathbf{x}) \): electric field (V/m)
- \( \mathbf{H}(t, \mathbf{x}) = (H_x, H_y, H_z)(t, \mathbf{x}) \): magnetic field (A/m)
- \( \mathbf{D}(t, \mathbf{x}) = (D_x, D_y, D_z)(t, \mathbf{x}) \): electric displacement (C/m²)
- \( \mathbf{B}(t, \mathbf{x}) = (B_x, B_y, B_z)(t, \mathbf{x}) \): magnetic induction (T)
Principles of light propagation

Propagation of light through media described by Maxwell Equations

Vector variables:

- \( \mathbf{E}(t, \mathbf{x}) = (E_x, E_y, E_z)(t, \mathbf{x}) \): electric field (V/m)
- \( \mathbf{H}(t, \mathbf{x}) = (H_x, H_y, H_z)(t, \mathbf{x}) \): magnetic field (A/m)
- \( \mathbf{D}(t, \mathbf{x}) = (D_x, D_y, D_z)(t, \mathbf{x}) \): electric displacement (C/m\(^2\))
- \( \mathbf{B}(t, \mathbf{x}) = (B_x, B_y, B_z)(t, \mathbf{x}) \): magnetic induction (T)

Transparent material usually no magnetic properties: \( \mathbf{B} = \mu_0 \mathbf{H}, \mu_0 = 4\pi 10^{-7} \text{H/m} \)
Principles of light propagation

Propagation of light through media described by Maxwell Equations

Vector variables:

- \( \mathbf{E}(t, \mathbf{x}) = (E_x, E_y, E_z)(t, \mathbf{x}) \): electric field (V/m)
- \( \mathbf{H}(t, \mathbf{x}) = (H_x, H_y, H_z)(t, \mathbf{x}) \): magnetic field (A/m)
- \( \mathbf{D}(t, \mathbf{x}) = (D_x, D_y, D_z)(t, \mathbf{x}) \): electric displacement (C/m\(^2\))
- \( \mathbf{B}(t, \mathbf{x}) = (B_x, B_y, B_z)(t, \mathbf{x}) \): magnetic induction (T)

Transparent material usually no magnetic properties: \( \mathbf{B} = \mu_0 \mathbf{H}, \mu_0 = 4\pi 10^{-7}\) H/m

Linear isotropic media: \( \mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_0 \varepsilon_r \mathbf{E}, \varepsilon_0 = 1/(36\pi 10^9)\) F/m
Principles of light propagation

Propagation of light through media described by Maxwell Equations

Vector variables:

- \( \mathbf{E}(t, \mathbf{x}) = (E_x, E_y, E_z)(t, \mathbf{x}) \): electric field (V/m)
- \( \mathbf{H}(t, \mathbf{x}) = (H_x, H_y, H_z)(t, \mathbf{x}) \): magnetic field (A/m)
- \( \mathbf{D}(t, \mathbf{x}) = (D_x, D_y, D_z)(t, \mathbf{x}) \): electric displacement (C/m\(^2\))
- \( \mathbf{B}(t, \mathbf{x}) = (B_x, B_y, B_z)(t, \mathbf{x}) \): magnetic induction (T)

Transparent material usually no magnetic properties: \( \mathbf{B} = \mu_0 \mathbf{H}, \mu_0 = 4\pi 10^{-7}\) H/m

Linear isotropic media: \( \mathbf{D} = \epsilon \mathbf{E} = \epsilon_0 \epsilon_r \mathbf{E}, \epsilon_0 = 1/(36\pi 10^9)\) F/m

Relative permittivity \( \epsilon_r \) typically between 1 and 10,
- function of frequency (color) \( \rightarrow \) dispersion.
- varies with space(/time) \( \rightarrow \) (time-varying) refraction and reflection.
Principles of light propagation

Propagation of light through media described by Maxwell Equations

Vector variables:

- \( \mathbf{E}(t, \mathbf{x}) = (E_x, E_y, E_z)(t, \mathbf{x}) \): electric field (V/m)
- \( \mathbf{H}(t, \mathbf{x}) = (H_x, H_y, H_z)(t, \mathbf{x}) \): magnetic field (A/m)
- \( \mathbf{D}(t, \mathbf{x}) = (D_x, D_y, D_z)(t, \mathbf{x}) \): electric displacement (C/m\(^2\))
- \( \mathbf{B}(t, \mathbf{x}) = (B_x, B_y, B_z)(t, \mathbf{x}) \): magnetic induction (T)

Transparent material usually no magnetic properties: \( \mathbf{B} = \mu_0 \mathbf{H}, \mu_0 = 4\pi 10^{-7}\) H/m

Linear isotropic media: \( \mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_0 \varepsilon_r \mathbf{E}, \varepsilon_0 = 1/(36\pi 10^9)\) F/m

Relative permittivity \( \varepsilon_r \) typically between 1 and 10,
- function of frequency (color) \( \rightarrow \) dispersion.
- varies with space(/time) \( \rightarrow \) (time-varying) refraction and reflection.

EM-wave fully characterized by:

\[
\mathbf{EM}(t, \mathbf{x}) = (\mathbf{E}(t, \mathbf{x}), \mathbf{H}(t, \mathbf{x}))
\]
Principles of light propagation

For harmonic waves Maxwell equations simplify to *Helmholtz equation*:

\[
\mathbf{EM}(t, \mathbf{x}) = \mathbf{A}(\mathbf{x}) \cos(\omega t - \phi(\mathbf{x})) = \text{Re} \{ \mathbf{U}(\mathbf{x}) e^{j \omega t} \}
\]

where

\[
\mathbf{U}(\mathbf{x}) = \mathbf{A}(\mathbf{x}) e^{-j \phi(\mathbf{x})}
\]
Principles of light propagation

For harmonic waves Maxwell equations simplify to *Helmholtz equation*:

\[
EM(t, x) = A(x) \cos(\omega t - \phi(x)) = \text{Re} \{U(x)e^{j\omega t}\}
\]

where

\[
U(x) = A(x)e^{-j\phi(x)}
\]

then the *Helmholtz (wave) equation* specifies:

\[
\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} + \frac{\omega^2}{V^2} U = 0
\]

where \( V \) the wave propagation speed

\[
V = \frac{1}{\sqrt{\mu_0 \epsilon}} = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r}} = \frac{c}{n}
\]

where \( c = 1/\sqrt{\mu_0 \epsilon_0} \) the speed of light in vacuum and \( n = \sqrt{\epsilon_r} \) the refraction index.
Principles of light propagation

In general harmonic EM-waves with frequency $\omega$ need to satisfy:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} + \frac{\omega^2}{V^2} U = J(x, \omega)$$

+ boundary conditions

where $J(x, \omega)$ an excitation function.
Principles of light propagation

In general harmonic EM-waves with frequency $\omega$ need to satisfy:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} + \omega^2 V^2 U = J(x, \omega)$$

+ boundary conditions

where $J(x, \omega)$ an excitation function.

Special solutions of Helmholtz equation:

- Planar waves: $U(x) = A(x)e^{-jk^Tx}$.
- Spherical waves: $U(x) = A(x)e^{-jkr}, \quad r = \sqrt{x^Tx}$.

In finite volume at large distance from focus spherical waves with amplitude proportional to $1/r$ can be considered as planar with constant amplitude.
Principles of light propagation

Spherical wave

\[ U(x, y, z) = \frac{a}{\sqrt{x^2 + y^2 + z^2}} e^{\pm jk\sqrt{x^2 + y^2 + z^2}} \]
Principles of light propagation

Spherical wave

\[ U(x, y, z) = \frac{a}{\sqrt{x^2 + y^2 + z^2}} e^{\pm jk \sqrt{x^2 + y^2 + z^2}} \]

for large \(|z|\) compared to \(\sqrt{x^2 + y^2}\) we have:

\[ \sqrt{x^2 + y^2 + z^2} \approx |z| + \frac{1}{2|z|} \sqrt{x^2 + y^2} \]
Principles of light propagation

Spherical wave

\[
U(x, y, z) = \frac{a}{\sqrt{x^2 + y^2 + z^2}} e^{\pm jk \sqrt{x^2 + y^2 + z^2}}
\]

for large \(|z|\) compared to \(\sqrt{x^2 + y^2}\) we have:

\[
\sqrt{x^2 + y^2 + z^2} \approx |z| + \frac{1}{2|z|} \sqrt{x^2 + y^2}
\]

such that \(U(x, y, z)\) is approximated by

\[
U(x, y, z) \approx \frac{a}{|z|} e^{\pm jk \frac{x^2 + y^2}{2|z|}} e^{-jk|z|} \approx \frac{a}{|z|} e^{\pm jk|z|}
\]
Principles of light propagation

Spherical wave

\[ U(x, y, z) = \frac{a}{\sqrt{x^2 + y^2 + z^2}} e^{\pm jk \sqrt{x^2 + y^2 + z^2}} \]

for large \(|z|\) compared to \(\sqrt{x^2 + y^2}\) we have:

\[ \sqrt{x^2 + y^2 + z^2} \approx |z| + \frac{1}{2|z|} \sqrt{x^2 + y^2} \]

such that \(U(x, y, z)\) is approximated by

\[ U(x, y, z) \approx \frac{a}{|z|} e^{\pm jk \frac{x^2 + y^2}{2|z|}} e^{-jk|z|} \approx \frac{a}{|z|} e^{\pm jk|z|} \]

and at \(z + \Delta z\):

\[ U(x, y, z + \Delta z) \approx \frac{a}{|z + \Delta z|} e^{\pm jk|z + \Delta z|} \approx \frac{a}{|z|} e^{\pm jk|z + \Delta z|} \]
Principles — Fermat’s Principle

Fermat’s Principle:

*The path followed by light going from point A to point B is such that the transit time is stationary (minimum or maximum).*
Principles — Fermat’s Principle

Fermat’s Principle:

The path followed by light going from point A to point B is such that the transit time is stationary (minimum or maximum).

\[ t = \frac{\left( n_0 AI + n_1 IJ + n_0 JK + n_1 KL + n_0 LM + n_2 MN + n_0 NB \right)}{c} \]

\[ t' = \frac{\left( n_0 AI' + n_1 I'J' + n_0 J'K' + n_2 K'L' + n_0 L'M' + n_3 M'N' + n_0 N'B \right)}{c} \]
Fermat’s Principle:

The path followed by light going from point A to point B is such that the transit time is stationary (minimum or maximum).

\[ t = \frac{1}{c} \int_{A}^{B} n(x, y, z) \, ds \]

\[ t' = \frac{1}{c} \int_{A}^{B} n(\bar{x}, \bar{y}, \bar{z}) \, ds \]

\[ t = \frac{1}{c} \int_{A}^{B} n(x, y, z) \, ds \]

\[ t' = \frac{1}{c} \int_{A}^{B} n(\bar{x}, \bar{y}, \bar{z}) \, ds \]
Principles — Fermat’s Principle

Law of Reflection:

angle of reflection equals angle of incidence
Principles — Fermat’s Principle

Law of Refraction (Snell’s law)$^1$:

*The law of refraction states that the incident ray, the refracted ray, and the normal to the interface, all lie in the same plane. Furthermore,*

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$^1$http://farside.ph.utexas.edu/teaching/302l/lectures/node128.html
Principles — Fermat’s Principle

Law of Refraction (Snell’s law)\(^1\):

The law of refraction states that the incident ray, the refracted ray, and the normal to the interface, all lie in the same plane. Furthermore,

\[ n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \]

\[ t(x) = \left( \frac{n_1 \sqrt{y_1^2 + x^2} + n_2 \sqrt{y_2^2 + (x_2 - x)^2}}{c} \right) \]

\[ \frac{dt}{dx} = \left( \frac{n_1 x}{\sqrt{y_1^2 + x^2}} - \frac{n_2 (x_2 - x)}{\sqrt{y_2^2 + (x_2 - x)^2}} \right) / c, \]

\[ \frac{dt}{dx} = 0 \iff \frac{n_1 x}{\sqrt{y_1^2 + x^2}} = \frac{n_2 (x_2 - x)}{\sqrt{y_2^2 + (x_2 - x)^2}} \]

\(^1\)http://farside.ph.utexas.edu/teaching/302l/lectures/node128.html
Principles of light propagation

plane with constant phase

\[ n_2 > n_1 \]

\[ d = \frac{(n_2 - n_1)h}{n_0} \]

(optical path difference!)
Principles of light propagation

Plane with constant phase

$n_0$

$n_1$

$n_2$

$n_2 > n_1$

$h$

$d = \frac{(n_2 - n_1)h}{n_0}$

(optical path difference!)

Phase difference ($n_0 = 1$):

$$\Delta \phi(x) = -kh(n(x) - 1) = -\frac{2\pi h(n(x) - 1)}{\lambda}$$
Principles of light propagation

Concluding so far:

- In transparent nonmagnetic media $n = \sqrt{\varepsilon_r}$;
Principles of light propagation

Concluding so far:

- In transparent nonmagnetic media $n = \sqrt{\epsilon_r}$;
- Fermat’s principle: time from A to B is minimum or maximum;
Principles of light propagation

Concluding so far:

- In transparent nonmagnetic media $n = \sqrt{\varepsilon_r}$;
- Fermat’s principle: time from A to B is minimum or maximum;
- Variations in $n$ result in optical pathlength differences:

$$\Delta \phi(x, y) = -\frac{2\pi h(n(x) - 1)}{\lambda}$$
Principles of light propagation

Concluding so far:

- In transparent nonmagnetic media \( n = \sqrt{\epsilon_r} \);
- Fermat’s principle: time from A to B is minimum or maximum;
- Variations in \( n \) result in optical pathlength differences:

\[
\Delta \phi(x, y) = -\frac{2\pi h(n(x) - 1)}{\lambda}
\]

- Dependency of phase on index of refraction decreases for increasing \( \lambda \) (sub-millimeter waves only suffer from pointing error);
Concluding so far:

- In transparent nonmagnetic media $n = \sqrt{\varepsilon_r}$;
- Fermat's principle: time from A to B is minimum or maximum;
- Variations in $n$ result in optical pathlength differences:

\[
\Delta \phi(x, y) = -\frac{2\pi h(n(x) - 1)}{\lambda}
\]

- Dependency of phase on index of refraction decreases for increasing $\lambda$ (sub-millimeter waves only suffer from pointing error);
- Compensation can be accomplished by changing optical pathlength if refraction index is independent of wavelength (no dispersion).
Adaptive Optics system components

- exoplanet
- atmospheric turbulence
- ELT
- spectrograph (scientific instrument)
Adaptive Optics system components

- exoplanet
- atmospheric turbulence
- ELT
- spectrograph (scientific instrument)
Adaptive Optics system components

- Exoplanet
- Atmospheric turbulence
- ELT
- DM
- Spectrograph (scientific instrument)
- Control
- WFS
Adaptive Optics system components

- Adaptation of optical system components
- Alignment errors and temperature variations
- Force wind disturbance
- Exoplanet
- Atmospheric turbulence
- Wind disturbance force
- ELT
- Ground vibrations
- Spectrograph (scientific instrument)
- Control
- WFS
- DM

Scientific Instrument

Delft Center for Systems and Control

Delft University of Technology
Adaptive Optics system components

Generalized plant:

\[ d(k) \rightarrow S(z) \rightarrow \phi_d(k) \]
Adaptive Optics system components

Generalized plant:

\[ d(k) \rightarrow S(z) \rightarrow \phi_d(k) \rightarrow \phi_r(k) \]

\[ H(z) \rightarrow \phi_u(k) \]

\[ u(k) \rightarrow \]

\[ S(z) \]

\[ H(z) \]
Adaptive Optics system components

Generalized plant:

\[ S(z) \]

\[ H(z) \]

\[ G(z) \]

\[ d(k) \]

\[ u(k) \]

\[ s(k) \]

\[ \phi_r(k) \]

\[ \phi_u(k) \]

\[ \phi_d(k) \]
Adaptive Optics system components

Generalized plant:

\[ S(z) \]
\[ H(z) \]
\[ G(z) \]

\[ d(k) \rightarrow S(z) \rightarrow \phi_d(k) \rightarrow \phi_r(k) \]
\[ u(k) \rightarrow H(z) \rightarrow \phi_u(k) \rightarrow G(z) \rightarrow s(k) \]
Adaptive Optics system components

Generalized plant:

\[ d(k) \rightarrow S(z) \xrightarrow{\phi_d(k)} \phi_r(k) \]

\[ u(k) \rightarrow H(z) \xrightarrow{\phi_u(k)} G(z) \]

\[ s(k) \]

\[ C(z) \]
Adaptive Optics system components

Generalized plant:

\[ d(k) \]

\[ S(z) \]

\[ \phi_d(k) \]

\[ \phi_r(k) \]

\[ u(k) \]

\[ H(z) \]

\[ \phi_u(k) \]

\[ G(z) \]

\[ s(k) \]

Optimal control law (static DM with unit sample delay):

\[ C(z) : \quad u(k + 1) = -H^\dagger \hat{\phi}_d(k + 1|k) \]
Adaptive Optics system components

Generalized plant:

\[ d(k) \]

\[ S(z) \]

\[ \phi_d(k) \]

\[ \phi_r(k) \]

\[ u(k) \]

\[ H(z) \]

\[ \phi_u(k) \]

\[ G(z) \]

\[ s(k) \]

Optimal control law with control weighting:

\[ C(z) : \quad u(k + 1) = -(H^T H + \alpha I)^{-1} H^T \hat{\phi}_d(k + 1|k) \]
Adaptive Optics system components

Kolmogorov distributed disturbance measured at William Herschel Telescope, La Palma (Spain)
Adaptive Optics system components

Kolmogorov distributed disturbance measured at William Herschel Telescope, La Palma (Spain)

\[
\text{Power spectral density } [\text{rad}^2/\text{Hz}] \\
\text{Frequency } [\text{Hz}]
\]
Adaptive Optics system components

Quality of imaging system: Strehl ratio ($S_r$) (K. Strehl [1895]),

\[
S_r := \frac{I_m \text{ (measured peak intensity)}}{I_o \text{ (diffraction limited peak intensity)}} \\
\approx e^{-||\phi_r||_2^2}
\]

Minimize cost function: \( J := ||\phi_r||_2^2 \)
Adaptive Optics system components

- Star
- No turbulence (bright picture of star)
- Shack Hartmann Lenslet array
- CCD camera view

Diagram shows a star being imaged through an adaptive optics system, with no turbulence causing a bright, clear picture.
Adaptive Optics system components

Measurement: $s_x = \frac{\partial \phi_r}{\partial x}$ and $s_y = \frac{\partial \phi_r}{\partial y}$, $s = [s_x^T, s_y^T]^T$
Adaptive Optics system components

Wavefront sensors:
- Shack Hartmann sensor (local wavefront slopes)
- Shearing interferometer (local wavefront slopes)
- Curvature Wavefront Sensor (local second order spatial derivative)
- Pyramid sensor
- Pinhole (light intensity at single spot)

Adaptive Optics system components

\[ G \]

\[ \nabla x/y \]

\[ \int_{t}^{t+t_e} \]

\[ \mathbf{v} \]

\[ s(k) = G\phi_r(k - 1) + \mathbf{v}(k) \]

'Static' model:

Adaptive Optics system components

Deformable mirrors:

- Continuous membrane
  (Flexible Optical, TNO, ...)
  local influence function per actuator
Adaptive Optics system components

Deformable mirrors:

- Continuous membrane
  (Flexible Optical, TNO, ...)
  local influence function per actuator

- Segmented membrane
  (E-ELT primary mirror 1000 segments,
  Boston micromachines, ...)
  fully decoupled actuators
Adaptive Optics system components

Deformable mirrors:

- Continuous membrane
  (Flexible Optical, TNO, ...)
  local influence function per actuator

- Segmented membrane
  (E-ELT primary mirror 1000 segments,
  Boston micromachines, ...)
  fully decoupled actuators

Kilo-DM (Boston),
1020 actuators,
60kHz frame-rate
Adaptive Optics system components

Deformable mirrors:

- Continuous membrane
  (Flexible Optical, TNO, ...)
  local influence function per actuator

- Segmented membrane
  (E-ELT primary mirror 1000 segments,
  Boston micromachines, ...)
  fully decoupled actuators

Kilo-DM (Boston),
1020 actuators,
60kHz frame-rate

Piezoelectric DM
(Flexible Optical, Delft),
low cost
Adaptive Optics system components

Performance limitations:
- Limited spatial actuation (number/position/range actuators);
- Limited spatial sensing (number/position/range sensors);
- Non-minimum phase dynamics (e.g., delays);
- Sensor noise;
- (Anisoplanatism, e.g., in astronomy).

Limited spatial actuation and sensing $\Rightarrow$ fitting error.

Good design: often these errors are in same order of magnitude.

Trend: more actuators/sensors $\rightarrow$ fitting error $\downarrow$
but time-error and sensor noise become more dominant!
Applications

Applications of adaptive optics:
- Astronomy: high resolution ground based telescopes;
Applications of adaptive optics:

- Astronomy: high resolution ground based telescopes;

- Ophtalmology: inspection/compensation eye aberrations;
Applications

Applications of adaptive optics:

- Astronomy: high resolution ground based telescopes;

- Ophthalmology: inspection/compensation eye aberrations;

- Lithography: compensation of (time varying) errors;
Applications

Applications of adaptive optics:

- Astronomy: high resolution ground based telescopes;
- Ophthalmology: inspection/compensation eye aberrations;
- Lithography: compensation of (time varying) errors;
- Microscopy: compensation change diffraction index in living cells;
Applications

Applications of adaptive optics:

- Astronomy: high resolution ground based telescopes;

- Ophthalmology: inspection/compensation eye aberrations;

- Lithography: compensation of (time varying) errors;

- Microscopy: compensation change diffraction index in living cells;

- Long range cameras: compensation of turbulence in air;
Applications of adaptive optics:
- Astronomy: high resolution ground based telescopes;
- Ophthalmology: inspection/compensation eye aberations;
- Lithography: compensation of (time varying) errors;
- Microscopy: compensation change diffraction index in living cells;
- Long range cameras: compensation of turbulence in air;
- LASERS: increase coherence length / power;
Applications of adaptive optics:

- Astronomy: high resolution ground based telescopes;
- Ophthalmology: inspection/compensation eye aberrations;
- Lithography: compensation of (time varying) errors;
- Microscopy: compensation change diffraction index in living cells;
- Long range cameras: compensation of turbulence in air;
- LASERS: increase coherence length / power;
- ...

Applications — Ophtalmology

 Courtesy: D. Williams
Applications — Microscopy

Confocal microscope/2-photon microscope:
Confocal microscope/2-photon microscope:
Applications — Microscopy

Confocal microscope/2-photon microscope:

- LASER
- DETECTOR
- CONTROLLER
- DEFORMABLE MIRROR
Conclusions

- Babcock (1953), Keck (1999), E-ELT, ...
Conclusions

- Babcock (1953), Keck (1999), E-ELT, …
- Spherical and Planar wave propagation;
Conclusions

- Babcock (1953), Keck (1999), E-ELT, ...
- Spherical and Planar wave propagation;
- Propagation of light, Fermat’s principle;
Conclusions

- Babcock (1953), Keck (1999), E-ELT, ...
- Spherical and Planar wave propagation;
- Propagation of light, Fermat’s principle;
- Variations in refraction index lead to optical pathlength variations \( \Delta d(x) = n(x)h \);
Conclusions

- Babcock (1953), Keck (1999), E-ELT, …

- Spherical and Planar wave propagation;

- Propagation of light, Fermat’s principle;

- Variations in refraction index lead to optical pathlength variations \((\Delta d(x) = n(x)h)\);

- Pathlength variations lead to wavefront phase variations \((\Delta \phi(x) = -2\pi \Delta d(x)/\lambda)\);
Conclusions

- Babcock (1953), Keck (1999), E-ELT, ...

- Spherical and Planar wave propagation;

- Propagation of light, Fermat’s principle;

- Variations in refraction index lead to optical pathlength variations \( \Delta d(x) = n(x)h \);

- Pathlength variations lead to wavefront phase variations \( \Delta \phi(x) = -2\pi \Delta d(x)/\lambda \);

- A generalized plant description of an AO system consists of:
  - Disturbance model (wavefront phase variations);
  - Performance criterion (e.g., Strehl or Contrast);
  - Actuation to adjust optical pathlength (e.g., deformable mirror);
  - Device to measure the variation in optical pathlength or wavefront phase (e.g., a Shack Hartmann wavefront sensor).
Conclusions

- Babcock (1953), Keck (1999), E-ELT, …

- Spherical and Planar wave propagation;

- Propagation of light, Fermat’s principle;

- Variations in refraction index lead to optical pathlength variations 
  \( \Delta d(x) = n(x)h \);

- Pathlength variations lead to wavefront phase variations 
  \( \Delta \phi(x) = -2\pi \Delta d(x)/\lambda \);

- A generalized plant description of an AO system consists of:
  - Disturbance model (wavefront phase variations);
  - Performance criterion (e.g., Strehl or Contrast);
  - Actuation to adjust optical pathlength (e.g., deformable mirror);
  - Device to measure the variation in optical pathlength or wavefront phase (e.g., a Shack Hartmann wavefront sensor).

Study: Chartier Sec. 1.2, 1.3, parts Ch. 2, and Sec. 3.1, 3.2; Hardy Ch. 1.