Subspace Identification of Local 1D Homogeneous Systems

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Abstract: This paper studies the local subspace identification of 1D homogeneous networked systems. The main challenge lies at the unmeasurable interconnection signals between neighboring subsystems. Since there are many unknown inputs to the concerned local system, the corresponding identification problem is semi-blind. To cope with this problem, a nuclear norm optimization based subspace identification is presented, which is carried out for solving the Markov parameters of a locally lifted system, followed by determining the system matrices of a single subsystem. In the step of Markov parameter estimation, we form a nuclear norm regularized optimization problem which can well handle the adverse effects of the unknown system inputs as long as the number of unknown system inputs is relatively small. In the step of system realization, we again derive a nuclear norm regularized optimization formulation which can cope with the under-determinedness of the realization problem. In the end, the overall identification algorithm is summarized.

Keywords: Markov parameter, system realization, low rank constraint

1. INTRODUCTION

With the emergence of large-scale networks, the research on distributed systems has been intensively carried out in both control and identification areas. Although tremendous progress has been made in the field of distributed control, the identification of networked systems is less developed. From an engineering perspective, the system identification research is more relevant than that about system control, because system identification can provide for an estimate of the concerned system model which is essential for conducting model based control research.

In this paper, the subspace identification of 1D large-scale distributed systems is investigated. To date, several contributions have been reported on this topic; however, it has not been adequately addressed. By representing the system model in terms of transfer functions, parametric system identification approaches have been proposed in Ali et al. (2011); Dankers et al. (2013). Since they are required to solve nonlinear (usually non-convex) optimization problems, it is difficult to ensure the global optimality of the obtained solutions. For the transfer function represented networked systems, the interconnecting signals between neighboring subsystems can always be measurable, which greatly simplifies the identification problem.

In practical applications such as identifying distributed parameter systems described by PDEs, the interconnecting signals between neighboring subsystems are unmeasurable. By modeling the interconnecting signals in the network as the states of a state-space represented network, several identification methods have been developed. In Rice and Verhaegen (2011), the state-space system model having sequential, semi-separability (SSS) property is parameterized, and the associated identification problem is handled by solving a nonlinear (non-convex) optimization problem. For large-scale circulant systems, a subspace identification method, by exploiting the particular circularity property, has been developed in Massioni and Verhaegen (2008). In Massioni and Verhaegen (2009), the identification of a decomposable system with a general network topology is considered. Since the identification of system matrices involves solving a Bilinear Matrix Inequality (BMI), the obtained solution can be a local optimum.

In order to ensure the scalability of the identification methods for large-scale systems, only local system inputs and outputs can be used. As a result, the corresponding local system identification problem involves several unknown system inputs, which poses great challenge to the identification problem. In Haber and Verhaegen (2014), a subspace identification algorithm is presented, which tries to approximate the unknown system inputs (unmeasurable neighboring states) using a linear combination of local input and output data. This method suits the case for which the neighboring subsystems are weakly coupled. In Matni and Rantzer (2014), a nuclear norm optimization based identification method is provided, which tries to separate the local dynamics and global dynamics by exploiting order and rank distinctions. In this method, the unknown system inputs are assumed to be energy bounded

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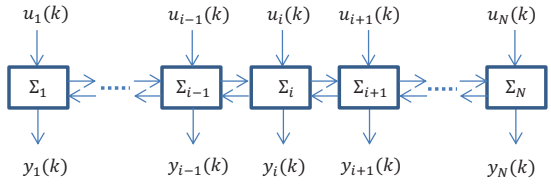

Fig. 1. Diagram of 1D distributed homogeneous system

disturbances, and they are considered as constraints to the nuclear norm optimization problem.

Inspired by the N2SID (nuclear norm subspace identification) method in Verhaegen and Hansson (2014), we present a subspace identification method to solve the local identification of large-scale 1D homogeneous networked systems. First, a nuclear norm regularized optimization problem is formed to estimate Markov parameters which exploits the low rank properties of the extended observability matrix and the unknown-input related term in the data equation, as well as the Toeplitz block Toeplitz structure of the Markov parameter matrix. After obtaining the Markov parameters of the local system, the system matrices of a single subsystem are estimated by solving another nuclear norm optimization problem which exploits the low rank property of a matrix constructed by the involved redundant parameters.

The paper is organized as follows. Section 2 formulates the identification problem. Section 3 provides an approach to estimate Markov parameters using local input and output data, followed by the system realization in Section 4. Section 5 summarizes the proposed identification algorithm. Section 6 concludes this contribution.

2. PROBLEM FORMULATION

We consider a 1D distributed homogeneous system consisting of N subsystems, as shown in Fig. 1. For a large-scale distributed system, we assume that $N \gg 0$. The i-th subsystem Σ_i, for $2 \le i \le N-1$, is represented by the state-space model as follows:

$$\begin{aligned} x_i(k+1) &= Ax_i(k) + Bu_i(k) + A_l x_{i-1}(k) + A_r x_{i+1}(k) \\ y_i(k) &= Cx_i(k) + e_i(k), \quad i = 1, \cdots, N, \end{aligned} \tag{1}$$

where $x_i(k) \in \mathbb{R}^{n\times 1}$, $u_i(k) \in \mathbb{R}^{m\times 1}$ and $y_i(k) \in \mathbb{R}^{p\times 1}$ are the state, input and output of the i-th subsystem, $x_{i-1}(k)$ and $x_{i+1}(k)$ are the neighboring states of the i-th subsystem.

For the above concerned large-scale system with only local system input and output data being available, we aim to identify the system matrices A, A_l, A_r, B, C up to a similarity transform; that is, the estimated system matrices satisfy that $\hat{A} = QAQ^{-1}$, $\hat{A}_l = QA_lQ^{-1}$, $\hat{A}_r = QA_rQ^{-1}$, $\hat{C} = CQ^{-1}$, and $\hat{B} = QB$ for some nonsingular ambiguity matrix $Q \in \mathbb{R}^{n\times n}$. Here, the notion "local system" around the i-th subsystem refers to a system combined by the subsystems $\{\Sigma_j\}_{j=i-R}^{i+R}$ with $R \ll N$.

In this paper, we assume that C is flat matrix, namely $p \le n$. Otherwise, the state in (1) can be represented in terms of the output. The local system model can then be recasted into an errors-in-variables (EIV) model, which can be solved using many classic identification methods, see Ljung (1999); Verhaegen and Verdult (2007).

3. MARKOV PARAMETER ESTIMATION

For the local system consisting of $\{\Sigma_j\}_{j=i-R}^{i+R}$, the corresponding spatially lifted state-space model reads:

$$\begin{aligned} \underline{x}_i(k+1) &= \underline{A}_R\underline{x}_i(k) + \underline{B}_R\underline{u}_i(k) + \underline{D}_R\underline{v}_i(k), \\ \underline{y}_i(k) &= \underline{C}_R\underline{x}_i(k) + \underline{e}_i(k), \end{aligned} \tag{2}$$

where $\underline{A}_R = \begin{bmatrix} A & A_r & & \\ A_l & \ddots & \ddots & \\ & \ddots & \ddots & A_r \\ & & A_l & A \end{bmatrix}$, $\underline{B}_R = \begin{bmatrix} B & & & \\ & B & & \\ & & \ddots & \\ & & & B \end{bmatrix}$, $\underline{C}_R = \begin{bmatrix} C & & & \\ & C & & \\ & & \ddots & \\ & & & C \end{bmatrix}$ with the subscript $_R$ indicating that $\underline{A}_R, \underline{B}_R, \underline{C}_R$ have R block rows. $\underline{x}_i(k) = \begin{bmatrix} x_{i-R}(k) \\ \vdots \\ x_{i+R}(k) \end{bmatrix}$, $\underline{u}_i(k) = \begin{bmatrix} u_{i-R}(k) \\ \vdots \\ u_{i+R}(k) \end{bmatrix}$, $\underline{y}_i(k) = \begin{bmatrix} y_{i-R}(k) \\ \vdots \\ y_{i+R}(k) \end{bmatrix}$, $\underline{D}_R = \begin{bmatrix} A_l & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & A_r \end{bmatrix}$, and $\underline{v}_i(k) = \begin{bmatrix} x_{i-R-1}(k) \\ x_{i+R+1}(k) \end{bmatrix}$.

In the sequel, we assume that the state-space model in (2) is minimal. The data equation of the state-space model (2) is given as:

$$\underline{Y}^i_{k,s,r} = \underline{\mathcal{O}}_s\underline{x}^i_{k,r} + \underline{\mathcal{T}}_s^{B_R}\underline{U}^i_{k,s,r} + \underline{\mathcal{T}}_s^{D_R}\underline{V}^i_{k,s,r} + \underline{E}^i_{k,s,r}, \tag{3}$$

where the block-Hankel matrix

$$\underline{Y}^i_{k,s,r} = \begin{bmatrix} \underline{y}_i(k) & \cdots & \underline{y}_i(k+r-1) \\ \vdots & \iddots & \vdots \\ \underline{y}_i(k+s-1) & \cdots & \underline{y}_i(k+r+s-2) \end{bmatrix}$$

with the superscript i being the spatial index, the subscripts k, s, r being the time indices of the top-left entry, the number of block rows and the number of block columns, respectively. The block-Hankel matrices $\underline{U}^i_{k,s,r}$, $\underline{V}^i_{k,s,r}$, $\underline{E}^i_{k,s,r}$ are defined similarly to $\underline{Y}^i_{k,s,r}$. The block-Toeplitz matrix $\underline{\mathcal{T}}_s^{B_R} = \begin{bmatrix} 0 & & & \\ \underline{C}_R\underline{B}_R & 0 & & \\ \vdots & \ddots & \ddots & \\ \underline{C}_R\underline{A}_R^{s-2}\underline{B}_R & \cdots & \underline{C}_R\underline{B}_R & 0 \end{bmatrix}$. The block-Toeplitz matrix $\underline{\mathcal{T}}_s^{D_R}$ and $\underline{\mathcal{T}}_s^{B_R}$ have similar definitions. The extended observability matrix $\underline{\mathcal{O}}_s = \begin{bmatrix} \underline{C}_R \\ \underline{C}_R\underline{A}_R \\ \vdots \\ \underline{C}_R\underline{A}_R^{s-1} \end{bmatrix}$ and $\underline{x}^i_{k,r} = [\underline{x}_i(k) \cdots \underline{x}_i(k+r-1)]$.

From the data equation (3), we can find that the terms $\underline{\mathcal{O}}_s \underline{x}^i_{k,r}$ and $\underline{\mathcal{T}}^{D_R}_s \underline{V}^i_{k,s,r}$ have low ranks.

Lemma 1. The sum $\underline{\mathcal{O}}_s \underline{x}^i_{k,r} + \underline{\mathcal{T}}^{D_R}_s \underline{V}^i_{k,s,r}$ in (3) has a rank satisfying

$$\text{rank}\left(\underline{\mathcal{O}}_s \underline{x}^i_{k,r} + \underline{\mathcal{T}}^{D_R}_s \underline{V}^i_{k,s,r}\right) \leq (2R+1)n + 2(s-1)n. \quad (4)$$

The result in the above lemma can be straightforwardly derived. It can be seen that when R or s is not too small, the sum $\underline{\mathcal{O}}_s \underline{x}^i_{k,r} + \underline{\mathcal{T}}^{D_R}_s \underline{V}^i_{k,s,r}$ has a lower rank with relation to the dimension of $\underline{Y}^i_{k,s,r}$. Using this low rank property and implementing the N2SID method in Verhaegen and Hansson (2014), we propose a nuclear norm regularized optimization for the local system identification:

$$\begin{aligned} \min_{\underline{\mathcal{T}}^{B_R}_s \in \mathcal{T}, \underline{\hat{Y}}^i_{k,s,r} \in \mathcal{H}} & \|\underline{\hat{Y}}^i_{k,s,r} - \underline{\mathcal{T}}^{B_R}_s \underline{U}^i_{k,s,r}\|_* \\ & + \lambda \sum_{t=k}^{k+r+s-2} \|\underline{\hat{y}}_i(t) - \underline{y}_i(t)\|^2, \end{aligned} \quad (5)$$

where λ is a regularization parameter to trade off the output fitting term and the nuclear norm term, $\mathcal{T}$ and $\mathcal{H}$ denote the sets of block-Toeplitz and block-Hankel matrices having the same structures of $\underline{\mathcal{T}}^{B_R}_s$ and $\underline{Y}^i_{k,s,r}$, respectively. The block-Hankel matrix $\underline{\hat{Y}}^i_{k,s,r}$ is constructed by the virtual noise-free output $\underline{\hat{y}}_i(t)$ satisfying that $\underline{\hat{y}}_i(t) = \underline{y}_i(t) - \underline{e}_i(t)$.

Solving the optimization problem (5) yields the estimated block-Toeplitz matrix $\underline{\mathcal{T}}^{B_R}_s$ and further augmented matrices $\underline{A}_R, \underline{B}_R, \underline{C}_R$ up to a similarity transform. However, the primal objective of this study is to estimate the system matrices A_l, A, A_r, B, C in (1) up to a similarity transform. By taking an insight into the structure of the block Toeplitz matrix $\underline{\mathcal{T}}^{B_R}_s$, we can find that each block Markov parameter in $\underline{\mathcal{T}}^{B_R}_s$ still has finer structures. Fig. 2 shows the structures of the block Markov parameters $\{\underline{M}_i = \underline{C}_R \underline{A}^i_R \underline{B}_R\}^3_{i=1}$, where $R = 3$ and each of its block element having size 2×2.

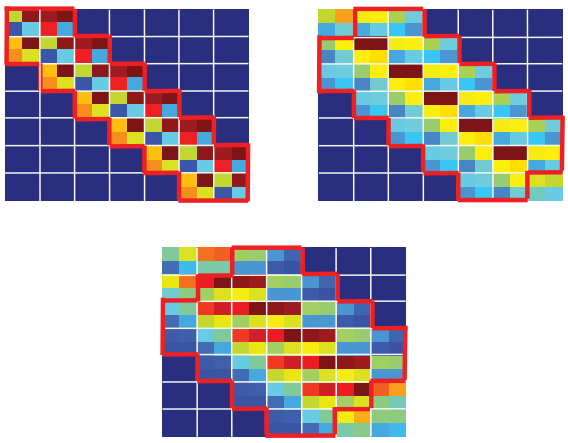

Fig. 2. Partial Toeplitz structure of $\{\underline{C}_R \underline{A}^i_R \underline{B}_R\}^3_{i=1}$. Top-left: $i = 1$; top-right: $i = 2$; bottom: $i = 3$. The blue entries are zeros. The parts surrounded by red curves have block-Toeplitz structures.

In equation (2), the system matrices $\underline{A}_R$ is a block tri-diagonal matrix, and $\underline{C}_R$ and $\underline{B}_R$ are block diagonal matrices. Note that $\underline{A}^i_R$ for $i \geq 2$ is not a block-Toeplitz matrix any more; however, it is partial Toeplitz matrix, as shown in Fig. 2.

Lemma 2. Based on the system matrices $\underline{A}_R, \underline{B}_R$ and $\underline{C}_R$ defined in (2), the matrix $\underline{M}_i = \underline{C}_R \underline{A}^i_R \underline{B}_R$ has the following features:

- $\underline{M}_i$ is a banded block matrix which has $2i+1$ non-zero block diagonal lines.
- For the matrix $\underline{M}_i$ with $i < 2R+1$, its submatrices with block row and column indices (j, p) satisfying $i+1 \leq j+p \leq 4R+3-i$ constitute a partial block Toeplitz region.

Lemma 3. Let the sequence of non-zero block entries from left to right of the partial Toeplitz region of $\underline{C}_R \underline{A}^i_R \underline{B}_R$ be denoted as $\{P_{i,-i}, P_{i,1-i}, \cdots, P_{i,i-1}, P_{i,i}\}$. Then these matrices satisfy the following equality:

$$\sum_{j=-i}^{i} P_{i,j} z^{-j} = C(A_l z^{-1} + A + A_r z)^i B, \quad (6)$$

where $z \in \mathbb{C}$.

The above lemmas can be derived by expressing $\underline{A}_R$ as $\underline{A}_R = I \otimes A + J_- \otimes A_l + J_+ \otimes A_r$, where J_- and J_+ are subdiagonal and supperdiagonal identity matrices, respectively.

By taking account of the partial Toeplitz structures of the Markov parameters $\{\underline{M}_i = \underline{C}_R \underline{A}^i_R \underline{B}_R\}^{s-2}_{i=0}$, the nuclear norm regularized optimization problem in (5) is solved by constraining the above mentioned two-layer Toeplitz structure of $\underline{\mathcal{T}}^{B_R}_s$. It is worth noting that imposing finer structures on $\underline{\mathcal{T}}^{B_R}_s$ will not destroy the convexity of (5); hence, the estimate of $\underline{\mathcal{T}}^{B_R}_s$ can be reliably obtained.

4. SYSTEM REALIZATION

After obtaining the estimate of the block entries of $\underline{\mathcal{T}}^{B_R}_s$, we shall develop an approach to realize the state-space model in (1), namely estimating the system matrices C, A_l, A, A_r, B up to a similarity transform. In order to focus on the essence of the realization method, we assume that $\underline{\mathcal{T}}^{B_R}_s$ is exactly known. In addition, for the sake of notational simplicity, we shall demonstrate the system realization method using the Markov parameters up to the fourth moment, i.e. $\{\underline{M}_i = \underline{C}_R \underline{A}^i_R \underline{B}_R\}^4_{i=0}$.

It is easy to see that $\underline{M}_0$ is a block diagonal matrix constructed by CB, $\underline{M}_1$ is a block tri-diagonal matrix determined by CA_lB, CAB and CA_rB. By induction, we can find that the Markov parameters $\{\underline{M}_i\}^4_{i=0}$ can be determined by the parameters: $CB, CA_lB, CAB, CA_rB, CA^2_lB$, $CA_lAB, \cdots, CA^4_rB$. In addition, with $\underline{\mathcal{T}}^{B_R}_s$ being available, the blocks $P_{i,j}$ for $0 \leq i \leq 4$ and $-i \leq j \leq i$, as shown in Lemma 3, are known as well. Let

$$\boldsymbol{\phi} = \begin{bmatrix} CB \\ CA_lB \\ CAB \\ CA_rB \\ CA_l^2B \\ CAA_lB \\ CA_lAB \\ CA^2B \\ CA_rAB \\ CAA_rB \\ CA_r^2B \\ \vdots \\ CA_r^4B \end{bmatrix} \text{ and } \mathbf{z} = \begin{bmatrix} P_{0,0} \\ P_{1,-1} \\ P_{1,0} \\ P_{1,1} \\ P_{2,-2} \\ P_{2,-1} \\ P_{2,0} \\ P_{2,1} \\ P_{2,2} \\ P_{3,1} \\ \vdots \\ P_{4,4} \end{bmatrix}. \tag{7}$$

Based on equation (6), we can find a constant matrix $\mathbf{T}$ such that

$$\mathbf{T}\boldsymbol{\phi} = \mathbf{z}. \tag{8}$$

In the above equation, $\mathbf{T}$ and $\mathbf{z}$ are known while $\boldsymbol{\phi}$ is to be determined. Since there are more variables than equations in (8), it is an under-determined linear estimation problem. By taking an insight into the parameter vector $\boldsymbol{\phi}$, we can find that the following matrix constructed by $\boldsymbol{\phi}$ has a low rank:

$$\begin{aligned} \mathbf{H}(\boldsymbol{\phi}) &= \begin{bmatrix} CB & CA_lB & \cdots & CA_r^2B \\ CA_lB & CA_l^2B & \cdots & CA_lA_r^2B \\ \vdots & \vdots & \ddots & \vdots \\ CA_r^2B & CA_r^2A_lB & \cdots & CA_r^4B \end{bmatrix} \\ &= \begin{bmatrix} C \\ CA_l \\ CA \\ CA_r \\ CA_l^2 \\ CA_lA \\ CA_lA_r \\ CAA_l \\ CA^2 \\ CAA_r \\ CA_rA_l \\ CA_rA \\ CA_r^2 \end{bmatrix} \begin{bmatrix} B^T \\ B^TA_l^T \\ B^TA^T \\ B^TA_r^T \\ B^T(A_l^2)^T \\ B^T(A_lA)^T \\ B^T(A_lA_r)^T \\ B^T(AA_l)^T \\ B^T(A^2)^T \\ B^T(AA_r)^T \\ B^T(A_rA_l)^T \\ B^T(A_rA)^T \\ B^T(A_r^2)^T \end{bmatrix}^T \end{aligned} \tag{9}$$

By combining equation (8) and the low rank property of the matrix $\mathbf{H}(\boldsymbol{\phi})$, we can form the following nuclear norm regularized optimization problem

$$\min_{\boldsymbol{\phi}} \quad \|\mathbf{T}\boldsymbol{\phi} - \mathbf{z}\|_2^2 + \alpha\|\mathbf{H}(\boldsymbol{\phi})\|_* \tag{10}$$

where α is a regularization parameter.

Solving the convex optimization problem in (10) yields the estimates of $\boldsymbol{\phi}$ and $\mathbf{H}(\boldsymbol{\phi})$. The SVD decomposition of $\mathbf{H}(\boldsymbol{\phi})$ can then be obtained as follows

$$\mathbf{H}(\boldsymbol{\phi}) = [\, U_s \;\; U_n \,] \begin{bmatrix} \Sigma_s & \\ & \Sigma_n \end{bmatrix} \begin{bmatrix} V_s^T \\ V_n^T \end{bmatrix}, \tag{11}$$

where U_s and V_s consists of n orthogonal columns, and $\Sigma_s \in \mathbb{R}^{n\times n}$ and Σ_n are real diagonal matrices with the diagonal entries of Σ_s being larger than those in Σ_n. Let's denote $\underline{O} = U_s$ and $\underline{C} = \Sigma_s V_s^T$. Then the estimates of C and B can be directly obtained as follows:

$$\begin{aligned} \hat{C} &= \underline{O}(1:p,:), \\ \hat{B} &= \underline{C}(:,1:m). \end{aligned} \tag{12}$$

Based on the obtained matrix $\underline{O}$ and its shift structure, the matrices A_l, A, A_r can be estimated as follows

$$\begin{aligned} \hat{A}_l &= [\underline{O}(1:4p,:)]^\dagger \begin{bmatrix} \underline{O}(p+1:2p,:) \\ \underline{O}(4p+1:5p,:) \\ \underline{O}(7p+1:8p,:) \\ \underline{O}(10p+1:11p,:) \end{bmatrix}, \\ \hat{A} &= [\underline{O}(1:4p,:)]^\dagger \begin{bmatrix} \underline{O}(2p+1:3p,:) \\ \underline{O}(5p+1:6p,:) \\ \underline{O}(8p+1:9p,:) \\ \underline{O}(11p+1:12p,:) \end{bmatrix}, \\ \hat{A}_r &= [\underline{O}(1:4p,:)]^\dagger \begin{bmatrix} \underline{O}(3p+1:4p,:) \\ \underline{O}(6p+1:7p,:) \\ \underline{O}(9p+1:10p,:) \\ \underline{O}(12p+1:13p,:) \end{bmatrix}. \end{aligned} \tag{13}$$

5. SUMMARY OF THE IDENTIFICATION METHOD

The presented identification method is divided into two steps: (a) estimate finer Markov parameters by solving the nuclear norm regularized optimization problem in (5); (b) realize the state-space model from the estimated Markov parameters using the method described in Section 4.

To ease the reference, the developed algorithm is summarized in **Algorithm 1**.

Algorithm 1: Subspace identification for 1D distributed systems	
Step 1	Solve the optimization problem (5) to obtain $\underline{\mathcal{T}}_s^{B_R}$;
Step 2	Estimate $\boldsymbol{\phi}$ by solving the optimization problem in (10);
Step 3	Take the SVD decomposition of $\mathbf{H}(\boldsymbol{\phi})$ as shown in (11);
Step 4	Determining C and B as shown in (12);
Step 5	Identifying A_l, A, A_r according to (13).

In the above proposed identification method, many parameters should be properly selected, such as R and s in (3), λ in (5), α in (10), and the number of Markov parameters adopted for system realization. For the parameters R and s, it is theoretically better to choose large values, which can result in significant low rank property of the sum $\underline{\mathcal{O}}_s \underline{x}_{k,r}^i + \underline{\mathcal{T}}_s^{D_R} \underline{V}_{k,s,r}^i$ in Lemma 1; however, larger s and R may lead to heavier computational burden when solving the optimization problem in (5). To select the regularization parameters λ and α, they can be empirically chosen from a sequence of grid points according to the cross-validation criterion in Verhaegen and Verdult (2007). For the number of adopted Markov parameters, if we choose a large number, the linear estimation equation in (8) will be more under-determined; however, the low rank property of the matrix $\mathbf{H}(\boldsymbol{\phi})$ becomes more significant. Hence, there exists a trade-off.

6. CONCLUSION

In this paper, we have provided a convex solution to the local system identification of 1D large-scale distributed systems. Two contributions have been made in this study: (a) the effects of unknown system inputs have been greatly suppressed by using the heuristic low-rank optimization method; (b) the under-determinedness in the system realization step has been dealt with by exploiting the low rank property of a matrix constructed by the associated redundant parameters. Since the algorithm development does not depend on the assumption of small unknown inputs

(neighboring states), the proposed algorithm can perform well even if the neighboring subsystems are strongly coupled. In our future work, the extension of the presented algorithm to the identification of high-dimensional large-scale systems will be explored.

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