Introduction by the Editors

A common way to represent the transfer function of linear time-invariant dynamical systems is to employ a series expansion representation of the form $G(z) = \sum_{k=1}^{\infty} g(k)z^{-k}$, where $\{g(k)\}$ refers to the impulse response of the system. Finite expansion representations are known as finite impulse response (FIR) models, and they are very useful dynamical models in many areas of engineering, ranging from signal processing, filter design, communication, to control design and system identification.

FIR models have many attractive properties, such as the fact that the coefficients $g(k)$ appear linearly in $G(z)$, leading to many computational and analytical advantages. However, a problem can be that for systems with high sampling rates and dominant high- and low-frequent dynamics, a very high number of terms in the expansion may be required to obtain an acceptable approximation of the true system dynamics.

As an alternative, generalized basis function models are considered of the form $G(z) = \sum_{k=1}^{\infty} c(k)F_k(z)$, for a predefined set of rational basis functions $\{F_k(z)\}$.

In this book, a full presentation is given of the rich theory and computational methods that are available for generalized rational basis function models, and in particular, in modelling of dynamical systems and in system identification.

When estimating dynamical models of processes on the basis of measured input and output signals, appropriate model structures are required to parameterize the models. A central problem is to select model structures that are sufficiently rich to contain many models so to be able to find accurate approximations of our process to be modelled (small bias). On the other hand, the model structures should be sufficiently parsimonious to avoid a large variance. This bias/variance trade-off is at the very heart of the model structure selection problem and therefore central to the problem area considered in this book. A wide variety of other issues will also be treated, all related to the construction, use, and analysis of model structures in the format of rational basis function expansions.

In the first introductory chapter, Bo Wahlberg, Brett Ninness, and Paul Van den Hof provide the basic motivation for the use of basis function representations in dynamic modelling through some simple examples. Starting from finite impulse response (FIR) representations and Laguerre models, generalized forms are shown, incorporating a level of flexibility – in terms of a selection of poles – that can be beneficially used by the model builder. Additionally, the development of basis function structures is set in a historical perspective.

Bo Wahlberg and Tomás Oliveira e Silva continue this setting in Chapter 2, *Construction and Analysis*, where the general theory and tools for the construction of generalized orthonormal basis functions (GOBF) are explained. Parallel developments are shown either through Gram-Schmidt orthonormalization of first-order impulse responses or through concatenation of balanced state space representations of all-pass systems. The standard tapped-delay line, as a simple representation and implementation for linear time-invariant systems, is shown to be generalized to a concatenation of all-pass systems with balanced state readouts.

In Chapter 3, *Transformation Analysis*, authored by Bo Wahlberg, an alternative construction of the basis functions is given, resulting from a bilinear mapping of the complex indeterminate z in the system's Z-transform $G(z)$. Similar in spirit to the bilinear mapping of the complex plane that occurs when relating continuous-time and discrete-time systems, these mappings can be used to arrive at attractive representations in transform domains, as *e.g.* representations that have fast decaying series expansions. By choosing the appropriate mapping, pole locations of the transform system can be influenced.

The application of orthonormal basis function model structures in system identification is addressed in Chapter 4, *System Identification with Generalized Orthonormal Basis Functions*. Paul Van den Hof and Brett Ninness outline the main results in a tutorial chapter on time domain prediction error identification. The benefits of linear-in-the-parameters output-error structures are discussed, and bias and variance results of the basis function model structures are specified.

In Chapter 5, *Variance Error, Reproducing Kernels, and Orthonormal Bases*, Brett Ninness and Håkan Hjalmarsson give a detailed analysis of the variance error in system identification. Relations between GOBF model structures and fixed denominator model structures are discussed. Existing (asymptotic) expressions for variance errors are improved and extended to finite model order expressions, showing the instrumental role of orthogonal basis functions in the specification of variance errors for a wide range of model structures, including the standard OE, ARMAX, and BJ model structures.

The same authors focus on numerical conditioning issues in Chapter 6, *Numerical Conditioning*, where the advantages of orthogonality in model structures for identification are discussed from a numerical perspective. A relation is also made with improved convergence rates in adaptive estimation algorithms.

In Chapter 7, *Model Uncertainty Bounding*, Paul Van den Hof discusses the use and attractive properties of linear-in-the-parameters output-error model structures when quantifying model uncertainty bounds in identification. This problem has been strongly motivated by the recent developments in identification for control and shows that linear model structures as GOBF models are instrumental in the quantification of model uncertainty bounds.

The material on system identification is continued in Chapters 8 and 9, where József Bokor and Zoltan Szabó present two chapters on frequency domain identification. In Chapter 8, *Frequency-domain Identification in* $\mathcal{H}_2$, results from a prediction error approach are formulated. Asymptotic (approximate) models are shown to be obtained from the unknown underlying process by an interpolation over a particular (basis-dependent) frequency grid, which is a warped version of the equidistant Fourier grid. In Chapter 9, *Frequency-domain Identification in* $\mathcal{H}_\infty$, the prediction error framework is replaced by an identification framework in a worst-case setting, relying on interpolation techniques rather than stochastic estimation. Results for GOBF models are presented in this setting, and extensions to (non-rational) wavelets bases are considered.

In Chapter 10, *Design Issues*, Peter Heuberger collects practical and implementation issues that pertain to the use of GOBF models in identification. Aspects of finite data, initial conditions, and number of selected basis poles come into play, as well as iterative schemes for basis poles selection. Also, the handling of multi-variable systems is discussed, including the use of multivariable basis functions.

Optimal basis selection is presented in Chapter 11, *Pole Selection in GOBF Models*, where Tomás Oliveira e Silva analyses the question concerning the best basis selection for predefined classes of systems. 'Best' is considered here in a worst-case setting. The classical result that an FIR basis (all poles in zero) is worst-case optimal for the class of stable systems is generalized to the situation of GOBF models.

In Chapter 12, *Transformation Theory*, authored by Peter Heuberger and Thomas de Hoog, the transform theory is presented that results when representing systems and signals in terms of GOBF models with repeating pole structures. The resulting Hambo transform of systems and signals is a generalization of the Laguerre transform, developed earlier, and can be interpreted as a transform theory based on transient signal/systems' responses. This in contrast to the Fourier transform being a transform based on a systems' steady-state responses. Attractive properties of representations in the transform domains are shown.

The transformation theory of Chapter 12 appears instrumental in solving the realization problem as presented by the same authors in Chapter 13, *Realiza-*

tion Theory. The topic here is the derivation of minimal state space representations of a linear system, on the basis of a (finite or infinite) sequence of expansion coefficients in a GOBF series expansion. It appears that the classical Ho-Kalman algorithm for solving such problems in FIR-based expansions can be generalized to include the GOBF expansions.

This book contains many results on the research efforts of the authors over the past 10 to 15 years. In this period the work on rational orthogonal basis functions has been very much inspiring to us, not in the least by the wide applicability of the developed theory and tools.
Finite impulse response models are extensively, and often routinely, used in signal processing, communication, control, identification, and many other areas, even if one often knows that infinite impulse response (IIR) models provide computational and performance advantages. The reason is often stability problems and implementation aspects with conventional IIR filters/representations. However, this is, as shown in this book, not a problem if one uses orthogonal basis function representations. Our mission has been to provide a framework and tools that make it very easy to evaluate how much one gains by using orthogonal basis function models instead of just FIR models.
We hope that with the material in this book, we have succeeded in convincingly showing the validity of the statement:

> *Almost all of what you can do using FIR models, you can do better for almost the same cost using generalized orthonormal basis function (GOBF) models!*

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