Evaluating techniques for redirecting turbine wake using SOWFA

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Abstract

Wind plant control is an active field of research in which controllers seek to maximize overall wind-plant performance in terms of power production, loading, or both. Such control strategies are often different from those that are optimal for an individual turbine. One type of wind-plant control method is to redirect the wake of an upstream turbine in order to avoid downstream turbines. In this paper, we investigate several possible methods for redirecting a turbine’s wake, including some existing and some novel approaches. The methods are compared in terms of their ability to redirect turbine wake, the effect on turbine power capture, and turbine loading using the high-fidelity wind plant simulation tool SOWFA.

1 Introduction

Wind turbine wakes are complex and difficult to model. When turbines are located together in wind power plants, wake interaction between turbines can decrease energy capture and increase turbine loads. Therefore, recent research has focused on the design of wind plant controllers to mitigate these effects. Often in the literature, the controllers are based on modifying an individual turbine’s axial induction factor by adjusting pitch and torque. Example studies that use this approach to optimize the global wind plant power capture include [1, 2, 3].

An alternative approach to wind plant power optimization is to redirect the wake using yaw misalignment rather than to optimize induction. In this method, when two turbines are aligned in the wind direction, the upstream turbine intentionally misaligns its yaw angle so as to apply skew to the wake such that it avoids the downstream turbine. This method has been studied experimentally in [4] and in computational fluid dynamics (CFD) simulation in [5] with encouraging results. In a similar way, vertical wake redirection obtained by changes in rotor tilt angle has been investigated in [6] using a CFD model. Note that both [5] and [6] use an actuator disk model of the turbine and [6] assumes laminar flow.

In this paper, we examine the potential of turbine controllers to redirect the turbine wake. This study is done through experiments with the high-fidelity wind plant modeling tool SOWFA.
ICOWES2013 Conference 17-19 June 2013, Lyngby

(Simulator for Off/Onshore Wind Farm Applications) [7]. SOWFA allows simulations to be performed that return performance results in terms of effects on the turbine (for example, power and loading) as well as on the flow (wake redirection).

Wake redirection methods to be examined include both the yaw- and tilt-angle based methods discussed above. We further add an additional novel, to the best of our knowledge, approach. The method attempts to employ individual pitch control (IPC) to achieve a horizontal or vertical wake skew by intentionally inducing a yaw or tilt moment. IPC is typically used to remove these moments, but we use it in reverse to apply them. The four techniques to be evaluated are illustrated in fig. 1.

The contributions of this paper are: first, we introduce a novel approach to achieving wake redirection (IPC). Second, we use high-fidelity simulations to evaluate the methods both in terms of their capability to redirect wakes and in terms of the effect on turbine power and loads.

Figure 1: Techniques considered for redirecting the wake. Note that arrow directions in yaw and tilt case correspond to positive increases.

The remainder of this paper is organized as follows. Section 2 describes the high-fidelity wind plant simulation tool, SOWFA, used in this study. Section 3 provides details of the simulation experiment setup. Section 4 provides the results and analysis of the experiment. Conclusions are given in section 5.

2 SOWFA

SOWFA [7] is a CFD tool coupled with the National Renewable Energy Laboratory’s (NREL’s) FAST turbine simulator tool [8] for studying wind plant behavior. The CFD solver is based on the OpenFOAM CFD toolbox [9]. Specifically, a large-eddy simulation (LES) is used, which directly resolves the larger, energy-containing turbulent scales, to simulate the atmospheric boundary layer and the turbulence contained within it. Then, actuator line turbine models are placed
in the flow to create wakes that interact with one another, and the actuator lines are coupled with FAST. Extensive details are given by Churchfield et al. [10], and are summarized here.

The flow is computed using an unstructured, collocated, variable, finite-volume formulation that is second-order accurate in time and space. The filtered momentum equation is solved along with an elliptic equation for pressure that enforces continuity. Buoyancy effects are included through a Boussinesq term in the momentum equations necessitating the solution of a temperature transport equation. Velocity-pressure decoupling that would normally occur with a collocated incompressible method is avoided through Rhie-Chow [11] interpolation and the Pressure-Implicit Splitting Operation algorithm [12] is used solve the equation set. The linear systems that arise when discretizing the implicit equations are solved using preconditioned iterative solvers.

Coriolis forces account for the Earth’s rotation. The lower surface boundary conditions based on Monin-Obukhov [13] similarity theory is used, which is common practice in the atmospheric LES community. The upper boundary is a stress-free, rigid lid. First, a laterally periodic atmospheric boundary layer precursor simulation with no turbines is performed to generate the turbulent atmospheric boundary layer. Once that simulation has reached quasi-equilibrium, planes of inflow data are saved every time step. These data are then used as inflow boundary conditions for the non-periodic wind turbine simulation, and the downstream boundaries are outflow.

Sørensen and Shen’s [14] actuator line method is used to model the interaction of the wind turbine blades with the wind. The basic idea is that each blade is represented as a line, and each line is discretized into segments. For each segment, the blade airfoil type, twist, and chord are known. The velocity vector experienced by that segment can be sampled from the LES flow field giving the velocity magnitude and angle of attack. Airfoil lift and drag tables are then used to compute the force vector at each actuator line segment. The forces are then projected, using a three-dimensional Gaussian at each actuator line segment, onto the flow field as volumetric body forces that enter the momentum equation. Large-scale structures like the rotor wake and blade tip, root, and bound vortices are resolved.

FAST is two-way loosely coupled to the actuator line model. The LES model samples the velocity along the actuator line segments and returns those values to FAST. FAST, which normally computes those velocities using blade element momentum theory, operates instead in blade-element mode because the LES solver computes induction caused by the rotor. The blade forces computed with FAST are returned to the LES solver and imposed as the body forces described above.

Validation of the SOWFA tool is an ongoing process. In [15], SOWFA was used to simulate the 48-turbine Liligrund wind plant, and the results were then compared with field data, with good agreement throughout the first five rows. Additionally, [10] includes documentation of experiments testing SOWFA’s capability to simulate the inertial range in the turbulent energy spectra and the log-layer mean flow.
3 Simulation experiment

In this study, numerous simulations are run within SOWFA of a single turbine subject to the four proposed methods shown in fig. 1, which are applied in individual simulations with varying settings of yaw misalignment, tilt angle, or IPC moment set-point. The wind inflow is the same for all simulations. From the simulations, we extract the turbine’s average power over the simulation, as well as the metrics of loading for several components. From the flow, we use a correlation method to identify the wake-center at all locations downstream from the turbine. The results allow for a trade-off analysis of wake redirection potential vs. turbine effects.

Note that this study is limited to a single turbine and an examination of the flow behind it. However, an important consideration is the effect of changes made by an upstream turbine on a downstream turbine. Additionally, it is also important to learn if a reduction in the power output of the upstream turbine is compensated for by an increase in the power output of the downstream turbine. These issues are addressed in a related paper [16].

We simulate an NREL 5-MW baseline turbine [17] in turbulent inflow. The inflow, which is based on the study reported in [10], is that of a neutral atmospheric boundary layer. This inflow was selected because it had previously been validated and represents a realistic scenario. The inflow is generated in a precursor atmospheric LES on a domain that is 3 km by 3 km in the horizontal and 1 km in height. The horizontally averaged wind speed is driven to 8 m/s at the turbine hub height and is controlled through a time-varying mean driving pressure gradient. The wind comes from the southwest ($300^\circ$) so that the elongated turbulent structures in the surface layer are not “trapped” by the periodic boundaries, continually cycling through in the same location. In the baseline case, the turbine rotor axis is aligned with the wind direction. The surface temperature flux is set to zero, although a capping inversion initially at 750 m above the surface is used both to slow boundary layer growth and because it is a real feature of atmospheric boundary layers. The surface aerodynamic roughness is set to 0.001 m, which is typical of flow over water. Details on the positioning of the turbine and meshing of the domain are given in fig. 2.

The yaw and tilt wake redirection strategies are tested for a range of settings. Each setting is tested in a simulation with 1,000s of simulated time. SOWFA requires significant computational power in order to run high-fidelity simulations: using a sample time of 0.02s, the time steps take an average 2.5s to calculate on the Sandia National Laboratories/NREL Red Mesa supercomputer [18], using distributed computation with 256 processors. This yields an execution time of 34.4h for each simulation.

In each case, the turbine uses the baseline controller defined in [17] for pitch and torque control. The IPC implementation is based on the design first presented in [19] using the parameters as specified in [20]. It is adapted so that IPC can be used in below-rated conditions and to induce an asymmetric moment, rather than remove one. Details on this IPC implementation are given in the Appendix.
Figure 2: Overview of the experimental setup in the baseline case.
4 Analysis and discussion

Following completion of the experiments, slices are extracted from the simulation outputs and a method is used to determine the mean wake center based on ideas of de Mare [21], which has been further developed at NREL. We take a horizontal slice of the mean velocity field at the turbine hub height and a vertical slice aligned with the mean wind and passing through the turbine centerline. From the horizontal slice, we take the mean velocity along lines within the slice plane and perpendicular to the flow at successive downstream locations. When plotted, the velocity along each line is a mean velocity profile. In the near wake, the velocity profiles are double-Gaussian in shape, and in the far wake, they resemble a normal Gaussian. The profiles at each downstream location are correlated with a Gaussian of similar width and depth. The point of maximum correlation is taken as the wake center position at each downstream location giving the lateral wake deflection. We follow the same process to find the vertical wake deflection using the vertical slice of mean velocity; however, we first subtract the vertical profile of mean velocity to remove the effect of vertical shear that is present in the atmospheric boundary layer. Fig. 3 shows the output of the wake center-line identification algorithm for several cases in the horizontal and vertical planes.

The cases shown in fig. 3 are representative of the collected results fully summarized in table 1. Wake center tracking results in the yaw and tilt simulations demonstrate significant displacements of wake center, in agreement with the previous literature. The IPC methods produce some redirection. While it does not achieve as much redirection as yaw or tilt angle adjustments, the skew is in some cases significant. Also noted, that while the intention of the IPC algorithms were to approximate yaw misalignment through an IPC-induced yaw moment, or tilt via a tilt moment, the results show that the largest vertical skew is given when a yaw moment is targeted and the largest horizontal skew for a high tilt moment.

In addition to measurements of the wake, data were collected from the FAST turbine output. The data included time series of output power, blade out-of-plane (OOP) bending moment, drivetrain torsion, tower fore-aft and side-side bending, and the yawing and tilting moments experienced at the yaw bearing. Using a root-sum-square combination, the separate tower and yaw moments are combined into a single moment. An average power output is computed, as well as the damage equivalent load (DEL) for each load signal. The DEL is a standard metric of fatigue damage; see [22] for an example implementation. These results are summarized in table 1. Note that the measurement of wake displacement is taken at 7 rotor diameters from the turbine, which is a typical location for a downstream turbine.

Reviewing table 1, there is a positive result for yaw-based wake skew. One can see that when the turbine yaws in the positive direction, wake redirection and load reduction for all components are simultaneously achieved for a number of operating points. Using a yaw misalignment to reduce turbine loads has been studied in the literature and these results are consistent with those findings. [23] There is a loss of power, however the intention is that this reduction should be compensated for by a larger gain in a downstream turbine. [24]
Table 1: Full results of experiment. Turbine wake redirection is summarized by the wake center 7 rotor diameters downstream from the turbine, bold indicates the larger offset.

<table>
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<th>Amount</th>
<th>Horizontal Wake-offset abs(x/D)</th>
<th>Vertical Wake-offset abs(x/D)</th>
<th>Power</th>
<th>Blade OOP</th>
<th>Drivetrain</th>
<th>Tower</th>
<th>Yaw Bearing</th>
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<td>+1.7%</td>
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(a) Tracking wake center in horizontal slices
(b) Tracking wake center in vertical slices

Figure 3: Tracking the wake center using Gaussian correlation. The magenta lines pass through the turbine hub and are parallel to the inflow direction to indicate a zero skew base.

Tilt similarly demonstrates potential for wake redirection with, mostly, positive load impacts. Observing the loads in the 12 degree case, all are reduced with the exception of blade bending, which has gone up 10%. It should be pointed out that currently there is no means of modifying tilt angle in the field. However, the effect of causing the higher-speed higher-altitude winds to be pulled downward might be rewarding enough to justify further investigation, although given that positive tilt angles would cause the blades to come closer to the tower on upwind turbines, this is appropriate more for downwind machines. [25]

The results for the IPC-based methods are mixed. Significant wake skew is achieved for some cases, however, because the method is maximizing an asymmetric rotor moment, the blade loads are substantially increased. This leads to the notion that while it may be possible to achieve wake redirection with IPC, this particular IPC algorithm is not a good method. Finding an IPC controller which achieves wake skew with reduced blade loads would be very useful because
IPC is already possible to implement on many existing turbines (unlike changes to tilt), and can be adjusted more quickly than yaw angle.

In considering the results, the authors now believe the initial concept for IPC-based wake redirection followed in this paper was fundamentally flawed. Specifically, while IPC can reproduce the rotor moments generated by yaw or tilt misalignment, this moment is not what can create skew. In the left of fig. 4, the conceptualization of yaw-misalignment induced wake redirection from [5] is redrawn. In it, the thrust force of the turbine is shown to act along the axis of the rotor shaft. When the wind inflow is at an angle to this direction, the thrust can be divided into components $f_x$ and $f_y$. $f_x$ is parallel to the flow and slows the wind, while $f_y$ is perpendicular and applies the force which causes wake redirection. A moment produced in IPC is based on an uneven plane of thrust force. This yields a moment from the turbine’s perspective, but no perpendicular force on the flow. However, IPC can cause a perpendicular net force on the flow. Observing the right section of fig. 4, when the blade torque through a rotor rotation is uneven (in the sense that rightward torque is not matched by leftward torque), a skew can result. This is because the reaction force on the flow is now also unbalanced, and a net perpendicular force is applied resulting in wake skew. Notice that the IPC configuration draw in fig. 4 will yield a tilt moment on the turbine (because the blade thrust is most different between the top and bottom azimuth positions) and a horizontal wake skew (because the flow reaction forces are most different in the horizontal force directions), which agrees with the results presented earlier.

![Figure 4: Demonstrating the difference between yaw/tilt and IPC-based methods for achieving wake skew. The figure on the left, based on the explanation given in [5], shows the thrust force decomposed into components. IPC, shown on the right, generates an asymmetric rotor torque which can yield a force perpendicular to flow direction.](image)

This analysis hopefully indicates that while the IPC algorithm first developed, which seeks to
apply skew by maximizing yaw or tilt moment, is problematic, an alternative implementation may be possible. This implementation, the subject of future work, should attempt to maximize the wake skew through torque imbalance while minimizing rotor moments.

5 Conclusions

In this paper, the NREL wind plant simulation tool, SOWFA, was used to simulate and investigate several methods for wake redirection. Wake redirection is one proposed method for improving wind plant overall performance. For yaw misalignment, simulations showed significant redirection effects coupled with reductions in loading across measured components, a positive result. Tilt angle adjustment was shown to also achieve wake redirection while reducing all turbine loads except for blade OOP bending. Although modifying tilt angle is not currently a controllable feature of wind turbines, knowledge of the capability of this effect might be useful, especially if the effect of pulling in faster wind yields greater overall power gains.

IPC-based methods also demonstrated an ability to affect wake skew, however this was achieved with a substantial increase in blade loading. Analysis presented indicates that the IPC algorithm employed in this paper, while enough to prove the concept that IPC can redirect wake, it too simplistic for actual implementation and future work will focus on the determination of more optimal designs.

6 Acknowledgements

The authors are very grateful to Wesley Jones and the NREL High Performance Computing team for their crucial help and support in completing this simulation study. This work was supported by the U.S. Department of Energy under Contract No. DE-AC36-08GO28308 with the National Renewable Energy Laboratory.

References


**Appendix**

**Appendix A: Implementation of IPC with induced yaw or tilt moments**

This section explains how individual pitch control (IPC) was implemented to allow yaw and tilt moments to be induced by the IPC. The implementation also can be used in below-rated operation with varying rotor speeds. Let $\varphi$ denote the rotor speed in rad/s, let $\{M_{y,i}\}_{i=1}^3$ denote blade root vibrations of each of the three blades, let $M_{r,yaw}, M_{r,tilt}$ denote setpoints for the induced yaw and tilt moments, and let $s$ denote the Laplace operator. Then the 1P and 2P IPC additive adjustments to the pitch, $\{\delta\theta_{i,jP}\}_{i=1}^3$, are given by:

$$
\begin{bmatrix}
\delta\theta_{1,jP} \\
\delta\theta_{2,jP} \\
\delta\theta_{3,jP}
\end{bmatrix} = \mathcal{L}(s) I_{3 \times 3} \mathcal{P}_j \left( \varphi + \delta_{jP} \right) \begin{bmatrix}
\frac{K_{jP,yaw}}{s} & 0 \\
0 & \frac{K_{jP,tilt}}{s}
\end{bmatrix} \times \left( \frac{2}{3} \mathcal{P}_j \left( \varphi \right) \mathcal{N}_j \left( s \right) I_{3 \times 3} \begin{bmatrix}
M_{y,1} \\
M_{y,2} \\
M_{y,3}
\end{bmatrix} - \begin{bmatrix}
M_{r,yaw} \\
M_{r,tilt}
\end{bmatrix} \right)
$$

for $j = 1, 2$, with Coleman transformation matrices:

$$
\mathcal{P}_j (\varphi) = \begin{bmatrix}
\cos (j \varphi) & \sin (j \varphi) \\
\cos (j (\varphi + 2\pi/3)) & \sin (j (\varphi + 2\pi/3)) \\
\cos (j (\varphi + 4\pi/3)) & \sin (j (\varphi + 4\pi/3))
\end{bmatrix},
$$

and with inverse notch filters $\mathcal{N}_j$, and low-pass filter $\mathcal{L}$:

$$
\mathcal{N}_j (s) = \mathcal{I}_{jP} \frac{2\zeta_j \omega_j s}{s^2 + 2\zeta_j \omega_j s + \omega_j^2}, \quad \mathcal{L}(s) = \frac{\omega_0^2}{s^2 + 2\zeta_L \omega_L s + \omega_L^2},
$$

with $\omega_j = j \varphi$, and parameters $K_j, \zeta_j, \omega_j, \delta_{jP}$ as specified in [20]. The filters are used in a Tustin discretized form with a sample time of 0.02s. The pitch angles are saturated to a 5 degree amplitude, and the pitch rates are limited to 8 deg/s. In the IPC induced moment test cases, $M_{r,yaw}$ or $M_{r,tilt}$ are chosen large enough such that the pitch angles vary with maximum amplitude, in order to find the maximum effect of IPC action on the wake.