A Maximum Power Point Tracking Approach for Wind Farm Control

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Abstract. This paper presents a new methodology for controlling wind farms in a decentralized framework, where the wind turbines exchange information with their neighbours, and update their own control parameters in order to optimize the total power produced by the wind farm in a time-efficient manner, taking into account effects of wake interaction. The method can be characterized as being model-free since it does not require a model of the aerodynamic interaction between the turbines. The total power optimization is performed using gradient-based optimization. The gradients are approximated based on information of the past control actions, the power response of the turbine itself, and the power response of neighbouring turbines. To achieve time-efficiency, only the influence on the nearest neighbouring turbine is taken into account. The method is tested on a simulation of the Princess Amalia Wind Park, a 60 turbine offshore wind farm in The Netherlands. Because of the improved time-efficiency of the control update scheme, the new method yields a faster convergence of the power optimization when compared to an existing model-free game-theoretic wind farm power optimization method.

1. Introduction
Behind each wind turbine in a wind farm, there exists a wake in which the average wind speed is lower than the free-stream wind speed in front of the turbine. Also, typically the turbulence level is higher in the near wake. When an individual wind turbine is controlled to extract the maximum amount of energy possible, wind turbines in its wake experience a power loss. Cooperative control strategies in which the turbines exchange information with each other, that take into account the interaction between the turbines through the wake aerodynamics, may be used to improve the total power production of the wind farm (see [1], for example). Developing model-based control strategies for the wind farm is difficult however, because of the complicated models that are needed to accurately describe the wake aerodynamics. Therefore, this paper develops a model-free control method, which means that the control algorithm does not need a model of the wind turbine dynamics or the wake aerodynamics to perform the power optimization. The control method that is chosen for this task is the Maximum Power Point Tracking (MPPT) method. The MPPT method can be used to optimize power of a single wind turbine in real-time by manipulating a wind turbine control parameter and acting on the resulting change of its own power production (see [2], for example). In this paper, the MPPT

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1 This paper was presented at the ‘The Science of Making Torque from Wind’ conference, October 9-11, 2012, Oldenburg, Germany.
method is extended in such a way that it can optimize the total power of a wind farm, by letting each turbine exchange information about their power production with other turbines in the farm. An important design criterion is that the optimization takes place in a time-efficient manner, in order to be able to track fluctuations of the wind speed and the wind direction.

2. MPPT Approach for Wind Farm Control

In this section, the MPPT wind farm control method is presented in two variants: the Fixed-Step MPPT (FS-MPPT) method and the Gradient-Descent MPPT (GD-MPPT) method. In Sections 2.1 and 2.2, FS-MPPT and GD-MPPT are explained for a single row of wind turbines. In Section 2.3, the two methodologies are extended in such a way that they can be used on any wind farm configuration.

2.1. Fixed-step MPPT control of a row of wind turbines

Consider a row of \( n \) wind turbines standing in the wake of each other (see Figure 1), with power productions \( \{P_i\}_{i=1}^n \) and certain control settings \( \{a_i\}_{i=1}^n \) that can be used to optimize power. In this work, \( a_i \) are set-points for axial induction factors, which can be tracked by controlling the blade pitch and rotor speed of a turbine [4]. Due to wake interaction, changing a control parameter \( a_i \) influences \( \{P_j\}_{j=i}^n \), the power productions not only of turbine \( i \), but also those of the turbines behind it. To maximize the total power production of the row of turbines, the control parameters \( \{a_i\}_{i=1}^n \) can be iteratively updated using a Fixed-Step Maximum Power Point Tracking (FS-MPPT) method, also known as hill-climbing optimization:

\[
a_i(k+1) = a_i(k) + K \text{sign} \left( \sum_{j=i}^{n} \frac{\partial P_j}{\partial a_i}(k) \right),
\]

for \( i = 1, \ldots, n \), with index \( k \) denoting the iterations, and a small scalar \( K > 0 \) defining the fixed size of the steps on \( a_i \). The gradients can be approximated through first-order backward differencing:

\[
\frac{\partial P_j}{\partial a_i}(k) \approx \frac{P_j(k) - P_j(k-1)}{a_i(k) - a_i(k-1)}.
\]

A difficulty is that it takes a substantial amount of time to find these gradients. For example, suppose \( a_1 \) is changed by a step \( K \), then to find the gradient \( \partial P_n/\partial a_1 \) one would have to wait until the wake of turbine 1 has reached turbine \( n \), the last turbine of the row. Because of the large distances between the turbines (typically 7 to 8 rotor diameters), the time this takes is very long for large wind farms. Considering that wind conditions change over time, and the fact that the aim is to perform the control updates in real-time, a more practical approach is to only take into account the influence of the control actions on the turbine itself, and the one behind...
This can be a good approximation in practical cases with substantial wake recovery. This then results in the following control update scheme:

\[ a_i(k+1) = a_i(k) + K \text{sign} \left( \sum_{j=i}^{i+1} \frac{\partial P_j}{\partial a_i} (k) \right), \]

(3)

2.2. Gradient-descent MPPT control of a row of wind turbines

Another approach is to use gradient-descent optimization to update the control variable:

\[ a_i(k+1) = a_i(k) + K \sum_{j=i}^{i+1} \frac{\partial P_i}{\partial a_i} (k), \]

(4)

with the scalar parameter \( K > 0 \) now being a scaling factor. This method may speed up convergence and reduce the oscillations around the optimal power. Higher-order optimization methods such as Newton’s method may further speed up the convergence, but they may be more sensitive to noise on the power production due to turbulence effects.

2.3. MPPT control of a wind farm

This section presents the scheme to perform MPPT on a full wind farm of an arbitrary, but known spatial configuration. Let \( F = \{1, 2, \ldots, N\} \) denote a set of indices that number the wind turbines in a wind farm, with \( N \) denoting the total number of turbines. Let \( G \subset F \) be the set of turbines that are directly influencing neighbouring downstream wind turbines through wake interaction, and let \( d(i) \) give the index of the nearest neighbour downstream turbine that a turbine \( i \in G \) is directly influencing. Further, \( L = \{i \in F | i \notin G\} \) is the set of turbines that are not influencing other turbines. Figure 2 gives an example of how to define the sets \( F, G, L \) and the mapping \( d(i) \) for a given wind farm configuration and wind direction. It is assumed that the sets \( F, G, L \) can be updated using information of the wind farm configuration and the wind directions in the wind farm. Notice that if each turbine rotor axis is aligned with the wind direction (through yaw control), the effective wind directions may be estimated from the yaw angle of each turbine.

Using the above definitions, the FS-MPPT control update law for the wind farm can be written as:

\[ \bar{a}_i \leftarrow a_i \quad \forall i \in G, \]

(5)

\[ a_i \leftarrow \bar{a}_i + K \text{sign} \left( \frac{\partial P_i}{\partial a_i} + \frac{\partial P_{d(i)}}{\partial a_i} \right) \quad \forall i \in G, \]

(6)

and the GD-MPPT control update law for the wind farm is:

\[ \bar{a}_i \leftarrow a_i \quad \forall i \in G, \]

(7)

\[ a_i \leftarrow \bar{a}_i + K \left( \frac{\partial P_i}{\partial a_i} + \frac{\partial P_{d(i)}}{\partial a_i} \right) \quad \forall i \in G. \]

(8)

In these update laws, the variable \( \bar{a}_i \) is used to store the previous control setting. Turbines in the set \( L \) are controlled to operate in such a way that the power of the turbine itself is maximized:

\[ a_i \leftarrow a_i^{\text{opt}} \quad \forall i \in L, \]

(9)

where \( a_i^{\text{opt}} \) is the control setting that will yield maximum power of the turbine \( i \) itself. In the case that \( a_i \) is the axial induction factor, \( a_i^{\text{opt}} = 1/3 \), [4]. After the control update has been
Figure 2. The above picture shows the top view of a 4-by-4 wind farm in a south-eastern wind flow. The dotted arrows show which turbine is directly influencing which other nearest neighbour downstream turbine through wake interaction. In this case, the set of indices numbering each turbine is $F = \{1, 2, \cdots , 16\}$. The indices of turbines $i$ that are influencing other turbines are collected in the set $G$, and the index of the neighbouring downstream turbine that they are directly influencing is given by $d(i)$, this mapping is given in the table below the picture. The indices of turbines that do not influence other turbines are collected in the set $L = \{4, 8, 12, 13, 14, 15, 16\}$.

performed, the gradient of the power of a turbine with respect to a change in its own control variable can be approximated with the first-order backward differencing method:

$$\frac{\partial P_i}{\partial a_i} \leftarrow \frac{P_i(t) - P_i}{a_i(t) - a_i^\tau} \quad \forall i \in G,$$

where $P_i$ is the power of the turbine before the control update has taken place, and $P_i(t)$ the power after the control update has had its effect. The first-order backward differencing method for approximating the gradient of the power of a neighbouring downstream turbine $d(i)$ with respect to a change in a control variable of a turbine $i$ is:

$$\frac{\partial P_{d(i)}}{\partial a_i} \leftarrow \frac{P_{d(i)}(t) - P_{d(i)}}{a_i - a_i^\tau} \quad \forall i \in G.$$  

(11)

After a control variable $a_i$ is updated, there is a delay before this change has an effect on the turbine $i$ itself, and a longer delay before the change has an effect on the neighbouring turbine $d(i)$. Notice that if each of the control variables is updated simultaneously, and if $d(i) \in G$, then $P_{d(i)}$ in eq. (11) has to be defined as the power after the update of $a_{d(i)}$ has had its effect, in order to only take into account the effect of the change of $a_i$. In Algorithm 1 the FS-MPPT control scheme is given. It shows how to schedule the updates of the control variables and the gradients in a way that accounts for the different delays. In this algorithm, $\Delta t$ denotes the interval between each sample time. In line 8 of Algorithm 1, the value of $\Delta t$ is employed to update a scalar time counter $\tau \in \mathbb{R}$ that schedules the updates of the gradients $\partial P_{d(i)}/\partial a_i$. The time interval $T_{s,local} \in \mathbb{R}$ is the largest settling time of the responses of the power $P_i$ of each
turbine $i \in G$ to the change of their own control variables $a_i$. Further, $T_{s,wake} \in \mathbb{R}$ is an upper bound for the time interval that it takes for each control variable $\{a_i | i \in G\}$ to have its full effect on the power of the neighbouring downstream turbine, $P_d(i)$. The interval $T_{s,wake}$ includes the maximum wake traveling time between a turbine $i \in G$ and its downstream neighbouring turbine $d(i)$. Therefore, the interval $T_{s,wake}$ can be assumed to be larger than $T_{s,local}$.

**Algorithm 1** The pseudocode below shows the fixed-step MPPT wind farm control algorithm. The gradient-descent MPPT control algorithm would be the same, except that the control update law in line 21 would be: $a_i \leftarrow \pi_i + K \left( \frac{\partial P_i}{\partial a_i} + \frac{\partial P_d(i)}{\partial a_i} \right)$.

1: given parameters $T_{s,wake}$, $T_{s,local}$, $K$ and sets $F$, $G$
2: $\tau \leftarrow 0$, LocalUpdatesDone $\leftarrow$ False
3: $a_i \leftarrow a_{i,\text{opt}} \forall i \in F$
4: $P_i \leftarrow P_i(t)$
5: $\pi_i \leftarrow a_i$
6: $a_i \leftarrow \pi_i - K \forall i \in G$
7: loop
8: $\tau \leftarrow \tau + \Delta t$
9: if $\tau > T_{s,local}$ and $\tau \leq T_{s,wake}$ and LocalUpdatesDone = False then
10: for all $i \in G$ do
11: $\frac{\partial P_i}{\partial a_i} \leftarrow P_i(t) - P_i(a_i(t) - \pi_i)$
12: $P_i(t) \leftarrow P_i(t)$
13: $P_d(i)(t) \leftarrow P_d(i)(t)$
14: end for
15: LocalUpdatesDone $\leftarrow$ True
16: else if $\tau > T_{s,wake}$ then
17: for all $i \in G$ do
18: $\frac{\partial P_d(i)}{\partial a_i} \leftarrow P_d(i)(t) - P_d(i)$
19: $P_i(t) \leftarrow P_i(t)$
20: $\pi_i \leftarrow a_i$
21: $a_i \leftarrow \pi_i + K \text{sign} \left( \frac{\partial P_i}{\partial a_i} + \frac{\partial P_d(i)}{\partial a_i} \right)$
22: end for
23: $\tau \leftarrow 0$
24: LocalUpdatesDone $\leftarrow$ False
25: end if
26: end loop

3. A benchmark wind farm control algorithm: the Game Theoretic approach

In the simulation example of Section 4, the MPPT approaches are compared to the Game Theoretic (GT) wind farm control approach from [3]. The GT approach optimizes power by making random perturbations to the control variables and holding the settings if they yield an improvement of the wind farm total power. Algorithm 2 gives the GT approach as implemented in the simulation example. The algorithm has two parameters that can be used to set the exploration rate of the randomized optimization:

- a scalar $E \in [0, 1]$ that defines the probability of using a new random setting for $a_i$, instead of keeping the old setting $\pi_i$,
- a scalar $K \in [0, 1]$ that defines the size of the interval in which to choose the random steps on the control settings.
The range $[a_{\text{min}}, a_{\text{max}}]$ is the set of all possible values of the control settings, which for the axial induction factor is given by $[0, 1/3]$. The algorithm is somewhat different than the one presented in [3], since it makes small random perturbations on the old settings $a_i$ to choose $a_i$, rather than taking random values in the full range $[a_{\text{min}}, a_{\text{max}}]$. This was done to improve the convergence speed of the algorithm, and reduce oscillations of the power.

Like the MPPT method, the GT approach is model-free, since it does not need any characterization of wind turbine dynamics or wind farm aerodynamics to track the point of maximum power. An important difference with the MPPT method is that the GT optimization has the total power as its objective function, and that the GT optimization does not use information on the layout of the wind farm. To evaluate the effect of each control variable change on the total power, the algorithm has to wait until the wake has traveled through the entire farm. This waiting time is denoted by $T_{s,farm}$.

### Algorithm 2

The pseudocode below shows a wind farm control algorithm with the Game Theoretic approach of [3]. The value of $r_1, r_2$ are drawn randomly using a uniform distribution.

```plaintext
1: given parameters $T_{s,farm}, K \in [0,1], E \in [0,1]$ and set $F$
2: $\tau \leftarrow 0$
3: $a_i \leftarrow a_i^{\text{opt}} \quad \forall \ i \in F$
4: $P \leftarrow \sum_{i=1}^{N} P_i(t)$
5: $\pi_i \leftarrow a_i$
6: loop
7: $\tau \leftarrow \tau + \Delta t$
8: if $\tau > T_{s,farm}$ then
9: if $\sum_{i=1}^{N} P_i(t) > P$ then
10: $\pi_i \leftarrow a_i \quad \forall \ i \in F$
11: $P \leftarrow \sum_{i=1}^{N} P_i(t)$
12: end if
13: for all $i \in F$ do
14: $r_1 \leftarrow \text{random value} \in [0,1]$
15: if $r_1 > E$ then
16: $r_2 \leftarrow \text{random value} \in [-a_{\text{max}}, a_{\text{max}}]$
17: $a_i \leftarrow \min \left( \max \left( \bar{a}_i + K r_2, a_{\text{min}} \right), a_{\text{max}} \right)$
18: else
19: $a_i \leftarrow \pi_i$
20: end if
21: end for
22: $\tau \leftarrow 0$
23: end if
24: end loop
```

### 4. Simulation example

In this case study, the convergence behavior of the different wind farm control algorithms are compared. To simulate the wind farm we use the Park model that was also used in [4]. Section 4.1 gives a short explanation of the Park model. Section 4.2 presents the results of the simulation.

#### 4.1. The Park wind farm model

The Park model is a relatively simple engineering model which gives an estimate of the velocity profile in the wind farm as a function of the incoming wind field and the set of axial induction
factors of each turbine $a = \{a_i | i \in F\}$. Consider a single turbine $i$ with a rotor diameter $D_i$, with its rotor axis aligned with the wind direction. Assume an incoming uniform wind field with a free-stream speed $V_\infty$. Let $(x, r)$ be a point in the wake of the turbine, where $x$ is the distance to the rotor disk plane of the turbine, and $r$ is the distance to the centerline of the wind turbine rotor axis (see Figure 3). The Park model then estimates the wind speed in this point to be:

$$V_{w,i}(x, r, a_i) = V_\infty (1 - \delta V_{w,i}(x, r, a_i)),$$

with the fractional velocity deficit $\delta V_{w,i}(x, r, a_i)$ given by:

$$\delta V_{w,i}(x, r, a_i) = \begin{cases} 2a_i \left( \frac{D_i}{D_{w,i}(x)} \right)^2 & \text{for } r \leq \frac{D_{w,i}(x)}{2}, \\ 0 & \text{for } r > \frac{D_{w,i}(x)}{2}, \end{cases}$$

where $D_{w,i}$ is the diameter of the wake, which is assumed to have a circular cross-section. The diameter is assumed to expand proportional to the distance $x$:

$$D_{w,i}(x) = D_i + 2\kappa x,$$

where parameter $\kappa$ represents a tunable wake expansion coefficient. In the simulation of Section 4.2, this parameter was tuned to $\kappa = 0.084$, to fit the offshore wind farm power data provided in [5].

The model can be extended to include multiple turbines with interacting wakes. Then the effective wind speed $V_j$ for a turbine $j \in F$ is calculated by summing the velocity deficit created by the wakes of each upstream turbine:

$$V_j(a) = V_\infty (1 - \delta V_j(a)),$$

with:

$$\delta V_j(a) = 2 \sqrt{\sum_{i \in F: x_i < x_j} \left( a_i \left( \frac{D_i}{D_i + 2\kappa (x_j - x_i)} \right)^2 \frac{A_{i,j}^{\text{overlap}}}{A_i} \right)^2},$$

where $A_j$ is the rotor swept area of turbine $j$, and $A_{i,j}^{\text{overlap}}$ is the overlapping area of the rotor swept disk of turbine $j$, and the wake generated by an upstream turbine $i$ at the rotor plane of turbine $j$ (see also Figure 3).

Figure 3. The wake expansion parameters in the Park model.
When the effective wind speed at each turbine is known, the power of each turbine can be calculated as, [4]:

\[ P_i(a_i) = \frac{1}{2} \rho A_i C_P(a_i) V_i(a)^3, \]  

(17)

where \( \rho \) is the air density and \( C_P \) is the power efficiency coefficient, which can be expressed as a function of the axial induction factor:

\[ C_P(a_i) = 4a_i(1 - a_i)^2. \]  

(18)

For more details on the Park model, refer to [3]. The Park model can be used to approximate the power and effective wind speeds for each turbine in a wind farm at each iteration step of the power optimization algorithms. In the above form, the Park model is a static model, in which a change in the axial induction factor has an immediate effect on the total power. For the case study of Section 4.2, the dynamics of the wake interaction were modeled by adding a delay structure to the Park model in order to let the axial induction factor \( a_i \) have a delayed effect on the power of the downstream turbines in a way that corresponds to how the wake travels through the configuration of the turbines in the wind farm. The explanation of this delay structure is omitted for brevity of the discussion.

To evaluate the time-efficiency of the optimization algorithms, an estimate of the time interval between each iteration has to be made. For the MPPT method this time is given by the largest turbine-to-turbine settling time \( T_{s,wake} \). An estimate for the wake travel time is made by assuming a constant speed in between the turbines that is equal to the wind speed halfway the distance between the turbines. This results in:

\[ T_{s,wake} \approx \max_{i \in G} \left( \frac{x_{d(i)} - x_i}{V_{w,i} \left( \frac{1}{2} (x_{d(i)} - x_i), 0, a_i \right)} \right). \]  

(19)

Under the same assumptions as used above, an estimate can be made for \( T_{s,farm} \) as used in the GT method, which denotes the largest time it takes for a change in a control variable \( a_i \) of a turbine \( i \in G \) to have its effect on the power of all of its downstream turbines, by summing each of the turbine-to-turbine wake travel times. Using the notation \( R_i \) for a set that includes the index of a turbine \( i \in G \) and the indices of the full row of turbines in the set \( G \) that are affected by that turbine \( i \), i.e., \( R_i = \{ i, d(i), d(d(i)), \ldots \} \), the approximation is given by:

\[ T_{s,farm} \approx \max_{i \in G} \left( \sum_{j \in R_i} \frac{x_{d(j)} - x_j}{V_{w,j} \left( \frac{1}{2} (x_{d(j)} - x_j), 0, a_j \right)} \right). \]  

(20)

The above estimates for the settling times are fairly rough, and can be used only to make a relative comparison of the time-efficiency of each optimization method. Notice that the response times of the turbine dynamics themselves are neglected, since they are small relative to the wake travel times, i.e., it is assumed the turbines respond instantly to changes in the local wind speeds, and to changes in their own control variable changes (also \( T_{s,local} \) is assumed to be zero).

4.2. Simulation Results

In wind farm simulations using the Park model, the power optimization was performed with the three presented methods with their parameters tuned to yield a fast convergence without too much oscillations around optimized power. This resulted in the FS-MPPT approach being used with the parameter \( K = 0.004 \), the GD-MPPT approach with \( K = 0.01 \), and the GT approach with \( K = 0.1 \) and \( E = 0.05 \). The wind farm configuration used in these simulations is that of the Princess Amalia Wind Park, an offshore wind farm that is located 23km off the coast of The
Netherlands (see Figure 4). It consists of 60 wind turbines with a rotor diameter of 80m and a rated power of 2MW. For each of the simulations, a constant incoming wind speed $V_\infty = 8\text{ms}^{-1}$ was assumed.

The results in Figure 5 show that each of the methods will improve the total power. The MPPT approaches converge to a slightly lower total power than the GT approach. This is because the MPPT approach for each turbine only considers the effect on the power of the neighbouring turbines, while the GT approach optimizes the total power. In other words, in this case the Game Theoretic approach can converge to a global optimum of the the total power, while the MPPT cannot. However, the MPPT approaches converge much faster than the GT approach. This is because the optimization considers the effect on the neighbouring turbine only, which makes that the algorithm can update the control settings more frequently, since the turbine-to-turbine settling time $T_{s,\text{wake}}$ is much shorter than the total wind farm settling time $T_{s,\text{farm}}$. Also, the MPPT methods use gradient information to converge to the local optimum in a faster way. Further, it can be seen that the GD-MPPT algorithm in this case study will reduce the oscillations of the power around the final optimum when compared to the FS-MPPT and the GT approach. It should however be considered that in the FS-MPPT approach these oscillations can be reduced by changing the step size $K$, although this may come at the cost of a longer convergence time.

5. Conclusions
In this paper, a cooperative wind farm power optimization control scheme is presented. In our simulation results that were generated using an engineering model for the wakes in the wind farm, the gradient-based optimization is performed in a time-efficient manner in which each of the turbines use information of its past control actions, the power response of the turbine itself, and the power response of the downstream nearest neighbouring turbine. These nearest neighbours can be found using knowledge of the wind farm layout and an estimate of the wind direction. Because of the improved time-efficiency of the control update scheme, the new method yields a much faster convergence of the power optimization when compared to the existing model-free game-theoretic wind farm power optimization method presented in [3]. If it is considered that the wind direction and other wind conditions change over time, which makes that also the optimal control settings change, the MPPT approach may be a more feasible method to optimize the wind farm power production in a real-time implementation.

In future work, the new control algorithm will be further evaluated using wind farm simulation models with more detailed wake models that are based on computational fluid dynamics, such as those presented in [6] and [7].

References
Figure 4. Locations of the wind turbines in the Princess Amalia Wind Park.

Figure 5. Results of power optimization control with GD-MPPT, FS-MPPT, and the GT approach on a wind farm simulation with the configuration of the Princess Amalia Wind Farm (shown in Figure 4), with an incoming wind under an angle of 25° that has a constant speed $V_\infty = 8\text{ms}^{-1}$. On the left, the markers on the curves correspond to the time instances of the control update steps. On the right, the results are shown on a larger time range to show the convergence of the GT approach. To show the effect of the randomization in the GT method, the distribution of the results of 100 experiments is shown.