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Brief Paper

Controller tuning freedom under plant identification uncertainty: double Youla beats gap in robust stability☆

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Abstract

In iterative schemes of identification and control one of the particular and important choices to make is the choice for a model uncertainty structure, capturing the uncertainty concerning the estimated plant model. Structures that are used in the recent literature encompass e.g. gap metric uncertainty, coprime factor uncertainty, and the Vinnicombe gap metric uncertainty. In this paper, we study the effect of these choices by comparing the sets of controllers that guarantee robust stability for the different model uncertainty bounds. In general these controller sets intersect. However in particular cases the controller sets are embedded, leading to uncertainty structures that are favourable over others. In particular, when restricting the controller set to be constructed as metric-bounded perturbations around the present controller, the so-called double Youla parametrization provides a set of robustly stabilizing controllers that is larger than corresponding sets that are achieved by using any of the other uncertainty structures. This is particularly of interest in controller tuning problems.

Keywords: Robust stability; Gap metric; Youla parametrization; Model uncertainty; Robust control; Simultaneous perturbation; Identification for control

1. Problem set-up

We consider linear time-invariant finite dimensional systems and controllers, in a feedback configuration depicted in Fig. 1, denoted by $H(G_0, C)$, where G_0 is the plant to be (modelled and) controlled, and C a present and known controller.

The closed-loop dynamics of $H(G_0, C)$ are described by the transfer matrix

$$T(G_0, C) = \begin{bmatrix} G_0 \\ I \end{bmatrix} (I + CG_0)^{-1} [C \;\; I],$$

which maps the vector of variables $col(r_1, r_2)$ into $col(y, u)$.[1] The closed-loop system is stable if and only if $T(G_0, C) \in \mathbb{R}H_\infty$.[2]

The problem that we consider is motivated by the following basic question:

> Consider an (unknown) plant G_0 controlled by a known controller C, redesign the controller so as to achieve a better control performance for the controlled plant G_0.

There are several different aspects that can be distinguished in this problem, as e.g.

- One can construct an identified (uncertainty) model of the plant G_0 on the basis of experimental data, e.g. composed of a nominal model and some norm-bounded model or parameter uncertainty. See e.g. Ninness and Goodwin (1995), Hakvoort and Van den Hof (1997), De Vries and

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[1] $col(a, b) = (a^{\mathrm{T}}, b^{\mathrm{T}})^{\mathrm{T}}$.

[2] $\mathbb{R}H_\infty$ is the set of real rational functions that are analytic in the closed right halfplane.

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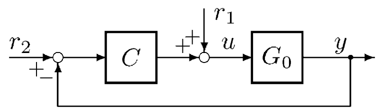

Fig. 1. Feedback interconnection $H(G_0, C)$.

Van den Hof (1995), Bombois, Gevers, and Scorletti (2000).
- The redesigning of the controller can be performed on the basis of a single model (nominal design possibly extended with robustness verifications), a (norm-bounded) uncertainty model (robust design), or on no model at all (as e.g. in iterative feedback tuning (Hjalmarsson, Gunnarsson, & Gevers, 1994)).

If in the controller redesign a (norm-bounded) uncertainty model is taken into account, then the worst-case performance of the newly designed control system can be optimized. This approach is e.g. followed in de Callafon and Van den Hof (1997) where the control design step is a robust control design optimizing the worst-case performance cost. If the identified uncertainty set contains the underlying real plant, guaranteed performance bounds will hold for the controlled real plant also. In this approach the control design utilizes all (uncertain) information on the plant that is available. The resulting control design algorithm becomes relatively complex (μ-synthesis in the work of de Callafon & Van den Hof, 1997). When in the controller redesign only a nominal model is taken into account for the design itself, and an uncertainty model for the plant is used a posteriori to verify the robustness of this design, there is a need for robustness tests concerning stability (and possibly performance).

In this contribution we focus on the latter situation, addressing the problem to characterize the freedom for the present controller C to be retuned/perturbed while maintaining robust stability of the closed-loop system. The characterization of this freedom for C to be perturbed will essentially depend on the uncertainty structure that is used to bound the uncertainty in the nominal plant model. The problem that is addressed in this paper is to consider the question whether sets of robustly stabilizing controllers that result from different choices of model uncertainty structures can be compared to each other.

In this contribution we will primarily consider gap metric uncertainty and uncertainty in terms of a dual-Youla representation. The different uncertainty structures for the plant model lead to different sets of robustly stabilizing controllers, which in general cannot be compared because they are intersecting. However, in particular cases, as will be shown in this paper, the sets of robustly stabilizing controllers are embedded; this leads to conclusions that one structure provides less conservative results than another. In particular, this will be shown to hold for the dual-Youla uncertainty structure, which is less conservative than the gap metric uncertainty in the situation that we restrict the controller set to be constructed as a metric-bounded perturbation of the present controller C. This result is of particular interest when considering problems of controller fragility and of controller tuning, as e.g. applied in Kammer, Bitmead, and Bartlett (2000). In a second stage the results are extended to cover also the Vinnicombe gap-metric and the so-called Λ-gap.

2. Preliminaries

A coprime factor framework will be used to represent plants and controllers, employing both right and left coprime factorizations:

$$G(s) = N(s)D^{-1}(s) = \tilde{D}^{-1}(s)\tilde{N}(s),$$

$$C(s) = N_c(s)D_c^{-1}(s) = \tilde{D}_c^{-1}(s)\tilde{N}_c(s), \tag{1}$$

where (N, D) and (N_c, D_c) are right coprime factorizations (rcf) and $(\tilde{N}, \tilde{D})$ and $(\tilde{N}_c, \tilde{D}_c)$ are left coprime factorizations (lcf) over $\mathbb{R}H_\infty$ (Vidyasagar, 1985). The coprime factorizations are normalized (nrcf), (nlcf) if they additionally satisfy $\bar{N}^*\bar{N} + \bar{D}^*\bar{D} = I$ and $\bar{\tilde{N}}\bar{\tilde{N}}^* + \bar{\tilde{D}}\bar{\tilde{D}}^* = I$, where $(\cdot)^*$ denotes complex conjugate transpose. The notation $\overline{(\cdot)}$ will be used to denote normalized factorizations. Let G and C have coprime factorizations as in (1) and let $\Lambda, \tilde{\Lambda} \in \mathbb{R}H_\infty$ be defined as

$$\Lambda = \bar{\tilde{N}}_c\bar{N} + \bar{\tilde{D}}_c\bar{D} \quad \tilde{\Lambda} = \bar{\tilde{N}}\bar{N}_c + \bar{\tilde{D}}\bar{D}_c, \tag{2}$$

then $H(G, C)$ is stable iff $\Lambda^{-1} \in \mathbb{R}H_\infty$ which is equivalent to the condition $\tilde{\Lambda}^{-1} \in \mathbb{R}H_\infty$ (Vidyasagar, 1985).

3. Robust stability results for double-Youla representations

Uncertainty on a model G_x can be described in very many different ways. In a norm-bounded formulation, there are options for additive, multiplicative, coprime-factor, gap-metric uncertainties, all having their particular robust stability tests. See e.g. de Callafon, Van den Hof, and Bongers (1996) for an overview in a rather uniform (coprime factor) framework.

When considering robust performance tests on norm-bounded uncertainty sets, it has been motivated in de Callafon and Van den Hof (1997) that for general classes of performance measures, norm-bounded uncertainty in a dual-Youla parametrization framework has particular advantages. In this parametrization, a norm-bounded plant uncertainty set is considered of the form:

$$\mathscr{G}_Y(G_x, C, Q, Q_c, \gamma_G) := \{G_\Delta = (\bar{N}_x + \bar{D}_c\Delta_R)(\bar{D}_x - \bar{N}_c\Delta_R)^{-1} \mid \|Q_c^{-1}\Delta_R Q\|_\infty \leqslant \gamma_G\}$$

with G_x a nominal model, C the present controller stabilizing G_x, and Q, Q_c stable and stably invertible weighting functions. The Youla parameter of a plant can directly

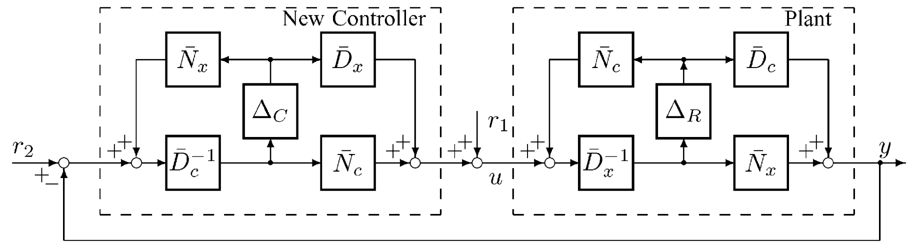

Fig. 2. Double-Youla parametrization.

be identified from closed-loop data. In fact, the identification becomes a 'standard' open-loop identification problem (Van den Hof, 1998). Any uncertainty bounding technique (e.g. Ninness & Goodwin, 1995; Hakvoort & Van den Hof, 1997; De Vries & Van den Hof, 1995; Bombois et al., 2000) can then be employed to obtain the bounded norm in the above definition. In terms of stability, the dual-Youla parametrization has the basic property that an element in $\mathscr{G}_{\mathrm{Y}}(G_x, C, Q, Q_{\mathrm{c}}, \gamma_G)$ is stabilized by C if and only if the corresponding Δ_R is stable.

Similar to characterizing plant uncertainty, a perturbation/retuning of the controller can be represented as a Youla-type "perturbation" on the present controller C. This results in the so-called double-Youla parametrization, indicated in Fig. 2, where

$$C_{\mathrm{new}} := C_\Delta = (\bar{N}_{\mathrm{c}} + \bar{D}_x \Delta_C)(\bar{D}_{\mathrm{c}} - \bar{N}_x \Delta_C)^{-1}.$$

The following stability results apply to this situation (Tay, Moore, & Horowitz, 1989; Schrama, Bongers, & Bosgra, 1992).

Proposition 1. *Let G_x and C have normalized coprime factorizations as described above, and let $H(G_x, C)$ be stable. Denote*

$$G_\Delta = (\bar{N}_x + \bar{D}_{\mathrm{c}} \Delta_R)(\bar{D}_x - \bar{N}_{\mathrm{c}} \Delta_R)^{-1}, \tag{3}$$

$$C_\Delta = (\bar{N}_{\mathrm{c}} + \bar{D}_x \Delta_C)(\bar{D}_{\mathrm{c}} - \bar{N}_x \Delta_C)^{-1}. \tag{4}$$

Then for $\Delta_R, \Delta_C \in \mathbb{R}H_\infty$

(a) *$H(G_\Delta, C_\Delta)$ is stable if and only if for some unimodular*[3] *$Q, Q_{\mathrm{c}} \in \mathbb{R}H_\infty$, $H(Q_{\mathrm{c}}^{-1}\Delta_R Q, Q^{-1}\Delta_C Q_{\mathrm{c}})$ is stable;*
(b) *$H(G_\Delta, C_\Delta)$ is stable if there exist some unimodular $Q, Q_{\mathrm{c}} \in \mathbb{R}H_\infty$ such that*

$$\|Q^{-1}\Delta_C Q_{\mathrm{c}}\|_\infty \cdot \|Q_{\mathrm{c}}^{-1}\Delta_R Q\|_\infty < 1.$$

Note that the Youla factors Δ_R and Δ_C are uniquely determined by expression 3 and 4 in terms of G_Δ and C_Δ and the normalized coprime factors of G_x and C, which are unique modulo unitary factors. The unimodular matrices Q and Q_{c} can be interpreted to reflect the freedom in choosing the coprime factorizations of G_x and C. Based on this result the next proposition can be formulated.

[3] $A \in \mathbb{R}H_\infty$ is unimodular if $A^{-1} \in \mathbb{R}H_\infty$.

Proposition 2. *Given a nominal model G_x and a nominal controller C, with nrcf's as described before, such that $H(G_x, C) \in \mathbb{R}H_\infty$. Define a set of plants $\mathscr{G}_{\mathrm{Y}}(G_x, C, Q, Q_{\mathrm{c}}, \gamma_G)$ and a set of controllers $\mathscr{C}_{\mathrm{Y}}(G_x, C, Q, Q_{\mathrm{c}}, \gamma_C)$ as*

$$\mathscr{G}_{\mathrm{Y}}(G_x, C, Q, Q_{\mathrm{c}}, \gamma_G) := \{G_\Delta = (\bar{N}_x + \bar{D}_{\mathrm{c}}\Delta_R)(\bar{D}_x - \bar{N}_{\mathrm{c}}\Delta_R)^{-1} \mid \|Q_{\mathrm{c}}^{-1}\Delta_R Q\|_\infty \leqslant \gamma_G\},$$

$$\mathscr{C}_{\mathrm{Y}}(G_x, C, Q, Q_{\mathrm{c}}, \gamma_C) := \{C_\Delta = (\bar{N}_{\mathrm{c}} + \bar{D}_x\Delta_C)(\bar{D}_{\mathrm{c}} - \bar{N}_x\Delta_C)^{-1} \mid \|Q^{-1}\Delta_C Q_{\mathrm{c}}\|_\infty < \gamma_C\}.$$

Then all plants in $\mathscr{G}_{\mathrm{Y}}(G_x, C, Q, Q_{\mathrm{c}}, \gamma_G)$ are stabilized by all controllers contained in the set $\mathscr{C}_{\mathrm{Y}}(G_x, C, Q, Q_{\mathrm{c}}, \gamma_C)$ if and only if $\gamma_G \cdot \gamma_C \leqslant 1$.

Proof. The result is direct by applying a small gain argument and employing Proposition 1. □

This Proposition serves as a means to specify the allowed perturbation of the controller C so as to guarantee robust stability with all models in the plant uncertainty set. Since the result is based on a small gain criterion, part (a) of Proposition 1 can be used to show that the resulting set of controllers is equal to the exclusive set of all controllers stabilizing the entire set $\mathscr{G}_{\mathrm{Y}}(G_x, C, Q, Q_{\mathrm{c}}, \gamma_G)$.

4. Gap metric results

When considering the gap metric as a measure for bounding plant uncertainty a similar analysis can be given as presented in the previous section. The gap metric distance between two systems G_x, G_Δ is defined by

$$\delta(G_x, G_\Delta) = \max\{\vec{\delta}(G_x, G_\Delta), \vec{\delta}G_\Delta, G_x)\},$$

where the *directed gap* is:

$$\vec{\delta}(G_x, G_\Delta) = \inf_{Q_\delta, Q_\delta^{-1} \in H_\infty} \left\| \begin{bmatrix} \bar{N}_x \\ \bar{D}_x \end{bmatrix} - \begin{bmatrix} \bar{N}_\Delta \\ \bar{D}_\Delta \end{bmatrix} Q_\delta \right\|_\infty, \tag{5}$$

where $(\bar{N}_x, \bar{D}_x)$ and $(\bar{N}_\Delta, \bar{D}_\Delta)$ are *nrcf's* of G_x and G_Δ. The stability result that is applicable to our problem set-up is the following.

Proposition 3 (Georgiou and Smith, 1990). *Let $H(G_x, C)$ be stable. Then $H(G_\Delta, C_\Delta)$ is stable if*

$$\delta(G_x, G_\Delta) + \delta(C, C_\Delta) < \|T(G_x, C)\|_\infty^{-1}.$$

This sufficient condition for stability leads to the following formulation in terms of stabilizing sets of controllers.

Corollary 1 (Georgiou and Smith, 1990). *Given a nominal model G_x and a nominal controller C such that $H(G_x, C) \in \mathbb{R}H_\infty$. The set $\mathscr{G}_\delta(G_x, \delta_G)$ defined as*

$$\mathscr{G}_\delta(G_x, \delta_G) := \{G_\Delta | \delta(G_x, G_\Delta) \leqslant \delta_G\}$$

is stabilized by all controllers contained in the set $\mathscr{C}_\delta(C, \delta_C)$ defined as

$$\mathscr{C}_\delta(C, \delta_C) := \{C_\Delta | \delta(C, C_\Delta) < \delta_C\}$$

if $\delta_C \leqslant \|T(G_x, C)\|_\infty^{-1} - \delta_G$.

In this proposition a sufficient condition for the retuning range (or the allowed "perturbation" from the present controller) is specified that is allowed under guarantee of robust stability. Unlike for Proposition 2, $\mathscr{C}_\delta(C, \delta_C)$ with δ_C equal to the above mentioned upper bound, does not contain all controllers stabilizing the entire set $\mathscr{G}_\delta(G_x, \delta_G)$. In Qui and Davison (1992) a slightly stronger result is given for robust stability under simultaneous plant and controller perturbations in gap-metric sense. Here we restrict attention to the original (and well known) results presented in Georgiou and Smith (1990).

5. Comparison of the two uncertainty structures

Theorem 1. *Given a set of plants $\mathscr{G}_\delta(G_x, \delta_G)$ and a set of controllers $\mathscr{C}_\delta(C, \delta_C)$ satisfying the gap stability condition of Corollary 1. Then for the sets of Proposition 2 with $Q = Q_c = I$, it holds that*

(a) $\mathscr{G}_Y(G_x, C, I, I, \bar{\gamma}_G) \supseteq \mathscr{G}_\delta(G_x, \delta_G)$, *with* $\bar{\gamma}_G = \delta_G \|T(G_x, C)\|_\infty (1 - \delta_G \|T(G_x, C)\|_\infty)^{-1}$
(b) $\mathscr{C}_Y(G_x, C, I, I, \bar{\gamma}_C) \supseteq \mathscr{C}_\delta(C, \delta_C)$, *with* $\bar{\gamma}_C = \delta_C \|T(G_x, C)\|_\infty (1 - \delta_C \|T(G_x, C)\|_\infty)^{-1}$
(c) $\bar{\gamma}_G \cdot \bar{\gamma}_C \leqslant 1$, *i.e. the two sets satisfy the stability condition of Proposition* 2.

Proof. The proof is provided in Appendix A.

The result of this theorem implies that even when embedding the gap uncertainty sets for plant and controller in (more conservative) sets in terms of Youla parametrizations, a simultaneous stabilization result remains valid. In other words: the resulting sets of plants and controllers can be compared on the basis of robust stability and it follows that the related robust stability test for the Youla-structured uncertainty is less conservative than the test for the gap metric.

In practice, the uncertainty set in terms of the Youla parametrization would be not be chosen as to enclose the set of the gap uncertainty but as to enclose the set of unfalsified plants. A direct consequence of the theorem in this respect is formulated in the next corollary.

Corollary 2. *Given a set of (unfalsified) plants $\mathscr{G}$, a gap uncertainty set $\mathscr{G}_\delta(G_x, \check{\delta}_G)$ and a Youla uncertainty set $\mathscr{G}_Y(G_x, C, I, I, \check{\gamma}_G)$, where $\check{\delta}_G, \check{\gamma}_G$ are the smallest values of δ_G, γ_G such that $\mathscr{G} \subset \mathscr{G}_\delta(G_x, \delta_G)$ and $\mathscr{G} \subset \mathscr{G}_Y(G_x, C, I, I, \gamma_G)$. Then the largest stabilizing controller sets resulting from Proposition 2 and Corollary 1, satisfy*

$$\mathscr{C}_\delta(C, \|T(G_x, C)\|_\infty^{-1} - \check{\delta}_G) \subset \mathscr{C}_Y(G_x, C, I, I, \check{\gamma}_G^{-1}).$$

Apparently, when describing plant uncertainty in either a gap metric bound or a norm bound in a dual-Youla representation, the latter format allows for a larger set of controllers that guarantee robust stability. The resulting set of controllers guaranteed to stabilize the set of unfalsified plants would still be larger when the freedom of applying weighting functions would be employed (cf. Proposition 1).

One of the principal differences in the two uncertainty structures is that a gap-metric distance between two plants is controller independent. A Youla formulation of the "distance" between two plants is taken under the presence of (and therefore dependent on) a particular controller. In the latter situation the closed-loop properties of the two plants can therefore be taken into account more particularly.

The formulation of the corollary technically allows that the sets are equal; in Section 7 an example is given in which the embedding is shown to be strict.

6. Extension to ν-gap and Λ-gap

The analysis as presented in this paper so far can readily be extended to other uncertainty structures as well, as e.g. the ν-gap and the Λ-gap. The Vinnicombe or ν-gap metric is defined as (Vinnicombe, 1993):

$$\delta_\nu(G_x, G_\Delta) = \begin{cases} \bullet \quad \left\| \begin{bmatrix} -\tilde{\bar{D}}_x & \tilde{\bar{N}}_x \end{bmatrix} \begin{bmatrix} \bar{N}_\Delta \\ \bar{D}_\Delta \end{bmatrix} \right\|_\infty, \\ \text{if } \det\left(\begin{bmatrix} \tilde{\bar{N}}_x^* & \tilde{\bar{D}}_x^* \end{bmatrix} \begin{bmatrix} \bar{N}_\Delta \\ \bar{D}_\Delta \end{bmatrix} \right) \neq 0 \ \forall \omega \\ \text{and} \\ W\left(\det\left(\begin{bmatrix} \tilde{\bar{N}}_x^* & \tilde{\bar{D}}_x^* \end{bmatrix} \begin{bmatrix} \bar{N}_\Delta \\ \bar{D}_\Delta \end{bmatrix} \right) \right) = 0 \\ \bullet \quad 1, \qquad \text{otherwise,} \end{cases} \tag{6}$$

where $W(g)$ denotes the winding number about the origin of $g(s)$ as s follows the standard Nyquist D-contour.

The Λ-gap $\vec{\delta}_\Lambda(G_x, G_\Delta)$ between two plants G_x and G_Δ is defined as (Bongers, 1991; Bongers, 1994; de Callafon et al., 1996)

$$\vec{\delta}_\Lambda(G_x, G_\Delta) = \inf_{Q_\Lambda, Q_\Lambda^{-1} \in \mathbb{R}H_\infty} \left\| \begin{pmatrix} \bar{N}_x \\ \bar{D}_x \end{pmatrix} \Lambda^{-1} - \begin{pmatrix} \bar{N}_\Delta \\ \bar{D}_\Delta \end{pmatrix} Q_\Lambda \right\|_\infty$$

with $(\bar{N}_x, \bar{D}_x)$ and $(\bar{N}_\Delta, \bar{D}_\Delta)$ *nrcf's* of G_x and G_Δ, and Λ as defined in (2).

The robust stability results—known from the literature—that can be exploited for our purpose of specifying a metric-bounded area around C under robust stability guarantees read as follows (Vinnicombe, 1993; Bongers, 1994).

Proposition 4. *Let $H(G_x, C)$ be stable. Then $H(G_\Delta, C_\Delta)$ is stable if*

(a) $\delta_\nu(G, G_\Delta) + \delta_\nu(C, C_\Delta) < \|T(G_x, C)\|_\infty^{-1}$ (*ν-gap condition*) *or*
(b) $\vec{\delta}_\Lambda(G_x, G_\Delta) + \vec{\delta}_\Lambda(C, C_\Delta) < 1$ (*Λ-gap condition*).

These sufficient conditions for stability lead to the following formulation in terms of stabilizing sets of controllers.

Corollary 3. *Given a nominal model G_x and a nominal controller C such that $H(G_x, C) \in \mathbb{R}H_\infty$. The set $\mathscr{G}_\nu(G_x, \delta_{\nu,G})$ defined as*

$$\mathscr{G}_\nu(G_x, \delta_{\nu,G}) := \{G_\Delta | \delta_\nu(G_x, G_\Delta) \leqslant \delta_{\nu,G}\}$$

is stabilized by all controllers contained in the set $\mathscr{C}_\nu(C, \delta_{\nu,C})$ defined as

$$\mathscr{C}_\nu(C, \delta_{\nu,C}) := \{C_\Delta | \delta_\nu(C, C_\Delta) < \delta_{\nu,C}\}$$

if $\delta_{\nu,C} \leqslant \|T(G_x, C)\|_\infty^{-1} - \delta_{\nu,G}$.

Corollary 4. *Given a nominal model G_x and a nominal controller C such that $H(G_x, C) \in \mathbb{R}H_\infty$. The set $\mathscr{G}_\Lambda(G_x, \delta_{\Lambda,G})$ defined as*

$$\mathscr{G}_\Lambda(G_x, \delta_{\Lambda,G}) := \{G_\Delta | \vec{\delta}_\Lambda(G_x, G_\Delta) \leqslant \delta_{\Lambda,G}\}$$

is stabilized by all controllers contained in the set $\mathscr{C}_\Lambda(C, \delta_{\Lambda,C})$ defined as

$$\mathscr{C}_\Lambda(C, \delta_{\Lambda,C}) := \{C_\Delta | \vec{\delta}_\Lambda(C, C_\Delta) < \delta_{\Lambda,C}\}$$

if $\delta_{\Lambda,C} \leqslant 1 - \delta_{\Lambda,G}$.

Note that $\mathscr{C}_\nu(C, \delta_{\nu,C})$ and $\mathscr{C}_\Lambda(C, \delta_{\Lambda,C})$ do not contain all controllers stabilizing the entire set $\mathscr{G}_\nu(G_x, \delta_{\nu,G})$ and $\mathscr{G}_\Lambda(G_x, \delta_{\Lambda,G})$, respectively. The use of the necessary and sufficient ν-gap condition (Vinnicombe, 1993)

$$\delta_{\nu,G} \leqslant \|T(G_x, C_\Delta)\|_\infty^{-1} \tag{7}$$

would result in a characterization of the exclusive set of all controllers stabilizing $\mathscr{G}_\nu(G_x, \delta_{\nu,G})$. This condition, however, does not allow for an explicit metric-bounded tuning range around a present controller and therefore is less suitable for studying controller fragility and controller tuning, where a perturbation of the controller around a nominal value is considered.

Based on these robust stability results one can now consider the same problem as is considered in the formulation of Theorem 1.

Theorem 2. *Given a set of plants $\mathscr{G}_\nu(G_x, \delta_{\nu,G})$ and a set of controllers $\mathscr{C}_\nu(C, \delta_{\nu,C})$ satisfying the ν-gap stability condition of Corollary 3. Then for the sets of Proposition 2 with $Q = Q_c = I$, it holds that*

(a) $\mathscr{G}_Y(G_x, C, I, I, \bar{\gamma}_{\nu,G}) \supseteq \mathscr{G}_\nu(G_x, \delta_{\nu,G})$, *with* $\bar{\gamma}_{\nu,G} = \delta_{\nu,G}\|T(G_x, C)\|_\infty(1 - \delta_{\nu,G}\|T(G_x, C)\|_\infty)^{-1}$
(b) $\mathscr{C}_Y(G_x, C, I, I, \bar{\gamma}_{\nu,C}) \supseteq \mathscr{C}_\nu(C, \delta_{\nu,C})$, *with* $\bar{\gamma}_{\nu,C} = \delta_{\nu,C}\|T(G_x, C)\|_\infty(1 - \delta_{\nu,C}\|T(G_x, C)\|_\infty)^{-1}$
(c) $\bar{\gamma}_{\nu,G} \cdot \bar{\gamma}_{\nu,C} \leqslant 1$, *i.e. the two sets satisfy the stability condition of Proposition 2.*

Theorem 3. *Given a set of plants $\mathscr{G}_\Lambda(G_x, \delta_{\Lambda,G})$ and a set of controllers $\mathscr{C}_\Lambda(C, \delta_{\Lambda,C})$ satisfying the Λ-gap stability condition of Corollary 4. Then for the sets of Proposition 2 with $Q = \Lambda^{-1}$ and $Q_c = \tilde{\Lambda}^{-1}$, with $\Lambda, \tilde{\Lambda}$ as defined in (2), it holds that*

(a) $\mathscr{G}_Y(G_x, C, \Lambda^{-1}, \tilde{\Lambda}^{-1}, \bar{\gamma}_{\Lambda,G}) \supseteq \mathscr{G}_\Lambda(G_x, \delta_{\Lambda,G})$, *with* $\bar{\gamma}_{\Lambda,G} = \delta_{\Lambda,G}(1 - \delta_{\Lambda,G})^{-1}$
(b) $\mathscr{C}_Y(G_x, C, \Lambda^{-1}, \tilde{\Lambda}^{-1}, \bar{\gamma}_{\Lambda,C}) \supseteq \mathscr{C}_\Lambda(C, \delta_{\Lambda,C})$, *with* $\bar{\gamma}_{\Lambda,C} = \delta_{\Lambda,C}(1 - \delta_{\Lambda,C})^{-1}$
(c) $\bar{\gamma}_{\Lambda,G} \cdot \bar{\gamma}_{\Lambda,C} \leqslant 1$, *i.e. the two sets satisfy the stability condition of Proposition 2.*

Proof. Proofs are provided in Appendix A.

These theorems show that, like the gap-metric uncertainty structure, also the ν-gap and Λ-gap uncertainty structures lead to controller sets that in the considered problem formulation are more conservative than the sets that are obtained by a double Youla-parametrization.

7. Example

An example is considered in which robust stability is guaranteed by the condition of Proposition 2, but not by the gap-metric conditions of Corollaries 1, 3 and 4. We consider a (physical model) of a rotating drive system G_0 and an identified model $\hat{G}$ based on input and output data. Fig. 3 shows that the model $\hat{G}$ is a fairly good model, although the error $|G_0 - \hat{G}|$ is quite large near the resonance frequencies. A controller C is designed, on the basis of the model $\hat{G}$, which stabilizes the system and reduces the influence of the resonance peaks beyond the bandwidth. A new controller C_{new} is designed to achieve a slightly larger bandwidth of

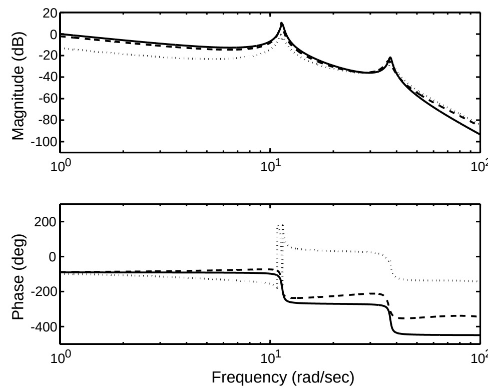

Fig. 3. Bode diagram of the (physical model of the) rotating drive system G_0 (solid), the estimate $\hat{G}$ (dashed) and the difference $G_0 - \hat{G}$ (dotted).

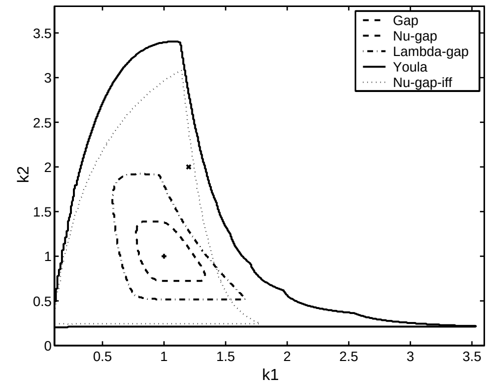

Fig. 4. Comparing the allowable range of controller perturbations of the form $C_{\text{new}}(k_1, k_2) = 1000k_2/(s/k_1 + 10)^3$ according to the robust stability conditions of Corollaries 1, 3 and 4, Proposition 2 and expression 7. The lines depict the boundaries outside which the associated controllers do not satisfy the respective robust stability conditions. The controllers C and $C_{\text{new}}(1.2, 2)$ are indicated by '+' and '×'.

the closed loop system. The respective transfer functions are given by

$$G_0 = \frac{182049.47}{s(s^2 + 0.377s + 130.5)(s^2 + 1.249s + 1395)},$$

$$\hat{G} = \frac{0.0051602(s^2 + 70.53s + 1634)(s^2 - 45.72s + 1.608^4)}{s(s^2 + 0.2671s + 128.3)(s^2 + 2.125s + 1348)},$$

$$C = \frac{1000}{(s + 10)^3}, \quad C_{\text{new}} = \frac{2000}{(s/1.2 + 10)^3}.$$

Before implementing the new controller the various robust stability tests are considered. We have the following numbers: $\|T(\hat{G}, C)\|_\infty^{-1} = 0.394$; $\delta(G_0, \hat{G}) = 0.235$; $\delta(C, C_{\text{new}}) = 0.417$; $\delta_\nu(G_0, \hat{G}) = 0.234$; $\delta_\nu(C, C_{\text{new}}) = 0.416$; $\delta_\Lambda(G_0, \hat{G}) = 0.492$; $\delta_\Lambda(C, C_{\text{new}}) = 0.976$; $\|\Delta_R\|_\infty = 0.381$; $\|\Delta_C\|_\infty = 1.455$.

Clearly $\delta_{(\nu)}(G_0, \hat{G}) + \delta_{(\nu)}(C, C_{\text{new}})$ is much larger than $\|T(\hat{G}, C)\|_\infty^{-1}$. Hence from Corollaries 1 and 3 it cannot be concluded that $H(G_0, C_{\text{new}})$ is stable. Moreover, as $\delta_{(\nu)}(C, C_{\text{new}}) > \|T(\hat{G}, C)\|_\infty^{-1}$, the gap and the ν-gap condition fail even to guarantee stability of $H(G_0, C)$. The Λ-gap condition of Proposition 4 does show stability of $H(G_0, C)$, but again stability of $H(G_0, C_{\text{new}})$ cannot be concluded as $\delta_\Lambda(G_0, \hat{G}) + \delta_\Lambda(C, C_{\text{new}}) > 1$. The Youla condition of Proposition 2 is satisfied and would even allow for larger perturbations as $\|\Delta_R\|_\infty \|\Delta_C\|_\infty$ is $0.554 < 1$.

A further indication of the differences between the robust stability conditions is provided by Fig. 4 which illustrates how the controller C could be retuned/perturbed while still be guaranteed by the various conditions to stabilize both models G_0 and $\hat{G}$. Perturbations are considered of the form

$$C_{\text{new}}(k_1, k_2) = \frac{1000k_2}{(s/k_1 + 10)^3}.$$

The figure is obtained by computation over a fine grid of k_1 and k_2 and the lines depict the boundaries outside which the associated controllers do not satisfy the respective robust stability conditions. The results illustrate that the Youla condition yields a much larger set of stabilizing controllers then the gap-conditions. The ν-gap and the gap are (practically) identical for the considered controllers. The Λ-gap is seen to be less conservative than the other two gaps. Finally, the necessary and sufficient ν-gap (7) is also evaluated. While the associated controller set necessarily embeds the gap and the sufficient ν-gap sets, it is seen that it cannot be compared with the Λ-gap as the two sets intersect. For the parameter range depicted here the Youla condition yields a larger set than the necessary and sufficient ν-gap condition. However outside this range the sets will intersect and cannot be compared.

8. Concluding remarks

The use of different uncertainty structures for specifying model uncertainty leads to different sets of robustly stabilizing controllers. These controller sets generally intersect, and therefore a "best" choice cannot be made. However, when restricting attention to controller sets that can be described as a metric-bounded perturbation of a nominal/present controller, the use of a double Youla parametrization for representing plant and controller uncertainty is shown to be less conservative than when using gap-metric type of uncertainties. This result can be fruitfully used for studying controller fragility and in controller (re-)tuning. An example has been provided to illustrate these results.

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Appendix A. Proof of Theorems 1–3

A.1. Gap metric (Theorem 1)

The proof of Theorem 1 consists of (1) showing that there exist $\bar{\gamma}_G$ and $\bar{\gamma}_C$ such that the sets $\mathscr{G}_{\mathrm{Y}}(G_x, C, I, I, \bar{\gamma}_G)$ and $\mathscr{C}_{\mathrm{Y}}(G, C, I, I, \bar{\gamma}_C)$, as defined in Proposition 2, embed sets $\mathscr{G}_\delta(G_x, \delta_G)$ and $\mathscr{C}_\delta(C, \delta_C)$ satisfying the gap stability condition of Corollary 1, respectively, and (2) that $\mathscr{G}_{\mathrm{Y}}(G_x, C, I, I, \bar{\gamma}_G)$ and $\mathscr{C}_{\mathrm{Y}}(G, C, I, I, \bar{\gamma}_C)$ satisfy the stability condition of Proposition 2, i.e $\bar{\gamma}_G \cdot \bar{\gamma}_C \leqslant 1$.

Lemma 1. *Given a nominal plant* $G_x = \bar{N}_x \bar{D}_x^{-1}$ *and controller* $C = \bar{N}_c \bar{D}_c^{-1}$ *such that* $H(G_x, C) \in \mathbb{R}H_\infty$. *Then every plant* G_Δ *stabilized by* C *and every controller* C_Δ *stabilized by* G_x *can be expressed in a (dual) Youla factorization (Vidyasagar, 1985), i.e.*

$$G_\Delta = (\bar{N}_x + \bar{D}_c \Delta_R)(\bar{D}_x - \bar{N}_c \Delta_R)^{-1} \text{ and}$$
$$C_\Delta = (\bar{N}_c + \bar{D}_x \Delta_C)(\bar{D}_c - \bar{N}_x \Delta_C)^{-1}$$

and the Youla parameters Δ_R *and* Δ_C *satisfy*

$$\|\Delta_R\|_\infty \leqslant \|\tilde{\Lambda}^{-1}\|_\infty \delta(G_x, G_\Delta)(1 - \|\Lambda^{-1}\|_\infty \delta(G_x, G_\Delta))^{-1}$$
$$\|\Delta_C\|_\infty \leqslant \|\Lambda^{-1}\|_\infty \delta(C, C_\Delta)(1 - \|\tilde{\Lambda}^{-1}\|_\infty \delta(C, C_\Delta))^{-1}.$$

Proof. Lemma 1 follows from exploiting the freedom in coprime factorizations with respect to a unimodular multiplication. Each G_Δ can be written in a Youla factorization and in terms of a coprime factorization related to the directed gap $\vec{\delta}(G_\Delta, G)$ of (5),

$$\begin{bmatrix} \bar{N}_\Delta \\ \bar{D}_\Delta \end{bmatrix} Q_G = \begin{bmatrix} \bar{N}_x + \bar{D}_c \Delta_R \\ \bar{D}_x - \bar{N}_c \Delta_R \end{bmatrix} Q. \tag{A.1}$$

Here Q_G denotes the minimizing argument of Q_δ in (5). Q_G is unique (modulo unitary factors) and unimodular (i.e. $Q_G, Q_G^{-1} \in \mathbb{R}H_\infty$) implying the left-hand side of (A.1) to represent a coprime factorization indeed (Sefton and Ober, 1993). The unimodular matrix Q accomplishes the equality and is uniquely defined (modulo unitary factors) due to the normalizations of the coprime factors. From (A.1) we can write:

$$\begin{bmatrix} -\bar{D}_c \\ \bar{N}_c \end{bmatrix} \Delta_R = \begin{bmatrix} \bar{N}_x \\ \bar{D}_x \end{bmatrix} - \begin{bmatrix} \bar{N}_x \\ \bar{D}_x \end{bmatrix} Q^{-1} + \left\{ \begin{bmatrix} \bar{N}_x \\ \bar{D}_x \end{bmatrix} - \begin{bmatrix} \bar{N}_\Delta \\ \bar{D}_\Delta \end{bmatrix} Q_G \right\} Q^{-1}. \tag{A.2}$$

This expression is simplified by multiplication from the left with $\tilde{\Lambda}^{-1}[-\bar{\tilde{D}}_x \;\; \bar{\tilde{N}}_x]$, using the fact that $\bar{\tilde{D}}_x \bar{N}_x = \bar{\tilde{N}}_x \bar{D}_x$, leading to

$$\Delta_R = \tilde{\Lambda}^{-1} \begin{bmatrix} -\bar{\tilde{D}}_x & \bar{\tilde{N}}_x \end{bmatrix} \times \left\{ \begin{bmatrix} \bar{N}_x \\ \bar{D}_x \end{bmatrix} - \begin{bmatrix} \bar{N}_\Delta \\ \bar{D}_\Delta \end{bmatrix} Q_G \right\} Q^{-1}, \tag{A.3}$$

where $\tilde{\Lambda}$ is as defined in (2).

Multiplication of (A.2) from the left with $\Lambda^{-1}[\bar{\tilde{N}}_c \;\; \bar{\tilde{D}}_c]$ yields

$$Q = I - \Lambda^{-1} \begin{bmatrix} \bar{\tilde{N}}_c & \bar{\tilde{D}}_c \end{bmatrix} \left\{ \begin{bmatrix} \bar{N}_x \\ \bar{D}_x \end{bmatrix} - \begin{bmatrix} \bar{N}_\Delta \\ \bar{D}_\Delta \end{bmatrix} Q_G \right\}. \tag{A.4}$$

Applying the singular value relation (Horn & Johnson, 1991) $\underline{\sigma}(A - B) \geqslant \underline{\sigma}(A) - \bar{\sigma}(B)$ to (A.4) and using the fact that $\bar{\sigma}(Q^{-1}(\omega)) = 1/\underline{\sigma}(Q(\omega))$ for all ω and that $[\bar{\tilde{N}}_c \;\; \bar{\tilde{D}}_c]$ is co-inner, results in

$$\bar{\sigma}(Q^{-1}(\omega)) \leqslant (1 - \|\Lambda^{-1}\|_\infty \vec{\delta}(G, G_\Delta))^{-1} \quad \forall \omega$$
$$\|Q^{-1}\|_\infty \leqslant (1 - \|\Lambda^{-1}\|_\infty \vec{\delta}(G, G_\Delta))^{-1}. \tag{A.5}$$

The bounds of Lemma 1 then follow easily from (A.3) and (A.5) upon noting that $[-\bar{\tilde{D}}_x \;\; \bar{\tilde{N}}_x]$ is co-inner. The appearance of the gap itself in Lemma 1 follows from a similar derivation for the other directed gap $\vec{\delta}(G, G_\Delta)$ and the fact that the gap is the maximum of both. A similar derivation can be followed for the bound on $\|\Delta_C\|_\infty$. □

By the fact that the sets $\mathscr{G}_\delta(G_x, \delta_G)$ and $\mathscr{C}_\delta(C, \delta_C)$ satisfy $\delta_G + \delta_C \leqslant \|T(G_x, C)\|_\infty^{-1}$, the following holds:

i. Lemma 1 applies, as all $G_\Delta \in \mathscr{G}_\delta(G_x, \delta_G)$ are stabilized by C and all $C_\Delta \in \mathscr{C}_\delta(C, \delta_C)$ are stabilized by G_x.
ii. With the fact that $\|T(G_x, C)\|_\infty = \|\tilde{\Lambda}^{-1}\|_\infty = \|\Lambda^{-1}\|_\infty$ (Schrama et al., 1992), Lemma 1 shows that every plant $G_\Delta \in \mathscr{G}_\delta(G_x, \delta_G)$ has an associated Youla parameter with infinity norm bounded by $\bar{\gamma}_G = \delta_G \|T(G_x, C)\|_\infty (1 - \delta_G \|T(G_x, C)\|_\infty)^{-1}$ and every controller $C_\Delta \in \mathscr{C}_\delta(C, \delta_C)$ has an associated Youla parameter with infinity norm bounded by $\bar{\gamma}_C = \delta_C \|T(G_x, C)\|_\infty (1 - \delta_C \|T(G_x, C)\|_\infty)^{-1}$.
iii. The values $\bar{\gamma}_G$ and $\bar{\gamma}_C$ satisfy $\bar{\gamma}_C \leqslant (1 - \delta_G \|T(G_x, C)\|_\infty)(\delta_G \|T(G_x, C)\|_\infty)^{-1} = \bar{\gamma}_G^{-1}$.

From this Theorem 1 is readily seen to hold.

A.2. ν-gap metric (Theorem 2)

For the ν-gap metric only a few specific steps will be highlighted as the line of proof is to a large extent similar to the proof of Theorem 1. From

$$\begin{bmatrix} \bar{N}_\Delta \\ \bar{D}_\Delta \end{bmatrix} Q^{-1} = \begin{bmatrix} \bar{N}_x + \bar{D}_c \Delta_R \\ \bar{D}_x - \bar{N}_c \Delta_R \end{bmatrix}, \tag{A.6}$$

with Q achieving the normalization, we have upon premultiplication with $[-\bar{\tilde{D}}_x \;\; \bar{\tilde{N}}_x]$,

$$\begin{bmatrix} -\bar{\tilde{D}}_x \\ \bar{\tilde{N}}_x \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \bar{N}_\Delta \\ \bar{D}_\Delta \end{bmatrix} Q^{-1} = \begin{bmatrix} -\bar{\tilde{D}}_x \\ \bar{\tilde{N}}_x \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \bar{N}_x + \bar{D}_c \Delta_R \\ \bar{D}_x - \bar{N}_c \Delta_R \end{bmatrix}$$
$$= (\bar{\tilde{N}}_x \bar{D}_x - \bar{\tilde{D}}_x \bar{N}_x) - (\bar{\tilde{D}}_x \bar{D}_c + \bar{\tilde{N}}_x \bar{N}_c) \Delta_R.$$

Therefore, by the definition of the ν-gap (6) and the fact that $\bar{\tilde{N}}_x \bar{D}_x - \bar{\tilde{D}}_x \bar{N}_x = 0$, it holds that

$$\|\Delta_R\|_\infty \leqslant \|\tilde{\Lambda}^{-1}\|_\infty \delta_\nu(G_\Delta, G_x)\|Q^{-1}\|_\infty. \tag{A.7}$$

From (A.6) and the fact that $[\bar{N}_\Delta \;\; \bar{D}_\Delta]^{\mathrm{T}}$ is inner it follows that

$$\|Q^{-1}\|_\infty \leqslant \left\|\begin{bmatrix}\bar{N}_x\\ \bar{D}_x\end{bmatrix}\right\|_\infty + \left\|\begin{bmatrix}\bar{D}_{\mathrm{c}}\\ -\bar{N}_{\mathrm{c}}\end{bmatrix}\right\|_\infty \|\Delta_R\|_\infty$$
$$\leqslant 1 + \|\tilde{\Lambda}^{-1}\|_\infty \delta_\nu(G_\Delta, G_x)\|Q^{-1}\|_\infty,$$

using expression (A.7) in the last step. For all $G_\Delta \in \mathscr{G}_\nu(G_x, \delta_{\nu,G})$ satisfying the Vinnicombe condition $\delta_{\nu,G} + \delta_{\nu,C} \leqslant \|T(G_x, C)\|_\infty$ of Corollary 3, it holds that $1 - \|\tilde{\Lambda}^{-1}\|_\infty \delta_\nu(P_\Delta, P) > 0$, and we have

$$\|Q^{-1}\|_\infty \leqslant (1 - \|\tilde{\Lambda}^{-1}\|_\infty \delta_\nu(G_\Delta, G_x))^{-1}. \tag{A.8}$$

Expressions (A.7) and (A.8) lead to norm bounds of the Youla parameter as in Lemma 1. The remainder of the proof is identical to the one of the previous section.

A.3. Λ-gap metric (Theorem 3)

The proof of Theorem 3 is only a minor adaption to the proof of Theorem 1, after replacing expression (A.2) by

$$\begin{bmatrix}-\bar{D}_{\mathrm{c}}\\ \bar{N}_{\mathrm{c}}\end{bmatrix}\Delta_R \Lambda^{-1} = \begin{bmatrix}\bar{N}_x\\ \bar{D}_x\end{bmatrix}\Lambda^{-1} - \begin{bmatrix}\bar{N}_x\\ \bar{D}_x\end{bmatrix}\Lambda^{-1}Q^{-1}\Lambda^{-1}$$
$$+ \left\{\begin{bmatrix}\bar{N}_x\\ \bar{D}_x\end{bmatrix}\Lambda^{-1} - \begin{bmatrix}\bar{N}_\Delta\\ \bar{D}_\Delta\end{bmatrix}Q_G\right\}Q^{-1}\Lambda^{-1}.$$

Following the proof of Lemma 1 the bounds on the weighted infinity norm $\|\tilde{\Lambda}\Delta_R\Lambda^{-1}\|_\infty$ of the Youla parameter and on the norm $\|Q^{-1}\Lambda^{-1}\|_\infty$ can be formulated.

References

Bombois, X., Gevers, M., & Scorletti, G. (2000). A measure of robust stability for an identified set of parametrized transfer functions. *IEEE Transactions on Automatic Control*, *45*, 2141–2145.

Bongers, P.M.M. (1991). On a new robust stability margin. *Recent Advances in Mathematical Theory of Systems*, *Control*, *Networks and Signal Processing*, *Proceedings of the International Symposium MTNS*-91 (pp. 377–382).

Bongers, P.M.M. (1994). *Modeling and identification of flexible wind turbines and a factorizational approach to robust control*. Ph.D. thesis, Delft Univ. of Techn., Mech. Eng. Systems and Control Group.

de Callafon, R. A., Van den Hof, P. M. J., & Bongers, P. M. M. (1996). A unified approach to stability robustness for uncertainty descriptions based on fractional model representations. *IEEE Transactions on Automatic Control*, *AC-41*, 723–727.

de Callafon, R. A., & Van den Hof, P. M. J. (1997). Suboptimal feedback control by a scheme of iterative identification and control design. *Mathematical Modelling of Systems*, *3*, 77–101.

De Vries, D. K., & Van den Hof, P. M. J. (1995). Quantification of uncertainty in transfer function estimation: A mixed probabilistic-worst-case approach. *Automatica*, *31*, 543–557.

Georgiou, T. T., & Smith, M. C. (1990). Optimal robustness in the gap metric. *IEEE Transactions on Automatic Control*, *AC-35*, 673–686.

Hakvoort, R. G., & Van den Hof, P. M. J. (1997). Identification of probabilistic system uncertainty regions by explicit evaluation of bias and variance errors. *IEEE Transactions on Automatic Control*, *AC-42*, 1516–1528.

Hjalmarsson, H., Gunnarsson, S., & Gevers, M. (1994). A convergent iterative restricted complexity control design scheme. *Proceedings of the 33rd Conference Decision and Control*, Lake Buena Vista, FL (pp. 1735–1740).

Horn, R. A., & Johnson, C. R. (1991). *Topics in matrix analysis*. England: Cambridge University Press.

Kammer, L. C., Bitmead, R. R., & Bartlett, P. L. (2000). Direct iterative tuning via spectral analysis. *Automatica*, *36*, 1301–1307.

Ninness, B. M., & Goodwin, G. C. (1995). Estimation of model quality. *Automatica*, *31*, 1771–1797.

Qui, L., & Davison, E. J. (1992). Feedback stability under simultaneous gap metric uncertainties in plant and controller. *Systems and Control Letters*, *18*, 9–22.

Schrama, R.J.P., Bongers, P.M.M., & Bosgra, O.H. (1992). Robust stability under simultaneous perturbations of linear plant and controller. *Proceedings of the 31st IEEE Conference Decision and Control*, Tucson, AZ (pp. 2137–2139).

Sefton, J. A., & Ober, R. J. (1993). On the gap metric and coprime factor perturbations. *Automatica*, *29*, 723–734.

Tay, T. T., Moore, J. B., & Horowitz, R. (1989). Indirect adaptive techniques for fixed controller performance enhancement. *International Journal of Control*, *50*, 1941–1959.

Van den Hof, P. M. J. (1998). Closed-loop issues in system identification. *Annual Reviews in Control*, *22*, 173–186.

Vidyasagar, M. (1985). *Control systems synthesis: A factorization approach*. Cambridge, USA: MIT Press.

Vinnicombe, G. (1993). Frequency domain uncertainty and the graph topology. *IEEE Transactions on Automatic Control*, *AC-38*, 1371–1383.

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