Dynamic network structure identification with prediction error methods - basic examples

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Abstract: Analysis of dynamical properties of highly complex and interconnected systems becomes important in different fields of science. When identifying the structure and dynamics of a network of dynamical systems, including cause-effect relations, there is a tendency to use nonparametric or FIR models of the output error type. In this paper it is shown, and illustrated by some simple examples, that appropriate attention should be given to using flexible noise models, in order to allow consistent identification of the dynamics, while the use of external excitation/probing signals may reduce this need. It is a first step towards using prediction error identification tools to identify the structure of a network.

Keywords: System identification; Identifiability; Network topologies; Complex systems; Prediction error methods.

1. INTRODUCTION

In the field of system identification one is used to identify a system (and possibly noise dynamics) in a system/measurement/excitation structure that is clearly well defined a priori. One knows upfront where the excitation takes place, where causal transfer functions appear, and where noise sources are assumed to be present. Typically one restricts to well-defined a priori chosen structures:

- Open-loop system with possible excitation on the input, and additive noise disturbance on the output;
- The previous situation, but with noisy measurements of the input also (errors-in-variables approach);
- Feedback configuration, with or without external excitation and having disturbances at particular locations.
- All this can be done scalar (SISO) and/or multivariable (MIMO).

From this standard situation there are two different directions for natural extension of the problem.

(1) If the data generating system is a complex network of connected dynamic subsystems, one would like to be able to identify the dynamics of the subsystems.
A conceptually simple solution could be to write the system back to a centralized form and into one of the classical open-loop or closed-loop schemes and apply standard multivariable identification methods. In this situation the structural properties of the MIMO models would become central. As a result new model concepts of structured models should be explored, in their use, suitability and consequences for identification purposes. Some contributions to identification of structured systems can be found in Gudi and Rawlings [2006], Leskens and Van den Hof [2007], Wahlberg et al. [2009], Massioni and Verhaegen [2008].
(2) If the interconnection structure of the subsystems is not known a priori and should be identified also on the basis of measurement data, there appears a problem of structure or topology identification. The basic question then becomes, under which conditions can we identify the connection structure of individual subsystems that form an interconnected system.

In this paper we are going to elaborate on this second problem, that seems to have not been addressed extensively in the identification literature.

Early contributions to this problem date back to Anderson and Gevers [1982], Gevers and Anderson [1981, 1982] who on the basis of the work of Granger [1969] and Caines and Chan [1975], Caines [1976] address the question whether an open-loop or closed-loop structure is present between two measured signals u and y. The conclusion was that for particular structures (two measured signals driven by two independent noise sources of similar dimensions), a unique separation of forward (system) and backward (feedback) path could be determined under fairly general conditions. This basically led to the joint i-o approach of closed-loop identification.

More generalized system configurations do not seem to have been addressed in the identification literature. How-

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ever currently interest has been renewed in trying to detect the topology of more complex networks based on measurements of the system (Friedman et al. [2010], Yuan et al. [2010], Sanandaji et al. [2011], Materassi and Innocenti [2010], Materassi et al. [2011] among many others).

In these papers the data generating system is assumed to be described by a node-and-link style of network, as shown in Fig. 1. The goal is to recover the topology using only time records of the variables $w_i, i = 1, \ldots, L$. The output of each node is:

$$w_i = \sum_{\substack{j=1 \\ j \neq i}}^{L} G_{ij}(q) w_j + v_i \tag{1}$$

with v_i a (stochastic) noise disturbance acting on each node, and q is the standard shift operator, $qw_i(t) = w_i(t+1)$.

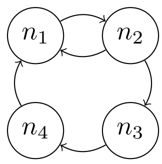

Fig. 1. The node-and-link style network used in topology identification. Each node n_i represents a signal w_i, while the arrows represent causal (transfer function) relationships.

Most often nonparametric or FIR output error models are used to model the dynamics of the links $G_{ij}(q, \theta)$ Friedman et al. [2010], Yuan et al. [2010], Sanandaji et al. [2011], Materassi and Innocenti [2010], Materassi et al. [2011]. Materassi and Innocenti [2010] identify a tree-based structure, where in each node an additive noise disturbance is present, and only passive data is taken from the network. Non-causal Wiener filters determine the dynamic components in the tree structure. This work is extended in Innocenti and Materassi [2008], Materassi et al. [2011] towards a more general interconnection structure, and to the use of causal Wiener filters as models. In Sanandaji et al. [2011] a similar structure is considered but FIR models are employed. Sometimes there are assumptions on the present excitation of each node, and the possibility to perform experiments (Timme [2007]).

In this paper we will consider a general approach to detect the network topology, similar to the approach in Materassi et al. [2011], and we will analyze some consequences of the use of the particular model structure (1).

First in Section 2 we will sketch the identification approach, and then in Sections 3 and 4 we will show some of its consequences for a simple cascaded system topology. In Section 5 a closed loop structure will be discussed. Some discussion on the benefits of external excitation signals will follow in Section 6. Finally some remarks and discussion will conclude the paper.

2. IDENTIFICATION SETUP

Consider a data generating system which is a network of nodes as shown in Fig. 1. For a network with L nodes, the output of node i can be written as:

$$w_i = \sum_{\substack{j=1 \\ j \neq i}}^{L} G_{ij}(q) w_j + v_i. \tag{2}$$

with v_i a stochastic noise disturbance, determined by

$$v_i(t) = H_i(q) e_i(t)$$

where for every $i = 1, ..L$, H_i is a monic stable filter with a stable inverse, and e_i is a white noise process, while e_i and e_j are independent for $i \neq j$. The variance of v_i is denoted by $\sigma_{v_i}^2$.

The structure of a network is considered as the interconnections between the subsystems of the network. Since the measurements are assumed to have been taken at the outputs of the dynamic subsystems, detecting the structure is equivalent to detecting non-zero dynamics between the given measurements. If it is decided in some manner that the transfer function from signal j to signal i is in fact zero, then the conclusion is that there is no connection between these two nodes, implicitly identifying the topology.

In order to estimate these transfers, we model this process with a standard one-step ahead prediction model (Ljung [1999]):

$$\hat{w}_i(t|t-1, \theta) = H_i^{-1}(q, \theta) \big(\sum_{\substack{j=1 \\ j \neq i}}^{L} G_{ij}(q, \theta) w_j \big) + \\ + (1 - H_i^{-1}(q, \theta)) w_i. \tag{3}$$

and we analyze the use of a mean square error approach, by considering the identification criterion:

$$\bar{V}_i(\theta) = \lim_{N \to \infty} \frac{1}{N} \sum_{t=0}^{N-1} E[\big(w_i(t) - \hat{w}_i(t|t-1, \theta)\big)^2], \tag{4}$$

of which the minimization leads to the (asymptotic) parameter estimate $\hat{\theta}$.

In Materassi et al. [2011] this problem has been analyzed on the basis of (causal) Wiener filter estimates of the transfers functions G_{ij}. It can be shown that in the prediction-error framework the optimal causal Wiener filter is a special case of the optimal FIR model estimate, with the model order tending to infinity, in case of an Output Error (OE) model structure, i.e. fixing the noise model $H(q, \theta)$ equal to 1. As a result, we will proceed using the prediction error framework, and the analysis tools available in this framework, but all the results apply to the causal Wiener filter approach as well. For a proof of this statement see the appendix.

Our investigation will consist of studying some very simple networks, and studying what the effects of certain assumptions are on the ability of the proposed method to recover the network topology. The first simple network considered will be an open-loop cascaded network. The second will be a closed loop network.

3. A OPEN-LOOP CASCADE EXAMPLE - OE MODELS

Consider a data generating system to be composed of a concatenation of two systems, according to:

$$w_1 = G_1 r + v_1 \tag{5}$$

$$w_2 = G_2 w_1 + v_2, \tag{6}$$

with G_1, G_2 proper transfer functions.

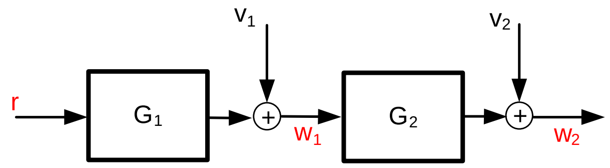

Fig. 2. Example cascade system with measured signals in red.

We are going to identify this system, but without any a priori information on its structure. We will only use the information that we have 3 measured signals w_1, w_2, r, with r being persistently exciting of a sufficiently high order.

We will focus our attention to identifying the dynamics between measurement w_1 as the output, and measurements w_2 and r as the inputs using the predictor:

$$\hat{w}_1(t|t-1,\theta) = G_{12}(q,\theta)w_2 + G_{1r}(q,\theta)r.$$

This predictor is the most interesting: since we are using a causal predictor, we expect the estimated $G_{12}(q,\theta)$ between w_1 and w_2 to be zero, and $G_{1r}(q,\theta)$ between w_1 and r to be non zero.

To find the asymptotical estimates of G_{12} and G_{1r} we analyze the prediction error $\varepsilon_1(t,\theta)$ for signal w_1:

$$\begin{aligned}\varepsilon_1(t,\theta) &= w_1(t) - \hat{w}_1(t|t-1,\theta)\\ &= w_1(t) - G_{12}(\theta)w_2(t) - G_{1r}(\theta)r(t)\\ &= [G_1 - G_{12}(\theta)G_2G_1 - G_{1r}(\theta)]r(t) +\\ &\quad +(1-G_{12}(\theta)G_2)H_1e_1(t) - G_{12}(\theta)H_2e_2(t).\end{aligned} \tag{7}$$

Next we will consider three different cases:

- G_1 and G_2 are strictly proper and v_1 and v_2 are white.
- G_1 and G_2 are strictly proper and v_1 and v_2 are colored.
- G_1 and G_2 are not strictly proper and v_1 and v_2 are white.

3.1 Case 1: G_1 and G_2 are strictly proper and v_1 and v_2 are white

In this case $H_1 = H_2 = 1$. Note that since G_2 is strictly proper, the smallest value of e_1-dependent term in the power of (7), is achieved when the filter that operates on e_1 is equal to 1. With $H_1 = 1$ it is clear then that the minimum power of the prediction error is achieved for

$$G_{12}(\hat{\theta}) = 0 \tag{8}$$

$$G_{1r}(\hat{\theta}) = G_1 \tag{9}$$

provided that these choices are present in the considered model set, and provided that $\sigma_{v_1}, \sigma_{v_2} > 0$.

When considering all possible situations of values of $\sigma_{v_1}, \sigma_{v_2}$ we obtain the results as indicated in the following table:

σ_{v_1} / σ_{v_2}	>0 / >0	>0 / $=0$	$=0$ / >0	$=0$ / $=0$
$G_{12}(\hat{\theta})$	0	0	0	nonunique
$G_{1r}(\hat{\theta})$	G_1	G_1	G_1	nonunique

Table 1. Asymptotic results in Case 1 (strictly proper transfers and white noises).

As a conclusion we can observe that as long as one of the two noise sources is present, the dynamics is consistently estimated, and the correct structure is identified.

3.2 Case 2: G_1 and G_2 are strictly proper and v_1 and v_2 are colored

In this case H_1 and H_2 are not equal to 1. When minimizing the power of (7), then the term with v_1 is not necessarily minimized for $G_{12}(\hat{\theta}) = 0$. In that situation $G_{12}(\hat{\theta})$ will be determined to minimize the sum of the two noise terms, and $G_{1r}(\hat{\theta})$ is then adjusted to equate the first term to 0. As a result $G_{12}(\hat{\theta})$ will be dependent on the relation between the noise powers σ_{v_1} and σ_{v_2}, and therefore it will generally be not consistent. The estimate becomes consistent though if $\sigma_{v_1} = 0$, since then the e_2-dependent term will force $G_{12}(\hat{\theta})$ to become 0.

σ_{v_1} / σ_{v_2}	>0 / >0	>0 / $=0$	$=0$ / >0	$=0$ / $=0$
$G_{12}(\hat{\theta})$	biased	biased	0	nonunique
$G_{1r}(\hat{\theta})$	biased	biased	G_1	nonunique

Table 2. Asymptotic results in Case 2 (strictly proper transfers and colored noises).

As a conclusion we can observe that in general the estimation results will be biased, which will limit the determination of the topology. Only in the situation of absence of the noise source v_1, the structure is correctly retrieved.

3.3 Case 3: G_1 and G_2 are not strictly proper and v_1 and v_2 are white

In this case again $H_1 = H_2 = 1$. If G_2 is not strictly proper, then a choice $G_{12}(\hat{\theta}) = G_2^{-1}$ would actually allow the e_1-dependent term in the power of (7) to become 0. However since $G_{12}(\theta)$ appears also in the e_2-term, the estimate $G_{12}(\hat{\theta})$ will generally be biased, and dependent on the relation between the two noise powers of e_1 and e_2.

σ_{v_1} / σ_{v_2}	>0 / >0	>0 / $=0$	$=0$ / >0	$=0$ / $=0$
$G_{12}(\hat{\theta})$	biased	G_2^{-1}	0	nonunique
$G_{1r}(\hat{\theta})$	biased	0	G_1	nonunique

Table 3. Asymptotic results in Case 3 (not strictly proper transfers and white noises).

As a conclusion we can observe that in general the estimation results will be biased, which will limit the determination of the correct topology. Only in the situation of absence of the noise source e_1, the structure is correctly retrieved.

3.4 Discussion

The results of this example show that the correct structure can be retrieved from the measured signals, in the situation that the dynamic links are strictly proper, and the noise sources are correctly modelled. In the prediction error framework, this latter condition is formulated as: the system should be included in the model set. Whereas in classical open-loop system identification it is possible to consistently identify plant transfers with output error models, even if the measured signals are disturbed by colored noises, apparently this property disappears in the structure identification problem.

4. A OPEN-LOOP CASCADE EXAMPLE - PARAMETRIZED NOISE MODELS

A simple remedy to the problem indicated in the previous section is the inclusion of appropriate noise models in the considered model structure, leading to

$$\begin{aligned}\varepsilon_1(t,\theta) = H_1(\theta)^{-1}\left[(G_1 - G_{12}(\theta)G_2G_1 - G_{1r}(\theta))r + \right.\\ \left. + (1 - G_{12}(\theta)G_2)H_1e_1 - G_{12}(\theta)H_2e_2\right].\end{aligned}$$

For Case 1 (white noise disturbance signals), the situation will essentially stay the same, but for Case 2, where the noise processes are non-white, the flexibility in $H_1(\theta)$ now allows the minimizing solution $G_{12}(\hat{\theta}) = 0$ (which minimizes the power of the e_2-term), and $H_1(\hat{\theta}) = H_1$ (which minimizes the power of the e_1-term). The new results for Case 2 now become:

σ_{v_1} / σ_{v_2}	>0 / >0	>0 / $=0$	$=0$ / >0	$=0$ / $=0$
$G_{12}(\hat{\theta})$	0	0	0	nonunique
$G_{1r}(\hat{\theta})$	G_1	G_1	G_1	nonunique

Table 4. Asymptotic results in Case 2 (strictly proper transfers and colored noises) - with parametrized noise models.

Clearly the incorporation of parametrized noise models allows the detection of the correct structure also when colored noises are present.

The results for Case 3, not strictly proper transfer functions G_1 and G_2, however stays the same as before. In this case no correct structure is identified.

5. A CLOSED-LOOP EXAMPLE

By adding a third link to our data generating system, we can move to a closed-loop structure, as depicted in Figure 3:

The system equations only have one additional term, in comparison to the cascade example from the previous section.

$$w_1 = G_1 r + v_1 - G_3 w_2 \quad (10)$$

$$w_2 = G_2 w_1 + v_2. \quad (11)$$

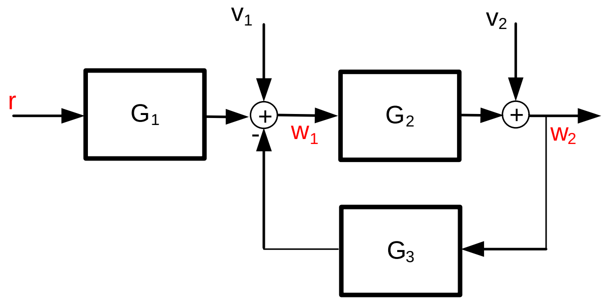

Fig. 3. Example closed-loop system.

With the sensitivity function:

$$S := \frac{1}{1 + G_3G_2}$$

the system equations read:

$$\begin{aligned}w_1 &= G_1Sr + Sv_1 - G_3Sv_2\\ w_2 &= G_2G_1Sr + G_2Sv_1 + Sv_2.\end{aligned}$$

Using the same model structure as in the previous section, the prediction error for signal w_1 now becomes:

$$\begin{aligned}\varepsilon_1(t,\theta) &= H_1(\theta)^{-1}\left[w_1(t) - G_{12}(\theta)w_2(t) - G_{1r}(\theta)r(t)\right]\\ &= H_1(\theta)^{-1}\left[(G_1S - G_{12}(\theta)G_2G_1S - G_{1r}(\theta))r + \right.\\ &\quad \left. + S(1 - G_{12}(\theta)G_2)H_1e_1 - S(G_3 + G_{12}(\theta))H_2e_2\right]\end{aligned}$$

For strictly proper transfers G_1, G_2, G_3 and colored noises v_1, v_2 we now arrive at the following results:

σ_{v_1} / σ_{v_2}	>0 / >0	>0 / $=0$	$=0$ / >0	$=0$ / $=0$
$G_{12}(\hat{\theta})$	biased	nonunique	$-G_3$	nonunique
$G_{1r}(\hat{\theta})$	biased	nonunique	G_1	nonunique

Table 5. Asymptotic results in Case 2 (strictly proper transfers and colored noises), with FIR/OE models.

σ_{v_1} / σ_{v_2}	>0 / >0	>0 / $=0$	$=0$ / >0	$=0$ / $=0$
$G_{12}(\hat{\theta})$	$-G_3$	nonunique	$-G_3$	nonunique
$G_{1r}(\hat{\theta})$	G_1	nonunique	G_1	nonunique

Table 6. Asymptotic results in Case 2 (strictly proper transfers and colored noises), with parametrized noise models.

In the current situation the estimate $G_{12}(\hat{\theta}) = -G_3$ and $G_{1r}(\hat{\theta}) = G_1$ corresponds to the correct structure. It is correctly identified in the situation that all noise sources are present, only when parametrized noise models are applied. With OE-models, a bias occurs in the estimates. A second opportunity for estimating the correct structure is when there is no noise present on the considered signal ($\sigma_{v_1} = 0$).

6. POTENTIAL BENEFIT OF EXCITATION SIGNALS

In our examples and analysis presented so far we have not made explicit use of the fact that the excitation signal r is at our disposal for probing the network. Actually in all the results presented in the previous sections, r is actually handled as just any other node signal.

However when we have an external probing signal available for probing the network, then this can provide additional information. Actually this phenomenon comes down to inserting the prior knowledge that the particular node signal only acts as input in the network and not as output.

In closed-loop identification the benefit of excitation signals is well known (Van den Hof [1998], Forssell and Ljung [1999]). It allows us to identify plant dynamics without having the necessity to model all noises correctly, through indirect methods. In a related phrase, it allows us to apply instrumental variable types of techniques, where the external excitation signals can act as external (instrumental) signals that are uncorrelated to all noise sources in the network (Gilson and Van den Hof [2005]). In order to utilize this in a network identification problem, a different model structure and a different identification criterion has to be considered.

As an example we use the closed-loop situation of Section 5, and we consider the problem of identifying the transfer G_{21}.

Consider now the predictor

$$\hat{w}_2(t|t-1,\theta) = G_{21}(\theta)w_1$$

which means that we have removed the information from the excitation signal r.

The prediction error then becomes:

$$\begin{aligned}\varepsilon_2(t,\theta) = {} & (G_1G_2S - G_{21}(\theta)G_1S)r + \\ & +S(G_2 - G_{21}(\theta))v_1 + S(1+G_{21}(\theta)G_3)v_2.\end{aligned}$$

Applying an instrumental variable (IV) method for estimation now comes down to setting:

$$\hat{\theta} = \mathrm{sol}_\theta \lim_{N\to\infty} \frac{1}{N}\sum_{t=0}^{N-1} \varepsilon_2(t,\theta)r(t-k) = 0 \quad \text{for } k = 1,2,3....$$

If r is an external excitation signal, and uncorrelated with the noise signals v_1, v_2, the solution will be determined by

$$(G_1G_2S - G_{21}(\hat{\theta})G_1S) = 0$$

implying that $G_{21}(\hat{\theta}) = G_2$. This correct result is obtained without any modelling of the (colored) noises v_1 and v_2.

It is conjectured that this mechanism can provide appropriate information for detecting the network structure, while refraining from extensive noise modelling. Results of this approach will be presented in future work.

7. CONCLUSIONS

In this paper we have touched upon the problem of identifying the network topology of a network of dynamical systems, when using classical identification tools, i.c. prediction error methods. FIR/Output error models or equivalently causal Wiener filter models, that are often used for this purpose, are shown to lead to biased results for the dynamic transfers in the network, if the noise contributions are non-white. And biased model estimates seem to be a weak basis for detecting the network structure. A solution exists in the use of parametrized noise models. The results of this have been illustrated in two simple examples. Additionally the particular role of external excitation/probing signals has been discussed, showing the opportunity to identify the network structure while circumventing parametrized noise modelling.

APPENDIX

Proposition 1. Let the prediction model (3) for the ith node be the FIR predictor:

$$\hat{w}_i(t|t-1,\theta) = \sum_{\substack{j=1\\ j\neq i}}^{L} G_{ij}(q,\theta)w_j = \sum_{\substack{j=1\\ j\neq i}}^{L} W_j\theta_{ij}$$

where $G_{ij}(q,\theta) = \sum_{k=0}^{n_b}\theta_{ij}(k)q^{-k}$ and

$$W_j = \begin{bmatrix} w_j(0) & w_j(-1) & \dots & w_j(-n_b) \\ w_j(1) & w_j(0) & \dots & w_j(-n_b+1) \\ \vdots & \vdots & & \vdots \\ w_j(N-1) & w_j(N-2) & \dots & w_j(N-1-n_b)\end{bmatrix}.$$

Suppose that all the signals w_i are persistently exciting (i.e. each matrix W_i has full rank for any n_b). Moreover, assume that each signal w_i contains an external excitation that is independent of the other signals w_j, $j \neq i$. Then the asymptotically optimal $\hat{\theta}_i$ is a MISO causal wiener filter.

Proof. Due to the conditions assumed on the signals w_i, the matrix $W = [W_1 \; \dots \; W_L]$ has full rank for any value of n_b. The optimal θ satisfies the equations:

$$(W^TW)\hat{\theta} = W^Tw_1$$

taking the expected value, and letting $N \to \infty$ results in:

$$\lim_{N\to\infty}\frac{1}{N}\mathbb{E}[(W^TW)\theta] = \lim_{N\to\infty}\frac{1}{N}\mathbb{E}[W^Tw_1]$$

which in turn results in

$$\begin{bmatrix} R_{w_2,w_2} & \dots & R_{w_2,w_L} \\ \vdots & & \vdots \\ R_{w_L,w_2} & \dots & R_{w_L,w_L}\end{bmatrix}\begin{bmatrix}\hat{\theta}_{12} \\ \vdots \\ \hat{\theta}_{1L}\end{bmatrix} = \begin{bmatrix}\breve{R}_{w_2,w_1} \\ \vdots \\ \breve{R}_{w_L,w_1}\end{bmatrix}$$

where

$$R_{w_i,w_j} = \begin{bmatrix} R_{w_i,w_j}(0) & R_{w_i,w_j}(1) & \dots \\ R_{w_i,w_j}(1) & R_{w_i,w_j}(0) & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

$$\breve{R}_{w_i,w_1} = \begin{bmatrix} R_{w_i,w_1}(0) \\ R_{w_i,w_1}(1) \\ \vdots \end{bmatrix}$$

where $R_{x,y}(\tau)$ is the crosscorrelation function between the signals x and y.

These are the famous Wiener-Hopf Equations, and their solution is the multi-input, single-output Wiener Filter Benesty [2008].

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