Control Systems Lab (SC4070)
Inverted Wedge (Balance) Experiment

Description

The inverted wedge (also called balance) setup consists of a cart driven by a DC motor. The motor can steer the cart left and right on a track approximately one meter long. The track itself can freely rotate in the plane coinciding with the direction of motion of the cart. The objective is to control the motion of the cart such that the track is balanced at a desired angle. The schematic diagram in Figure 1 shows the construction of the system including all the relevant parameters and variables. Positive directions of variables are indicated by arrows.

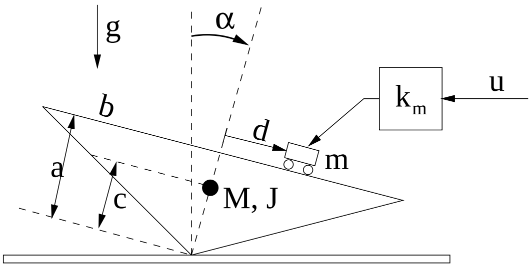

Figure 1: Schematic drawing of the inverted wedge.

This system has one control input u, which is the force that accelerates the cart left or right (delivered by the motor). This input is commanded from the computer and is scaled between -1 (corresponds to the maximal force moving the cart to the left) and +1 (corresponds to the maximal force moving the cart to the right).

There are two measured outputs: d – the position of the cart, and α – the angle of the track. These measurements are given in their physical units – meters and radians, respectively.

The physical parameters of the system are listed in Table 1. Most of the values can easily be determined by measuring the dimensions and masses (such as the height of the track, the mass of the cart, etc.). The input-to-force gain k_m can be computed from the motor specifications and the gains of the interface amplifiers.

The value of the damping coefficient b (including viscous friction and the back-emf of the motor) is not known a priori and can only be determined experimentally (an estimated range is given in Table 1. It is your task to devise and carry out an identification experiment to obtain a more accurate value for this damping coefficient b.

Another parameter that cannot be accurately measured is the distance from the center of gravity of the track to the point of rotation c (the position of the center of gravity is unknown). Another identification experiment would be needed to obtain an estimate for this parameter. As only limited time is available in the lab, a reasonable value is already provided in Table 1. You may of course think about a suitable experiment and if time permits you may verify the given value.

Table 1: Physical parameters and their values.

Symbol	Parameter	Value
g	acceleration due to gravity	9.81 ms^{-2}
a	height of track	0.11 m
c	distance from center of gravity to point of rotation	0.045 m
m	mass of cart	0.49 kg
M	mass of balance	3.3 kg
J	inertia of balance	0.42 kgm^2
k_m	input-to-force gain	5.0 N
b	damping coefficient	4 to 10 kgs^{-1}

Control Objective

Design a controller that makes the angle α of the balance track follow a specified reference trajectory. The controlled system should have zero steady state error in α and adequate disturbance rejection properties, i.e., it should be able to recover from a small tick against the track.

Physical Modeling

The nonlinear model equations are given below. They have been derived by using the Euler–Lagrange equations, neglecting rotational viscous friction, translational Coulomb friction and stiction and the dynamics of the motor electrical circuit (armature).

$$\ddot{d} = \frac{1}{m}\Big(k_m u - ma\ddot{\alpha} - b\dot{d} + md\dot{\alpha}^2 + mg\sin(\alpha)\Big) \quad (1)$$

$$\ddot{\alpha} = \frac{1}{J + ma^2 + md^2}\Big(-ma\ddot{d} - 2\dot{\alpha}md\dot{d} + mga\sin(\alpha) + mgd\cos(\alpha) + Mgc\sin(\alpha)\Big) \quad (2)$$

The term $ma^2 + md^2$ is a point mass approximation of the added inertia due to the cart.

Note that $\ddot{\alpha}$ depends on $\ddot{d}$ and vice versa: this is called an algebraic loop. Simulink will display warnings when you simulate these equations and the simulation will be slower (the algebraic loop must be solved numerically in each simulation step). It is therefore advised to break the algebraic loop either by neglecting the term $a\ddot{\alpha}$ in Equation 1 or by inserting a delay of one integration step (block called “Memory”, in the library of continuous-time blocks).

Simulink Template

A Simulink template `baltemplate.mdl` contains the necessary real-time interface blocks and some scopes. Make your own copy of this file and use it as a starting point for your experiments. Before starting the first simulation, define the sampling period h as a variable in the workspace of MATLAB. Always use the red button “Stop” to stop the system before you terminate a simulation. Use the other buttons to move the cart to a desired initial position.