# Written Exam Knowledge Based Control Systems (SC 4081)
# 26 June 2015 from 9:00 until 12:00

**Onderstaande aanwijzingen nauwkeurig lezen.**

- Schrijf je naam en studienummer duidelijk leesbaar op ieder blad dat je inlevert.
- Lees iedere vraag goed alvorens te antwoorden. Het is **niet** toegestaan om het dictaat, boeken, aantekeningen, en andere hulpmiddelen te gebruiken (behalve rekenmachines).
- Praat nooit met je buurman om welke reden dan ook: het tentamen wordt in dit geval meteen ingenomen.
- Het tentamen bestaat uit 7 vraagstukken. De vraagstelling is in het Engels, antwoorden mogen zowel in het Nederlands als in het Engels zijn. Motiveer elk antwoord, schrijf je redeneringen op papier. Bij elke vraag staat tussen haakjes het maximaal te behalen aantal punten aangegeven (totaal = 100). Het aantal punten bepaalt het cijfer.
- Succes!

**Read carefully the instructions below.**

- *Write clearly your name and student number on each sheet you hand in.*
- *Read carefully each question prior to formulating an answer. You are* **not** *allowed to use the course lecture notes, books, your own notes and other aids (except for calculators).*
- *Never talk to your fellow students: violation of this rule will result in an immediate termination of your exam.*
- *The exam consists of 7 questions. The questions are formulated in English, answers can be in English as well as in Dutch. Motivate each answer, write down your reasoning. For each question, the maximum number of points you can get is indicated in parentheses (total =100). The number of points gained will determine the final mark.*
- *Good luck!*

Delft University of Technology, Delft Center for Systems and Control

1. Fuzzy set $A$ is described by the following membership function on the finite discrete domain X = {−3,−2,−1, 0, 1, 2, 3}:

$$\mu_A(x) = \frac{5-|x|}{10}$$

a) Is $\mu_A(x)$ a valid membership function? Represent the above fuzzy set as:

i) a list of "membership degree/element" pairs **(3 p)**

ii) a list of "α / α-cut" pairs **(2 p)**

b) Defuzzify $A$ by the center-of-gravity method and the mean-of-maxima method. (5 **p**)

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2. Given is the following singleton fuzzy model:

1) **If** $x$ is Small **then** $y_m = b_S$
2) **If** $x$ is Large **then** $y_m = b_L$

with $b_S = 1$, $b_L = 2$ and the antecedent membership functions are as in Figure 1.

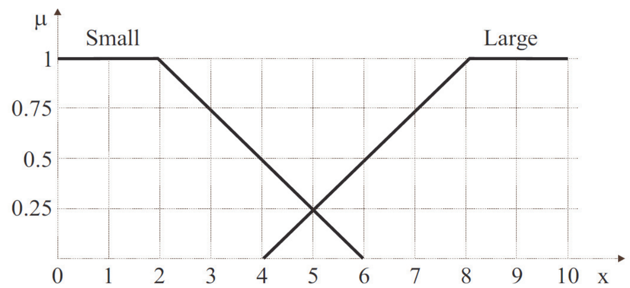


**Figure 1.** Membership functions.

Furthermore, the following input–output data pairs are given:

$(x_1, y_1) = (10, 2)$, $(x_2, y_2) = (5,1)$, $(x_3, y_3) = (1,0.5)$

a) State the formula to compute the output $y_m$ of the model. Explain **all** the symbols. **(5 p)**

b) Give the equation to compute the summed squared error in general (explain all symbols) and compute the error for the given data set. **(5 p)**

c) Draw a scheme of a fuzzy-neural network corresponding to the given singleton model. Explain the scheme. **(5 p)**

3.

a) Show how training and test errors behave as a function of the number of rules (R) in a Takagi-Sugeno model. Draw a plausible pair of training and test error curves as function or R. In the plot clearly mark regions of "R" where Takagi-Sugeno under-fits and where it over-fits. **(5 p)**

b) Consider the Takagi Sugeno model with 2 rules:

$$\textbf{\textit{If}}\ z \textit{ is low } \textbf{\textit{then}}\ \dot{x} = A_1 x$$

$$\textbf{\textit{If}}\ z \textit{ is high } \textbf{\textit{then}}\ \dot{x} = A_2 x$$

where $A_1 = \begin{pmatrix} -3 & 1 \\ 0 & -2 \end{pmatrix}$, $A_2 = \begin{pmatrix} -2 & 0 \\ 16 & -1 \end{pmatrix}$, $\mu_{\text{low}}(z) = -0.5z + 0.5$, $\mu_{\text{high}}(z) = 1 - \mu_{\text{low}}(z)$, $z \in [-1,1]$. Is the fuzzy model well-defined? Are the local models stable? Is the global model unstable? Explain your answers. **(10 p)**

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4. A nonlinear process is controlled by a linear proportional controller according the scheme given in Figure 2.

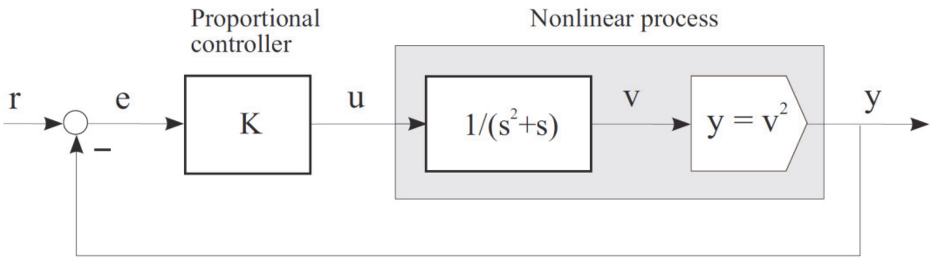


**Figure 2.** A nonlinear process controlled by a linear proportional controller.

Due to the quadratic nonlinearity, the closed-loop behavior strongly depends on the value of the output, which is not acceptable. Your task is to design a *fuzzy supervisory controller* in order to minimize the influence of the nonlinearity on the closed-loop performance.

a) Draw a block diagram of the closed loop system including the fuzzy supervisor. Discuss explicitly the function of the individual blocks and the rationale behind the choice of the supervisor's input and output variables. **(5 p)**

b) Describe the steps of the design procedure. What parameters must be determined/tuned and how would you do it? **(5 p)**

c) Give an example of three fuzzy rules in the supervisor's rule base, sketch the membership functions and give an indication for the choice of the corresponding consequent parameters. **(5 p)**

Give all details one would need for the implementation of the supervisor. Numerical values or ranges can be freely chosen.

5. Consider the following data set in $\mathbb{R}^2$

$$Z = \begin{bmatrix} -2 & -1 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

a) Give an example of a fuzzy partition matrix for two clusters. Give an example of a possibilistic partition matrix for the same number of clusters and explain the differences between the two partitions. Which one represents better the given data? **(5 p)**

b) State the objective functional of the fuzzy c-means algorithm. Explain all symbols and the rationale behind this functional, what does one actually minimize in fuzzy *c*-means?. **(5 p)**

c) Outline the three steps of the fuzzy *c*-means iteration. **(5 p)**

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6. a) Draw a diagram and give the input–output equation of a radial basis function (RBF) network. Explain all the symbols. **(5 p)**

b) Consider the neural network given in Figure 3. In the diagram, Σ denotes summation, $\sigma$ is a nonlinear differentiable activation function and $a$, $b$, $c$ and $d$ are adjustable weights.

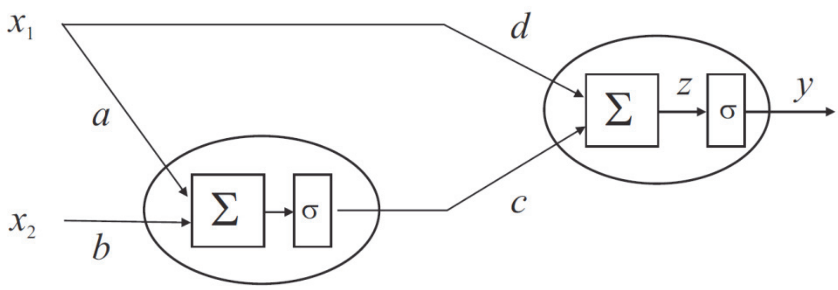


**Figure 3.** Artificial neural network.

Give the input–output equation of this network. **(5 p)**

c) Explain using words and equations how the backpropagation algorithm works. You may use the network in Figure 3 as an example. **(5 p)**

7.

a) Explain the main idea behind reinforcement learning (RL). In the RL context, explain the terms 1) *environment*, 2) *state*, 3) *Markov property*, 4) *agent*, 5) *policy*, 6) *immediate and cumulative reward*, 7) *discount rate*, 8) *exploration*. Use formulas and equations to make your explanation clear. **(5 p)**

b) Consider the problem of finding the optimal route from home to the University, starting the trip always at 8:00am sharp, by bicycle. You want to minimize travel time but also burn some calories. Explain how RL could be applied to solve this problem by:

1) providing concrete examples for this problem of the 8 notions under question 7.a). **(5 p)**

2) and providing details of the RL algorithm that you would implement to solve this problem, **(5 p)**