

# Lecture 1: Introduction & Fuzzy Control I

*Jens Kober*   Robert Babuška

Knowledge-Based Control Systems (SC42050)

Cognitive Robotics

3mE, Delft University of Technology, The Netherlands

12-02-2018

# Lecture Outline

- ① General information about the course
- ② Introduction
- ③ Fuzzy control I

# Course Information

# Knowledge-Based Control Systems (SC42050)

- **Lecturers:**
  - Jens Kober, lectures 1-6
  - Tim de Bruin, lectures 7 & 8
  - Hans Hellendoorn, lecture 9
- **Assistants:** Thijs Greevink
- **Lectures:** (9 lectures = 18 hours)
  - Monday (15:45 – 17:30) in lecture hall Chip at EWI
  - Wednesday (15:45 – 17:30) in lecture hall Chip at EWI

# Knowledge-Based Control Systems (SC42050)

- **Examination:** (check yourself the dates and times!):
  - April 20th 2018, 9:00-12:00.
  - June 29th 2018, 9:00-12:00.

Exam constitutes 60% of the final grade, remaining 40% are two assignments: **Literature and Practical assignment**

- To obtain the credits of this course:  
**Each activity must be approved.**

# Practical Assignment

## Objectives:

- Get additional insight through Matlab/Python implementation.
- Apply the tools to practical (simulated) problems.

The assignment consists of three problems: fuzzy control, neural networks, and reinforcement learning.

Work in groups of two students, more information later.

Will be handed out on [February 19th 2018](#)

Report deadline [April 11th 2018](#)

# Literature Assignment

## Objectives:

- gain knowledge on recent research results through literature research
- learn to effectively use available search engines
- write a concise paper summarizing the findings
- present the results in a conference-like presentation

Deadlines – [March 21st](#), [March 28th](#), and [April 3rd 2018](#)

Symposium: Reserve the whole afternoon [Tuesday April 3rd 2018](#)

Work in groups of four students.

Choose subject via Brightspace → SC42050 → Literature assignment –  
Do it this week!

# Goals and Content of the Course

## knowledge-based and intelligent control systems

- 1 Fuzzy sets and systems
- 2 Data analysis and system identification
- 3 Knowledge based fuzzy control
- 4 Artificial neural networks
- 5 Gaussian Processes (new)
- 6 Control based on fuzzy and neural models
- 7 Basics of reinforcement learning
- 8 Reinforcement learning for control
- 9 Applications



# Course Material



- Lecture notes
- Items available for download at:  
[www.dcsc.tudelft.nl/~sc42050](http://www.dcsc.tudelft.nl/~sc42050)
  - Transparencies as PDF files
  - Demos, examples, assignments with Matlab/Simulink
- Brightspace

The entire content of the lectures and lecture notes<sup>1</sup> will be examined!

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<sup>1</sup>Chapters marked with '\*' are not relevant for the exam

## Where to run Matlab

- Own PC: Campus Licence.
- Computer rooms at 3mE.
- Computer rooms of other faculties (e.g., at Drebbelweg)

# Prerequisites, Background Knowledge

- Mathematical analysis
- Linear algebra
- Basics of control systems (e.g., Control Systems)

# Motivation for Intelligent Control

# Pro's and Con's of Conventional Control

- + systematic approach, mathematically elegant
- + theoretical guarantees of stability and robustness
  
- time-consuming, conceptually difficult
- control engineering expertise necessary
- often insufficient for nonlinear systems

# When Conventional Design Fails

- no model of the process available
  - mathematical synthesis and analysis impossible
  - experimental tuning may be difficult
  
- process (highly) nonlinear
  - linear controller cannot stabilize
  - performance limits

## Example: Stability Problems

$$\frac{d^3y(t)}{dt^3} + \frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} = y^2(t)u(t)$$

Use Simulink to simulate a proportional controller (nlpid.m)

## Example: Stability Problems

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Use Simulink to simulate a proportional controller (nlpid.m)

### Conclusions:

- stability and performance depend on process output
- re-tuning the controller does not help
- nonlinear control is the only solution



# Intelligent Control

techniques motivated by human intelligence

- fuzzy systems (represent human knowledge, reasoning)
- artificial neural networks (adaptation, learning)
- genetic algorithms (optimization)
- particle swarm optimization
- etc.

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  - etc.
- 
- Fuzzy knowledge-based control
  - Fuzzy data analysis, modeling, identification
  - Learning and adaptive control (neural networks)
  - Reinforcement learning

# Fuzzy Control I

# Outline

- ① Fuzzy sets and set-theoretic operations
- ② Fuzzy relations
- ③ Fuzzy systems
- ④ Linguistic model, approximate reasoning

# Fuzzy Sets and Fuzzy Logic

Relatively new methods for **representing** uncertainty and **reasoning** under uncertainty.

Types of uncertainty:

- chance, randomness (stochastic)
- imprecision, vagueness, ambiguity (non-stochastic)

# Classical Set Theory

A **set** is a collection of objects with a common property.

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Examples:

- Set of natural numbers smaller than 5:  $A = \{1, 2, 3, 4\}$

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- Unit disk in the complex plane:  $A = \{z | z \in \mathbb{C}, |z| \leq 1\}$



# Classical Set Theory

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Examples:

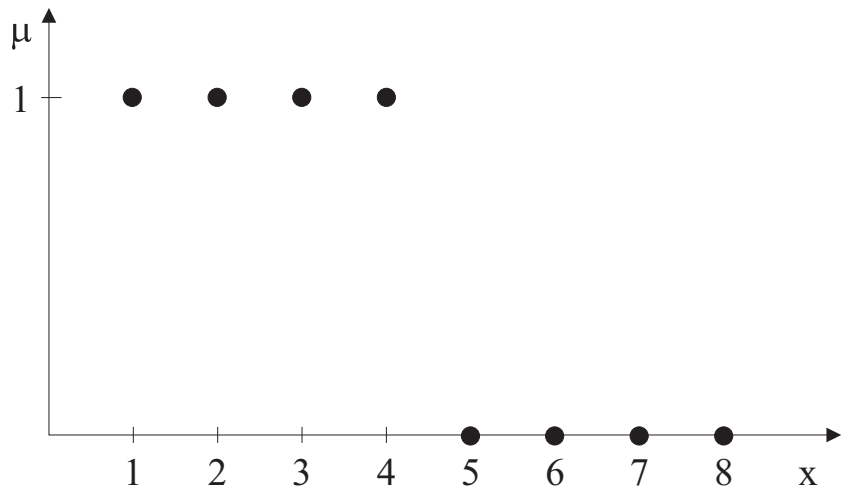
- Set of natural numbers smaller than 5:  $A = \{1, 2, 3, 4\}$
- Unit disk in the complex plane:  $A = \{z | z \in \mathbb{C}, |z| \leq 1\}$
- A line in  $\mathbb{R}^2$ :  $A = \{(x, y) | ax + by + c = 0, (x, y, a, b, c) \in \mathbb{R}\}$

# Representation of Sets

- Enumeration of elements:  $A = \{x_1, x_2, \dots, x_n\}$
- Definition by property:  $A = \{x \in X \mid x \text{ has property } P\}$
- Characteristic function:  $\mu_A(x) : X \rightarrow \{0, 1\}$

$$\mu_A(x) = \begin{cases} 1 & x \text{ is member of } A \\ 0 & x \text{ is not member of } A \end{cases}$$

## Set of natural numbers smaller than 5



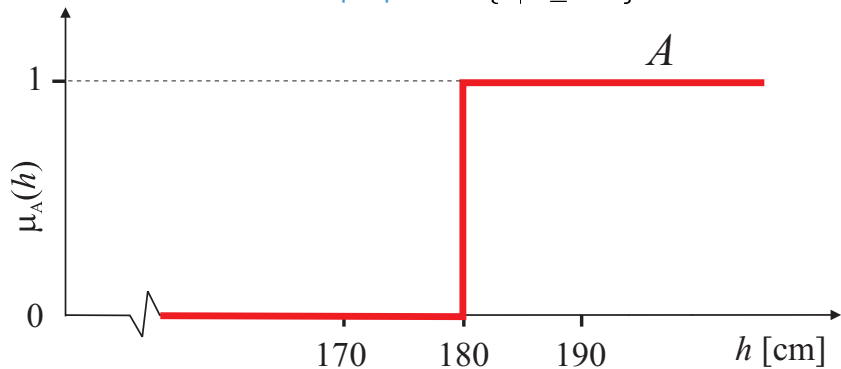
# Fuzzy sets

# Why Fuzzy Sets?

- Classical sets are good for well-defined concepts (maths, programs, etc.)
- Less suitable for representing commonsense knowledge in terms of vague concepts such as:
  - a tall person, slippery road, nice weather, . . .
  - want to buy a big car with moderate consumption
  - If the temperature is too low, increase heating a lot

# Classical Set Approach

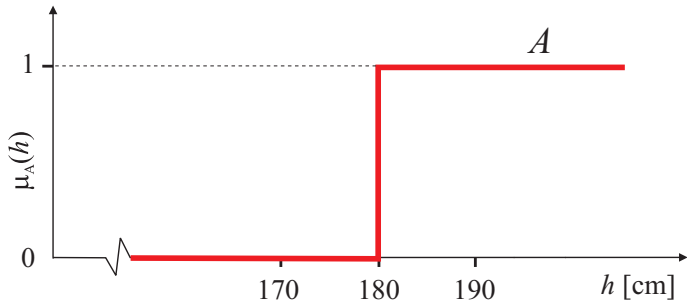
set of tall people  $A = \{h|h \geq 180\}$



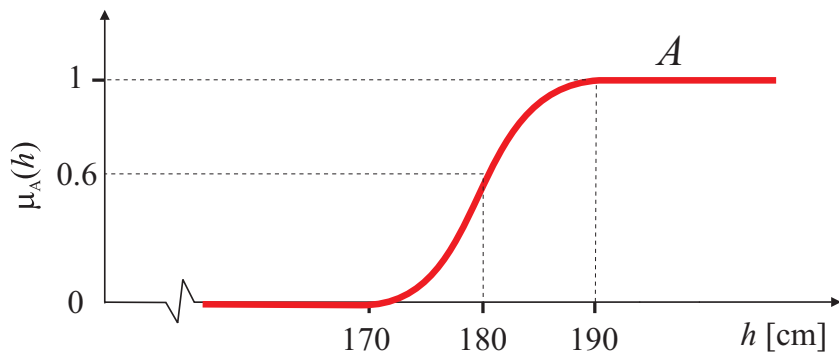
# Logical Propositions

“John is tall” ... true or false

John's height:  $h_{John} = 180.0$        $\mu_A(180.0) = 1$  (true)  
 $h_{John} = 179.5$        $\mu_A(179.5) = 0$  (false)



## Fuzzy Set Approach



$$\mu_A(h) = \begin{cases} 1 & h \text{ is full member of } A & (h \geq 190) \\ (0, 1) & h \text{ is partial member of } A & (170 < h < 190) \\ 0 & h \text{ is not member of } A & (h \leq 170) \end{cases}$$



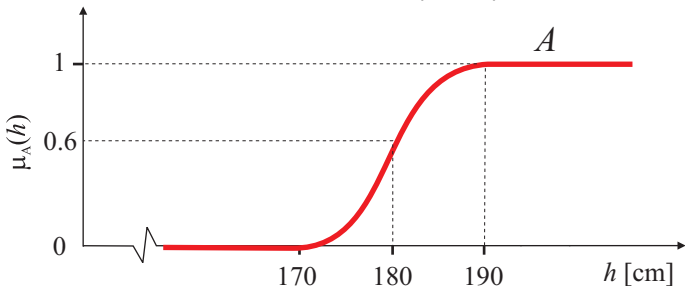
# Fuzzy Logic Propositions

“John is tall” ... degree of truth

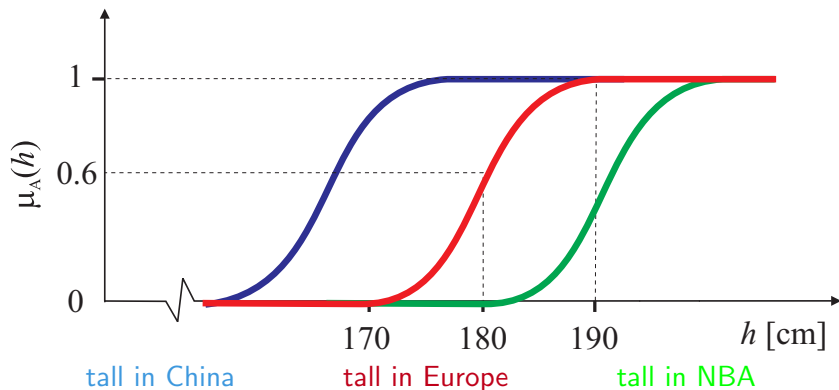
John's height:  $h_{John} = 180.0$        $\mu_A(180.0) = 0.6$

$h_{John} = 179.5$        $\mu_A(179.5) = 0.56$

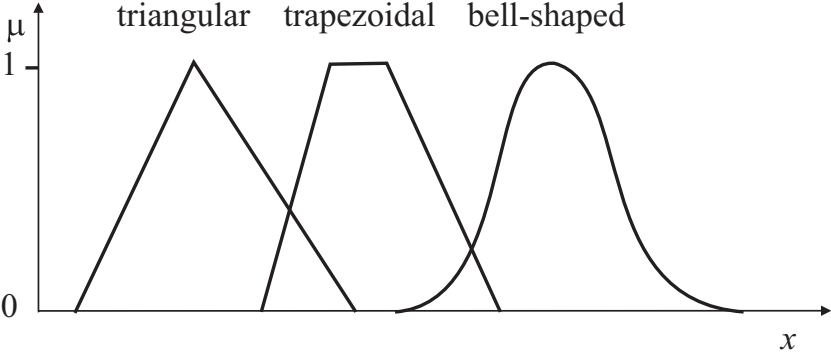
$h_{Paul} = 201.0$        $\mu_A(201.0) = 1$



## Subjective and Context Dependent



# Shapes of Membership Functions



# Representation of Fuzzy Sets

- Pointwise as a list of membership/element pairs:

$$A = \{\mu_A(x_1)/x_1, \dots, \mu_A(x_n)/x_n\} = \{\mu_A(x_i)/x_i \mid x_i \in X\}$$

- As a list of  $\alpha$ -level/ $\alpha$ -cut pairs:

$$A = \{\alpha_1/A_{\alpha_1}, \alpha_2/A_{\alpha_2}, \dots, \alpha_n, A_{\alpha_n}\} = \{\alpha_i/A_{\alpha_i} \mid \alpha_i \in (0, 1)\}$$

# Representation of Fuzzy Sets

- Analytical formula for the membership function:

$$\mu_A(x) = \frac{1}{1+x^2}, \quad x \in \mathbb{R}$$

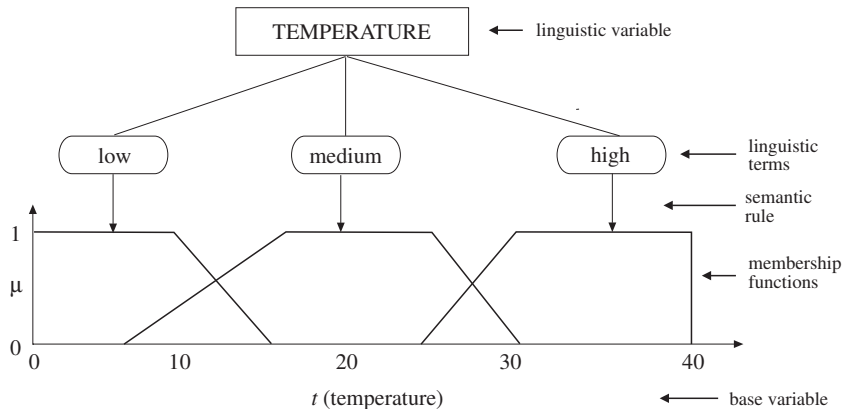
or more generally

$$\mu(x) = \frac{1}{1+d(x, v)}.$$

$d(x, v)$  ... dissimilarity measure

Various shorthand notations:  $\mu_A(x) \dots A(x) \dots a$

# Linguistic Variable

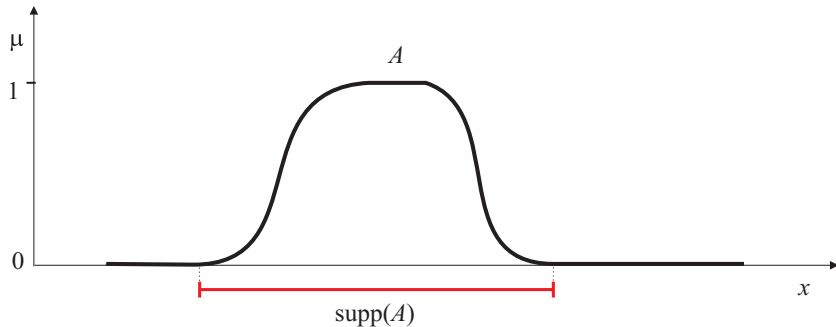


Basic requirements: coverage and semantic soundness

# Properties of fuzzy sets

## Support of a Fuzzy Set

$$\text{supp}(A) = \{x \mid \mu_A(x) > 0\}$$

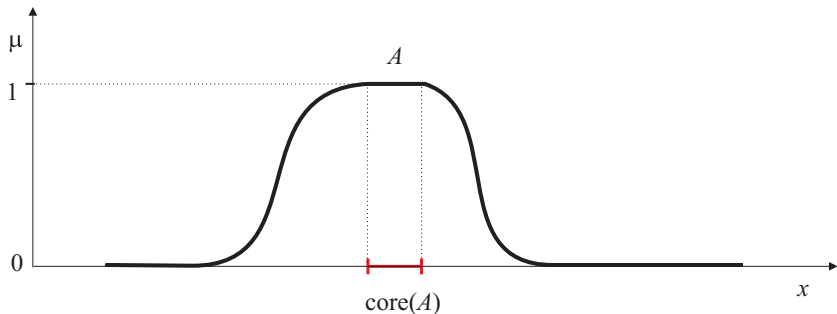


support is an *ordinary set*



## Core (Kernel) of a Fuzzy Set

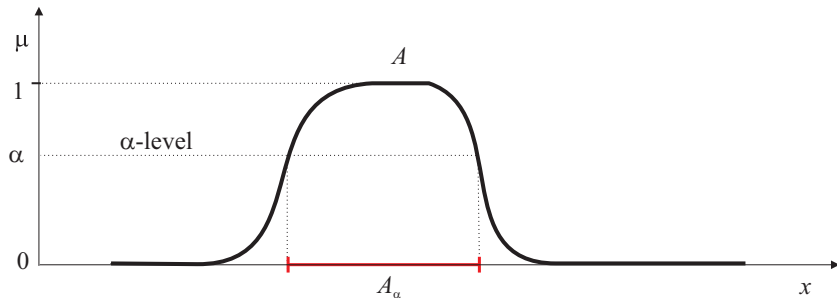
$$\text{core}(A) = \{x \mid \mu_A(x) = 1\}$$



core is an *ordinary set*

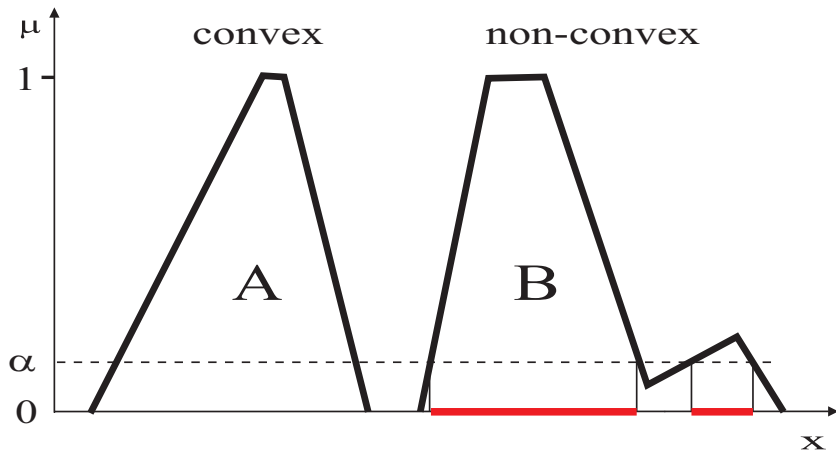
## $\alpha$ -cut of a Fuzzy Set

$$A_\alpha = \{x | \mu_A(x) > \alpha\} \quad \text{or} \quad A_\alpha = \{x | \mu_A(x) \geq \alpha\}$$



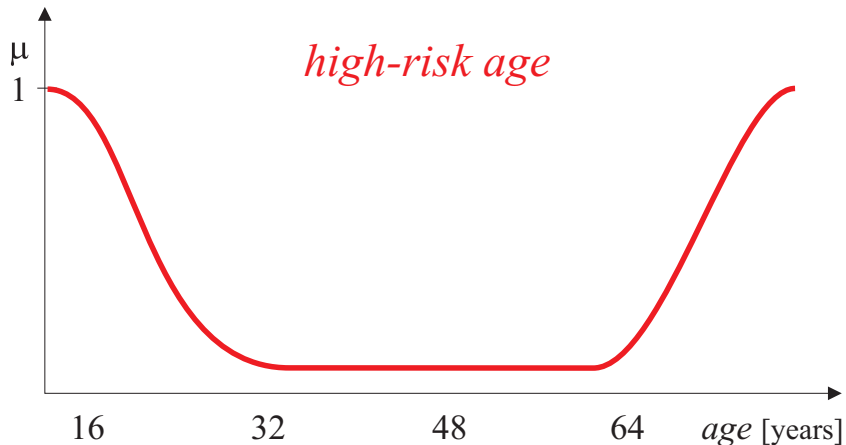
$A_\alpha$  is an *ordinary set*

## Convex and Non-Convex Fuzzy Sets



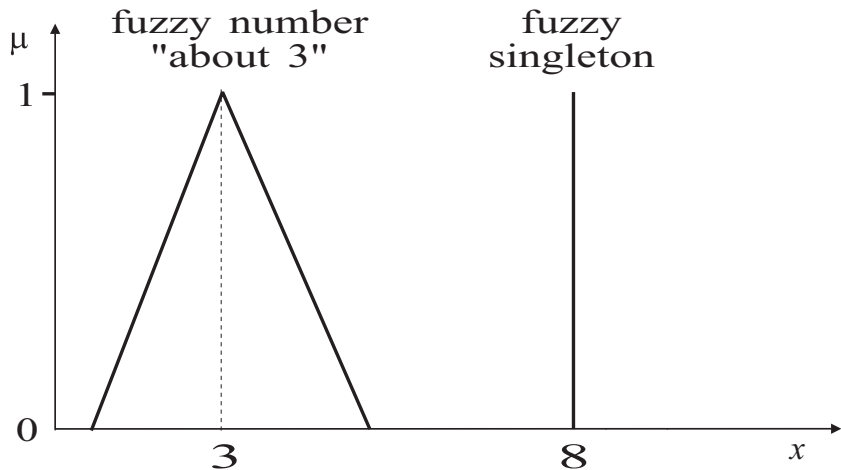
A fuzzy set is **convex**  $\Leftrightarrow$  all its  $\alpha$ -cuts are convex sets.

## Non-Convex Fuzzy Set: an Example



High-risk age for car insurance policy.

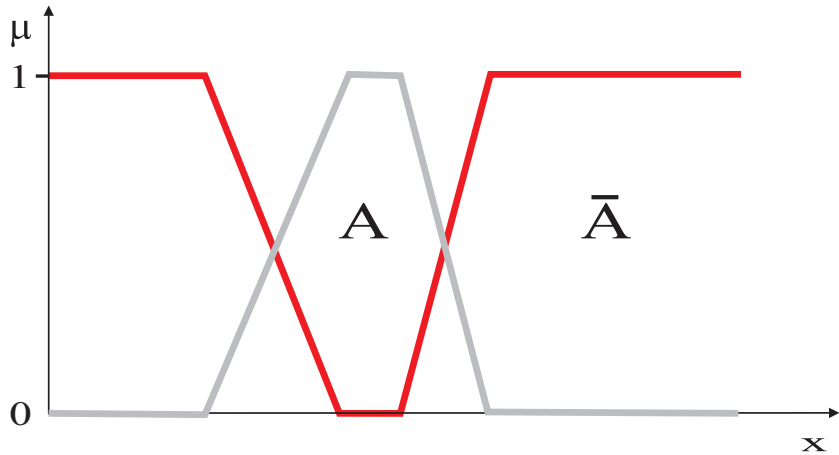
## Fuzzy Numbers and Singletons



Fuzzy linear regression:  $y = \tilde{3}x_1 + \tilde{5}x_2$

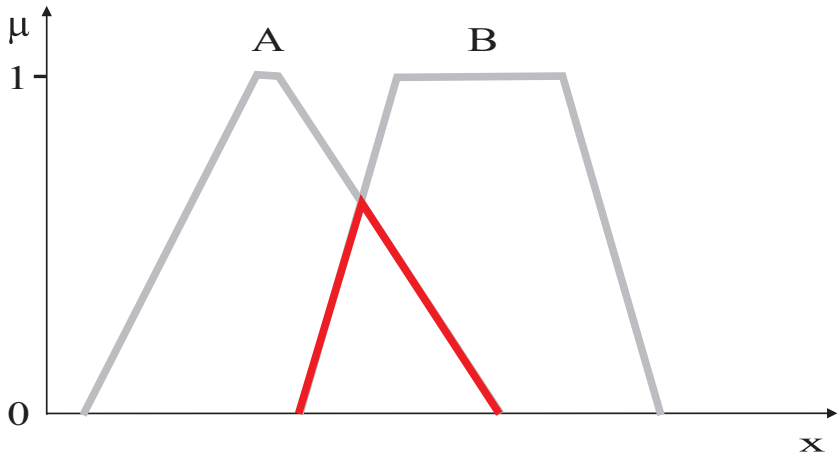
# Fuzzy set-theoretic operations

## Complement (Negation) of a Fuzzy Set



$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

# Intersection (Conjunction) of Fuzzy Sets



$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$$



## Other Intersection Operators (T-norms)

Probabilistic “and” (product operator):

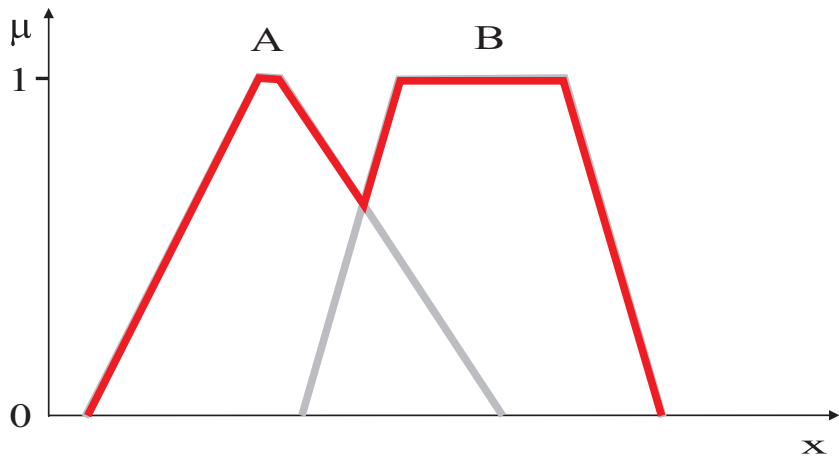
$$\mu_{A \cap B}(x) = \mu_A(x) \cdot \mu_B(x)$$

Łukasiewicz “and” (bounded difference):

$$\mu_{A \cap B}(x) = \max(0, \mu_A(x) + \mu_B(x) - 1)$$

Many other t-norms  $\dots [0, 1] \times [0, 1] \rightarrow [0, 1]$

## Union (Disjunction) of Fuzzy Sets



$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$$

## Other Union Operators (T-conorms)

Probabilistic “or”:

$$\mu_{A \cup B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$$

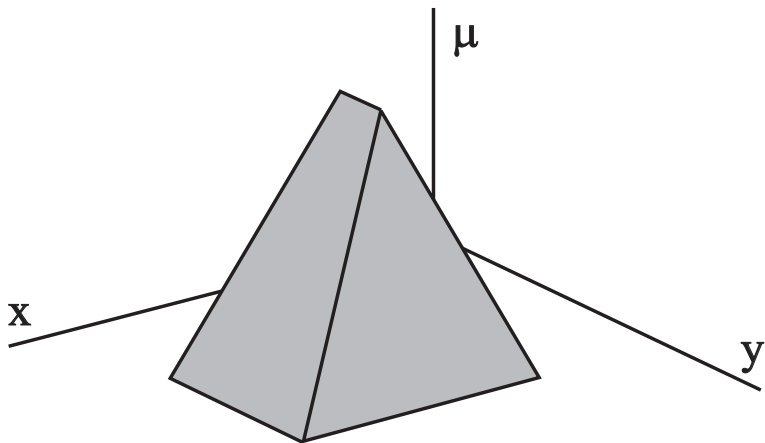
Łukasiewicz “or” (bounded sum):

$$\mu_{A \cup B}(x) = \min(1, \mu_A(x) + \mu_B(x))$$

Many other t-conorms  $\dots [0, 1] \times [0, 1] \rightarrow [0, 1]$

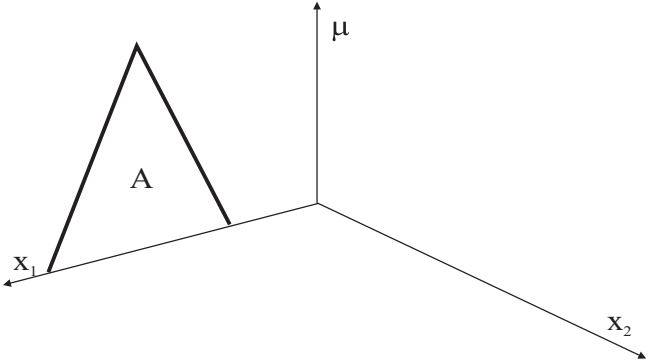
# Demo of a Matlab tool

## Fuzzy Set in Multidimensional Domains

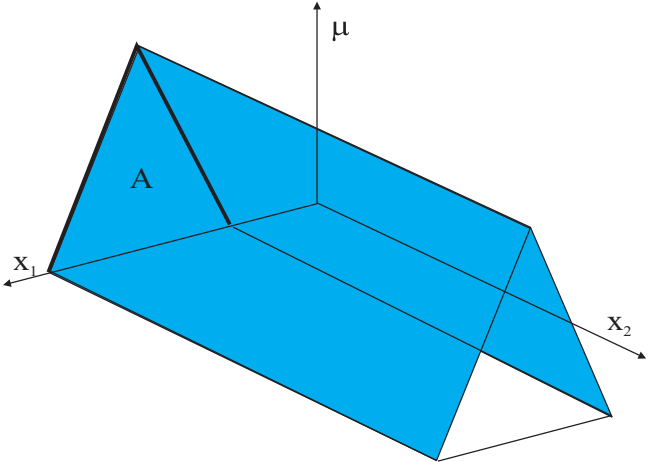


$$A = \{\mu_A(x, y) / (x, y) \mid (x, y) \in X \times Y\}$$

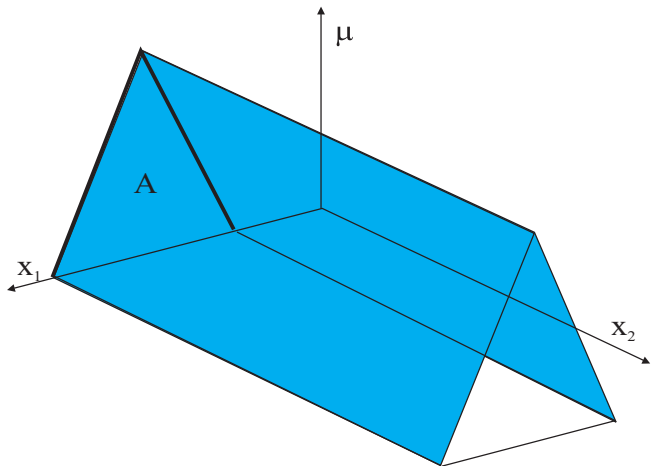
# Cylindrical Extension



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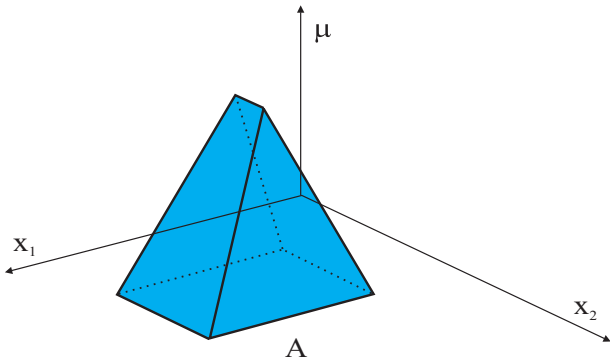
## Cylindrical Extension



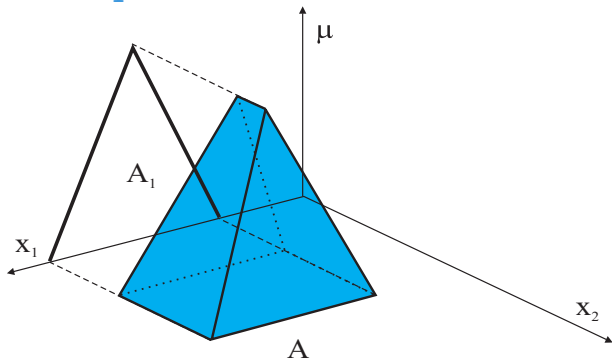
$$\text{ext}_{x_2}(A) = \{\mu_A(x_1)/(x_1, x_2) \mid (x_1, x_2) \in X_1 \times X_2\}$$



# Projection

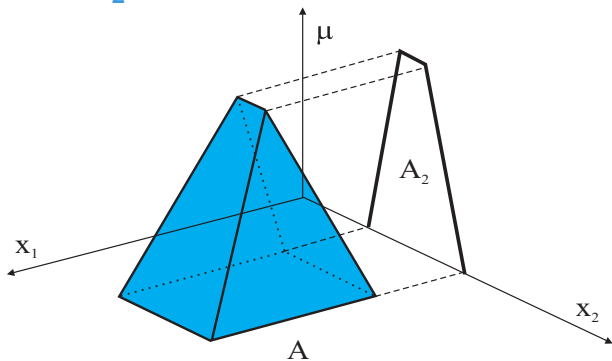


## Projection onto $X_1$



$$\text{proj}_{X_1}(A) = \left\{ \sup_{x_2 \in X_2} \mu_A(x_1, x_2) / x_1 \mid x_1 \in X_1 \right\}$$

## Projection onto $X_2$

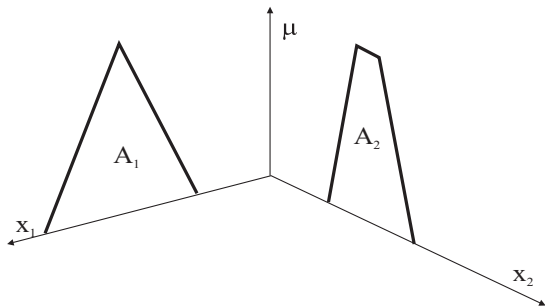


$$\text{proj}_{X_2}(A) = \left\{ \sup_{x_1 \in X_1} \mu_A(x_1, x_2) / x_2 \mid x_2 \in X_2 \right\}$$

## Intersection on Cartesian Product Space

An operation between fuzzy sets are defined in different domains results in a multi-dimensional fuzzy set.

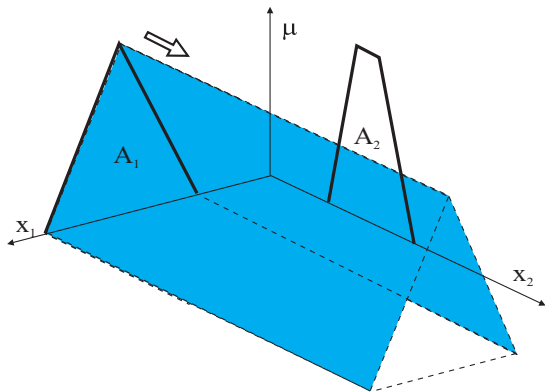
Example:  $A_1 \cap A_2$  on  $X_1 \times X_2$ :



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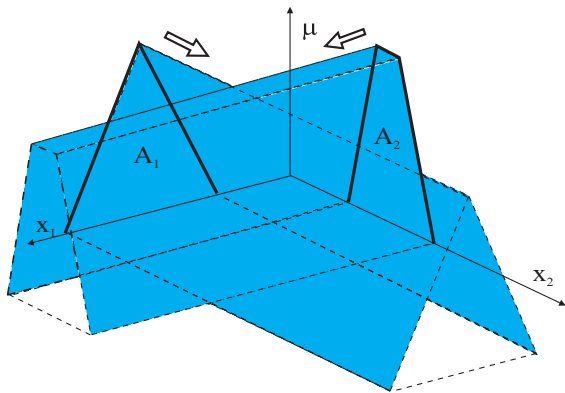
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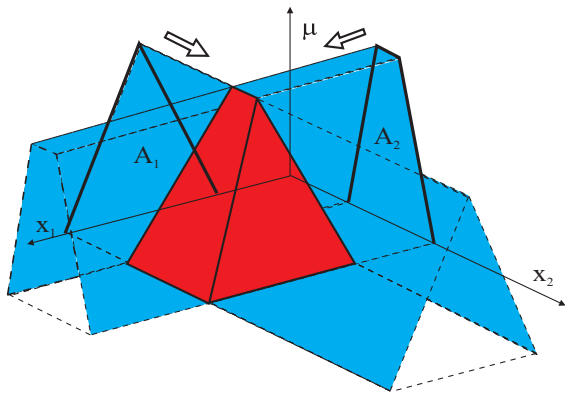
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Example:  $A_1 \cap A_2$  on  $X_1 \times X_2$ :



# Fuzzy Relations

Classical relation represents the presence or absence of interaction between the elements of two or more sets.

With **fuzzy relations**, the degree of association (correlation) is represented by membership grades.

An  $n$ -dimensional fuzzy relation is a mapping

$$R : X_1 \times X_2 \times X_3 \cdots \times X_n \rightarrow [0, 1]$$

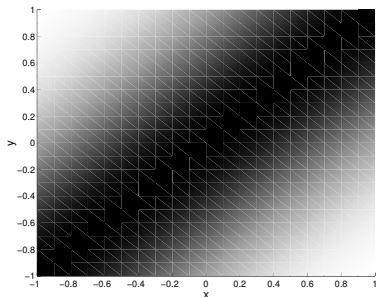
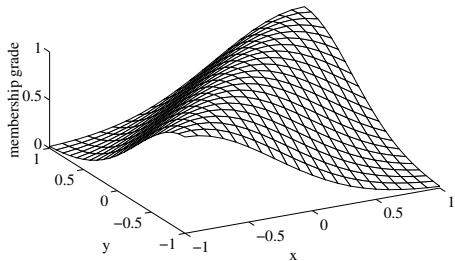
which assigns membership grades to all  $n$ -tuples  $(x_1, x_2, \dots, x_n)$  from the Cartesian product universe.



## Fuzzy Relations: Example

Example:  $R : x \approx y$  ("x is approximately equal to y")

$$\mu_R(x, y) = e^{-(x-y)^2}$$



## Relational Composition

Given fuzzy relation  $R$  defined in  $X \times Y$  and fuzzy set  $A$  defined in  $X$ , derive the corresponding fuzzy set  $B$  defined in  $Y$ :

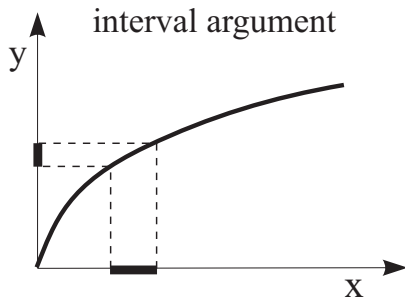
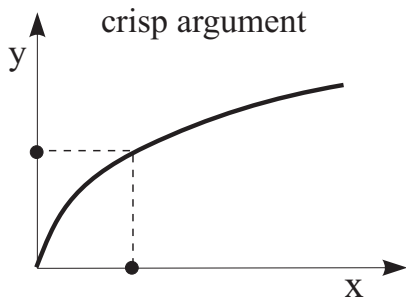
$$B = A \circ R = \text{proj}_Y(\text{ext}_{X \times Y}(A) \cap R)$$

max-min composition:

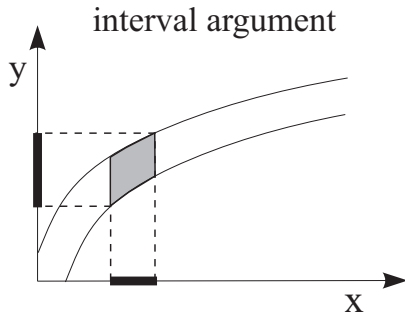
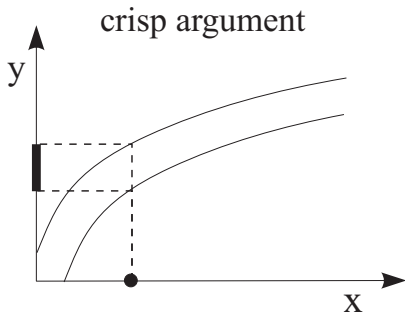
$$\mu_B(y) = \max_x \left( \min(\mu_A(x), \mu_R(x, y)) \right)$$

Analogous to evaluating a function.

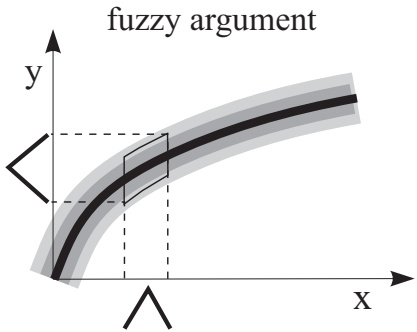
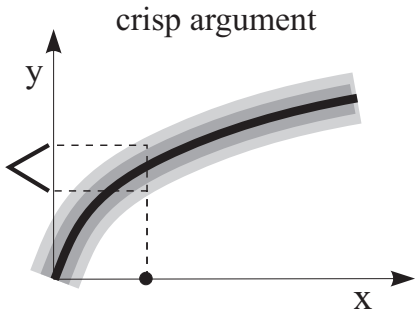
## Graphical Interpretation: Crisp Function



## Graphical Interpretation: Interval Function



# Graphical Interpretation: Fuzzy Relation



## Max-Min Composition: Example

$$\mu_B(y) = \max_x \left( \min(\mu_A(x), \mu_R(x, y)) \right), \quad \forall y$$

$$\begin{bmatrix} 1.0 & 0.4 & 0.1 & 0.0 & 0.0 \end{bmatrix} \circ \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.4 & 0.8 \\ 0.0 & 0.1 & 1.0 & 0.2 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.9 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 0.3 & 0.0 \end{bmatrix} =$$

## Max-Min Composition: Example

$$\mu_B(y) = \max_x \left( \min(\mu_A(x), \mu_R(x, y)) \right), \quad \forall y$$

$$\begin{bmatrix} 1.0 & 0.4 & 0.1 & 0.0 & 0.0 \end{bmatrix} \circ \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.4 & 0.8 \\ 0.0 & 0.1 & 1.0 & 0.2 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.9 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 0.3 & 0.0 \end{bmatrix} = \begin{bmatrix} 0.0 & 0.1 & 0.4 & 0.4 & 0.8 \end{bmatrix}$$

# Fuzzy Systems



# Fuzzy Systems

- Systems with fuzzy parameters

$$y = \tilde{3}x_1 + \tilde{5}x_2$$

- Fuzzy inputs and states

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = \tilde{2}$$

- Rule-based systems

*If the heating power is high  
then the temperature will increase fast*

# Rule-based Fuzzy Systems

- Linguistic (Mamdani) fuzzy model

**If  $x$  is  $A$  then  $y$  is  $B$**

- Fuzzy relational model

**If  $x$  is  $A$  then  $y$  is  $B_1(0.1), B_2(0.8)$**

- Takagi–Sugeno fuzzy model

**If  $x$  is  $A$  then  $y = f(x)$**

# Linguistic Model

**If  $x$  is  $A$  then  $y$  is  $B$**

$x$  **is**  $A$  – antecedent (fuzzy proposition)

$y$  **is**  $B$  – consequent (fuzzy proposition)

# Linguistic Model

**If  $x$  is  $A$  then  $y$  is  $B$**

$x$  **is**  $A$  – antecedent (fuzzy proposition)

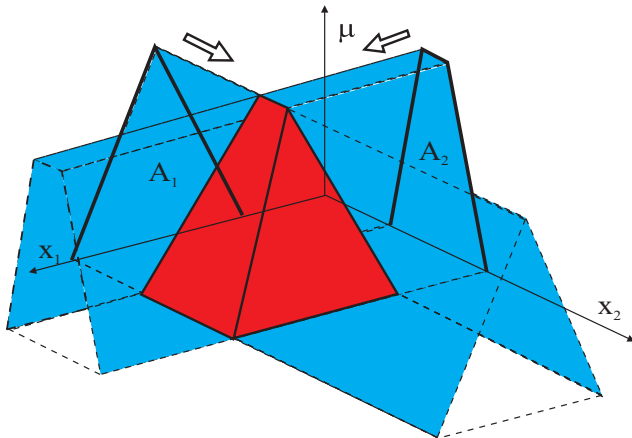
$y$  **is**  $B$  – consequent (fuzzy proposition)

Compound propositions (logical connectives, hedges):

**If  $x_1$  is very big and  $x_2$  is not small**

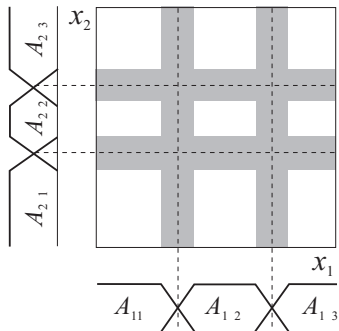
# Multidimensional Antecedent Sets

$A_1 \cap A_2$  on  $X_1 \times X_2$ :

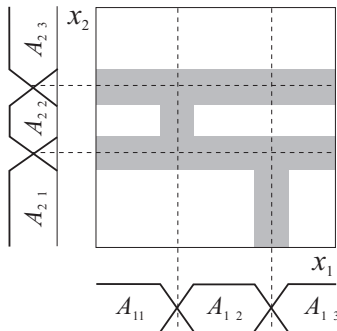


# Partitioning of the Antecedent Space

conjunctive



other connectives



# Inference Mechanism

Given the if-then rules and an input fuzzy set, deduce the corresponding output fuzzy set.

- Formal approach based on fuzzy relations.
- Simplified approach (Mamdani inference).
- Interpolation (additive fuzzy systems).

# Formal Approach

- 1 Represent each if–then rule as a fuzzy relation.
- 2 Aggregate these relations in one relation representative for the entire rule base.
- 3 Given an input, use *relational composition* to derive the corresponding output.



# Modus Ponens Inference Rule

Classical logic

**if**  $x$  is  $A$  **then**  $y$  is  $B$

$x$  is  $A$

---

$y$  is  $B$

Fuzzy logic

**if**  $x$  is  $A$  **then**  $y$  is  $B$

$x$  is  $A'$

---

$y$  is  $B'$

## Relational Representation of Rules

**If-then** rules can be represented as a *relation*, using implications or conjunctions.

Classical implication

$A$	$B$	$A \rightarrow B (\neg A \vee B)$
0	0	1
0	1	1
1	0	0
1	1	1

$A \backslash B$	0	1
0	1	1
1	0	1

$$R: \{0, 1\} \times \{0, 1\} \rightarrow \{0, 1\}$$

# Relational Representation of Rules

**If-then** rules can be represented as a *relation*, using implications or conjunctions.

Conjunction

$A$	$B$	$A \wedge B$
0	0	0
0	1	0
1	0	0
1	1	1

$A \setminus B$	0	1
0	0	0
1	0	1

$$R: \{0, 1\} \times \{0, 1\} \rightarrow \{0, 1\}$$

## Fuzzy Implications and Conjunctions

Fuzzy implication is represented by a fuzzy relation:

$$R: [0, 1] \times [0, 1] \rightarrow [0, 1]$$

$$\mu_R(x, y) = I(\mu_A(x), \mu_B(y))$$

$I(a, b)$  – implication function

"classical"	Kleene–Diene	$I(a, b) = \max(1 - a, b)$
	Łukasiewicz	$I(a, b) = \min(1, 1 - a + b)$
T-norms	Mamdani	$I(a, b) = \min(a, b)$
	Larsen	$I(a, b) = a \cdot b$

# Inference With One Rule

- 1 Construct implication relation:

$$\mu_R(x, y) = I(\mu_A(x), \mu_B(y))$$

## Inference With One Rule

- 1 Construct implication relation:

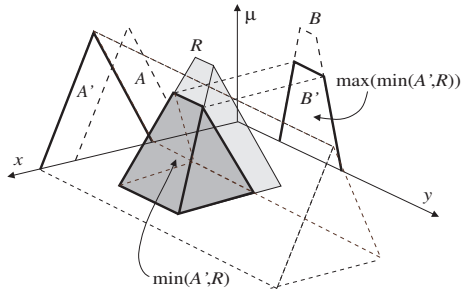
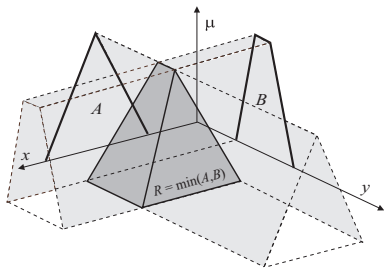
$$\mu_R(x, y) = I(\mu_A(x), \mu_B(y))$$

- 2 Use relational composition to derive  $B'$  from  $A'$ :

$$B' = A' \circ R$$

# Graphical Illustration

$$\mu_R(x, y) = \min(\mu_A(x), \mu_B(y)) \quad \mu_{B'}(y) = \max_x(\min(\mu_{A'}(x), \mu_R(x, y)))$$



## Inference With Several Rules

- 1 Construct implication relation for each rule  $i$ :

$$\mu_{R_i}(x, y) = I(\mu_{A_i}(x), \mu_{B_i}(y))$$

- 2 Aggregate relations  $R_i$  into one:

$$\mu_R(x, y) = \text{aggr}(\mu_{R_i}(x, y))$$

The aggr operator is the minimum for implications and the maximum for conjunctions.

- 3 Use relational composition to derive  $B'$  from  $A'$ :

$$B' = A' \circ R$$



## Example: Conjunction

- 1 Each rule

**If  $\tilde{x}$  is  $A_i$  then  $\tilde{y}$  is  $B_i$**

is represented as a fuzzy relation on  $X \times Y$ :

$$R_i = A_i \times B_i \quad \mu_{R_i}(\mathbf{x}, \mathbf{y}) = \mu_{A_i}(\mathbf{x}) \wedge \mu_{B_i}(\mathbf{y})$$

## Example: Conjunction, Aggregation

- 1 Each rule

**If  $\tilde{x}$  is  $A_i$  then  $\tilde{y}$  is  $B_i$**

is represented as a fuzzy relation on  $X \times Y$ :

$$R_i = A_i \times B_i \quad \mu_{R_i}(\mathbf{x}, \mathbf{y}) = \mu_{A_i}(\mathbf{x}) \wedge \mu_{B_i}(\mathbf{y})$$

- 2 The entire rule base's relation is the union:

$$R = \bigcup_{i=1}^K R_i \quad \mu_R(\mathbf{x}, \mathbf{y}) = \max_{1 \leq i \leq K} [\mu_{R_i}(\mathbf{x}, \mathbf{y})]$$

## Example: Conjunction, Aggregation, and Composition

- 1 Each rule

**If  $\tilde{x}$  is  $A_i$  then  $\tilde{y}$  is  $B_i$**

is represented as a fuzzy relation on  $X \times Y$ :

$$R_i = A_i \times B_i \quad \mu_{R_i}(\mathbf{x}, \mathbf{y}) = \mu_{A_i}(\mathbf{x}) \wedge \mu_{B_i}(\mathbf{y})$$

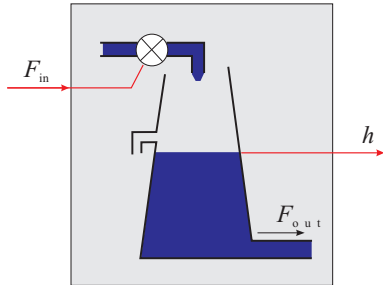
- 2 The entire rule base's relation is the union:

$$R = \bigcup_{i=1}^K R_i \quad \mu_R(\mathbf{x}, \mathbf{y}) = \max_{1 \leq i \leq K} [\mu_{R_i}(\mathbf{x}, \mathbf{y})]$$

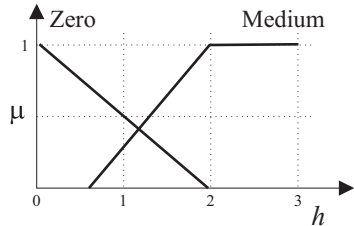
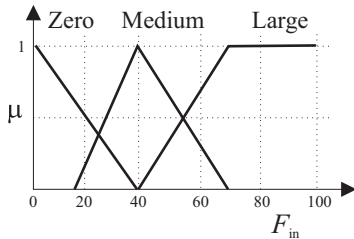
- 3 Given an input value  $A'$  the output value  $B'$  is:

$$B' = A' \circ R \quad \mu_{B'}(\mathbf{y}) = \max_X [\mu_{A'}(\mathbf{x}) \wedge \mu_R(\mathbf{x}, \mathbf{y})]$$

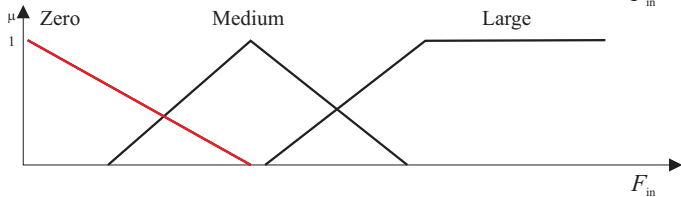
# Example: Modeling of Liquid Level



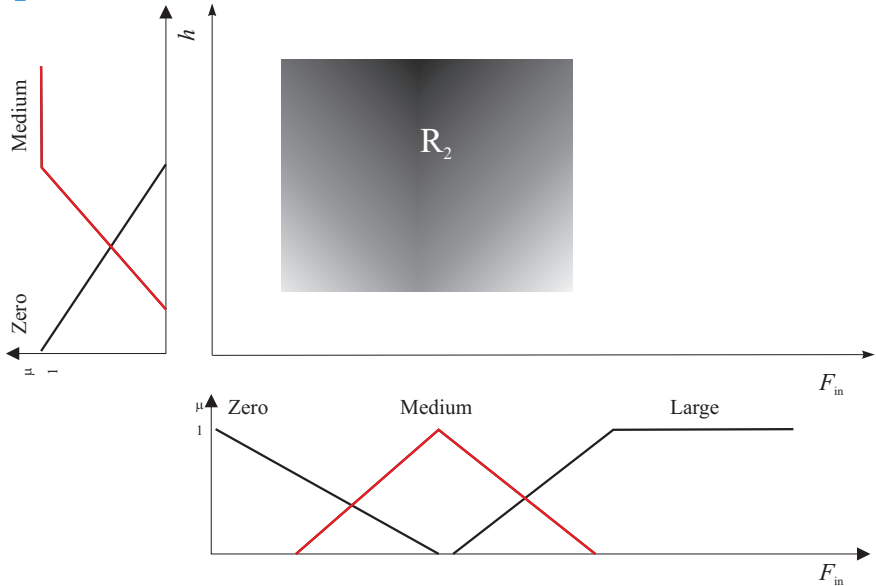
- If  $F_{in}$  is Zero then  $h$  is Zero
- If  $F_{in}$  is Med then  $h$  is Med
- If  $F_{in}$  is Large then  $h$  is Med



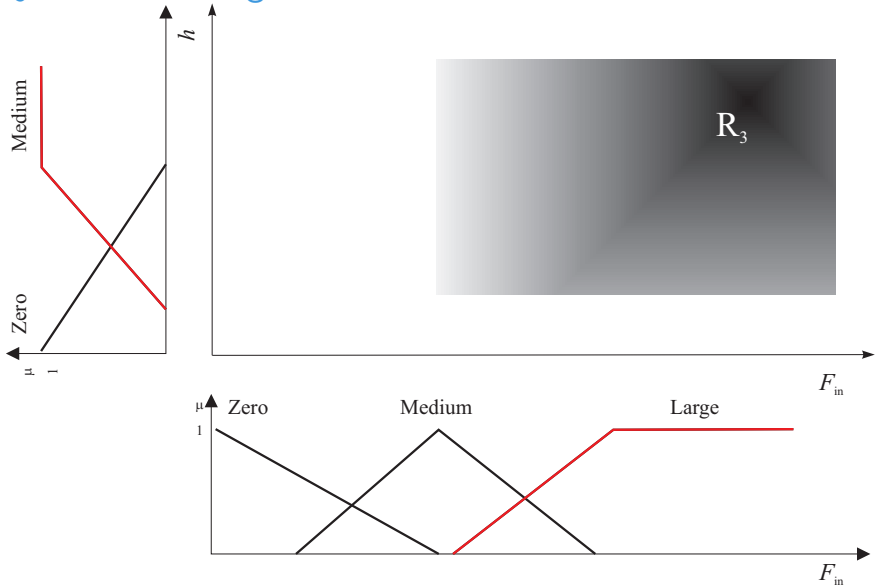
# $\mathcal{R}_1$ If Flow is Zero then Level is Zero



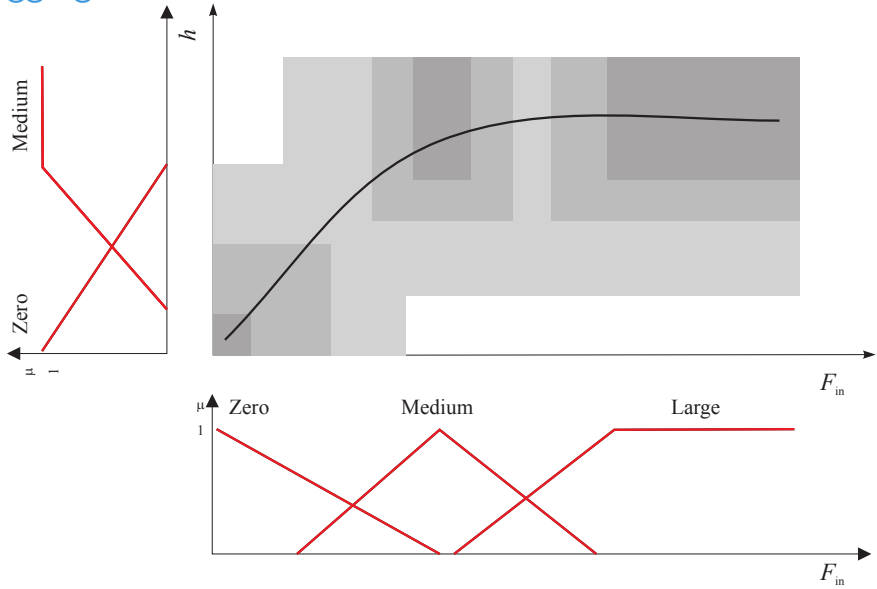
## $\mathcal{R}_2$ If Flow is Medium then Level is Medium



## $\mathcal{R}_3$ If Flow is Large then Level is Medium



# Aggregated Relation



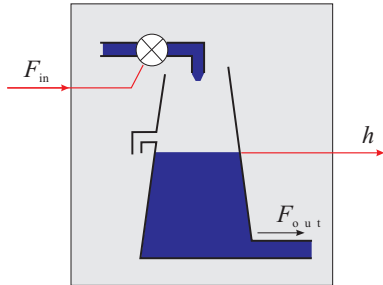


# Simplified Approach

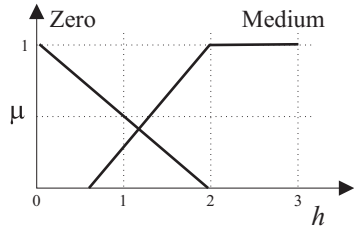
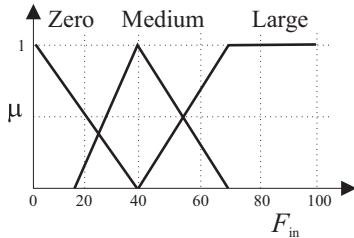
- ① Compute the match between the input and the antecedent membership functions (*degree of fulfillment*).
- ② Clip the corresponding output fuzzy set for each rule by using the degree of fulfillment.
- ③ Aggregate output fuzzy sets of all the rules into one fuzzy set.

This is called the *Mamdani* or *max-min* inference method.

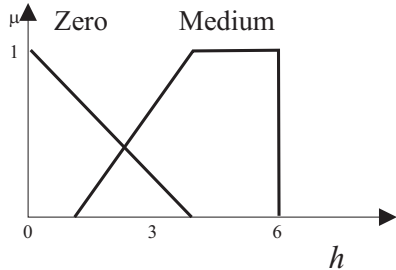
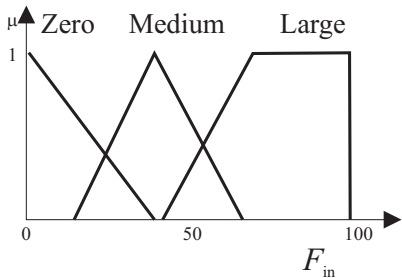
# Water Tank Example



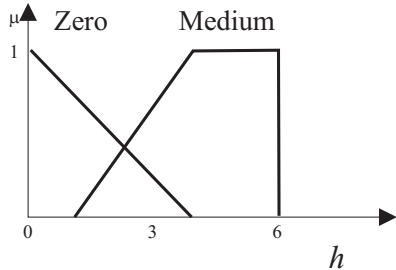
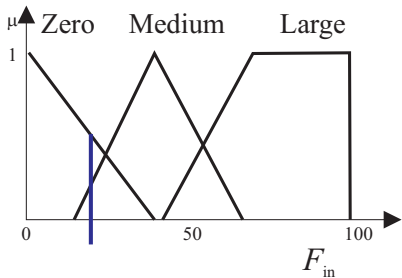
- If  $F_{in}$  is Zero then  $h$  is Zero
- If  $F_{in}$  is Med then  $h$  is Med
- If  $F_{in}$  is Large then  $h$  is Med



# Mamdani Inference: Example

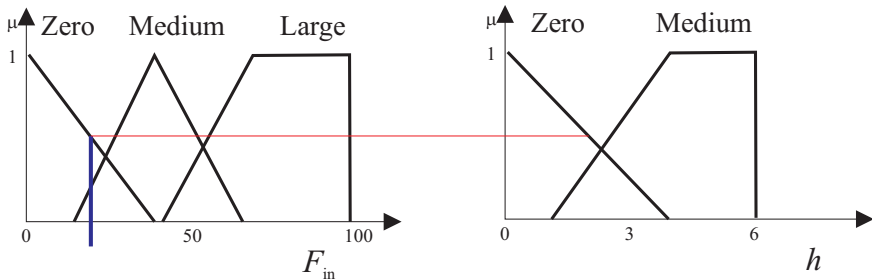


# Mamdani Inference: Example



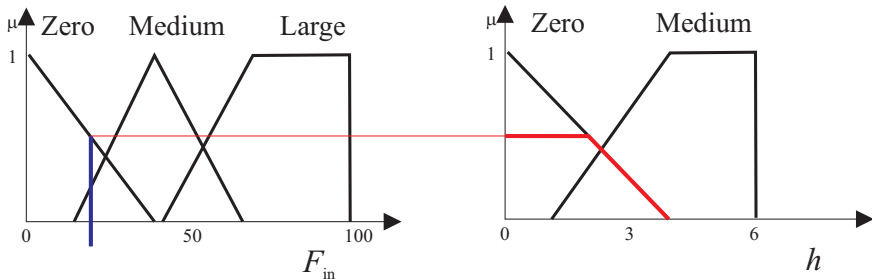
Given a crisp (numerical) input ( $F_{in}$ ).

If  $F_{in}$  is Zero then ...



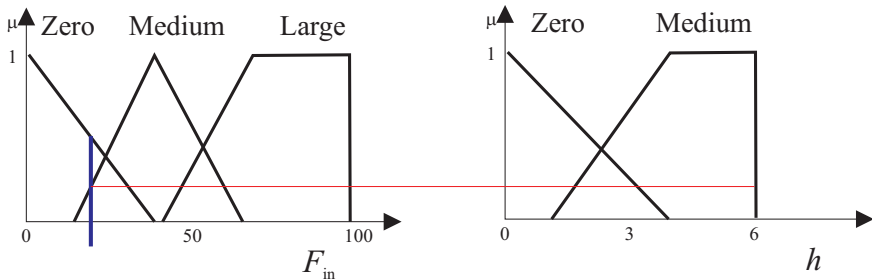
Determine the degree of fulfillment (truth) of the first rule.

If  $F_{in}$  is Zero then  $h$  is Zero



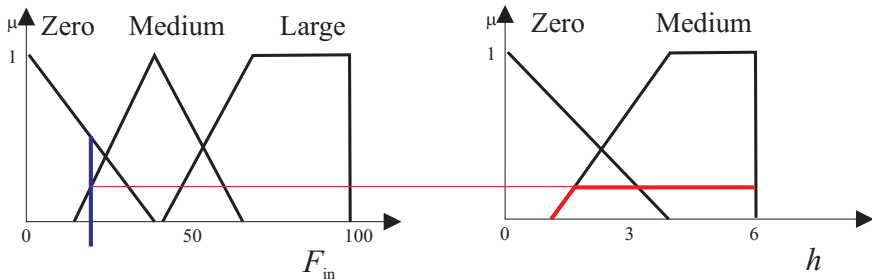
Clip consequent membership function of the first rule.

If  $F_{in}$  is Medium then ...



Determine the degree of fulfillment (truth) of the second rule.

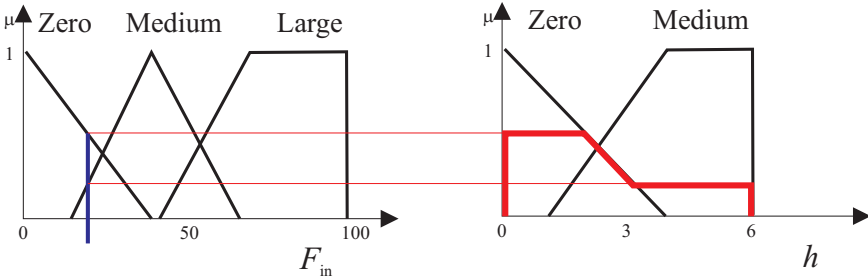
If  $F_{in}$  is Medium then  $h$  is Medium



Clip consequent membership function of the second rule.



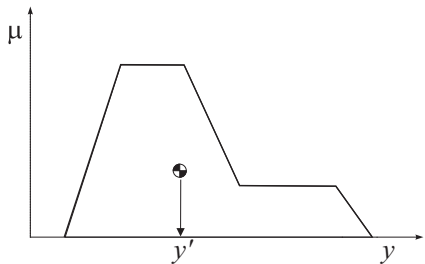
# Aggregation



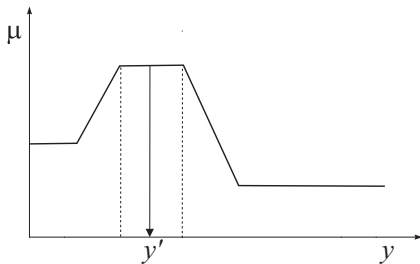
Combine the result of the two rules (union).

# Defuzzification

*conversion of a fuzzy set to a crisp value*



(a) center of gravity

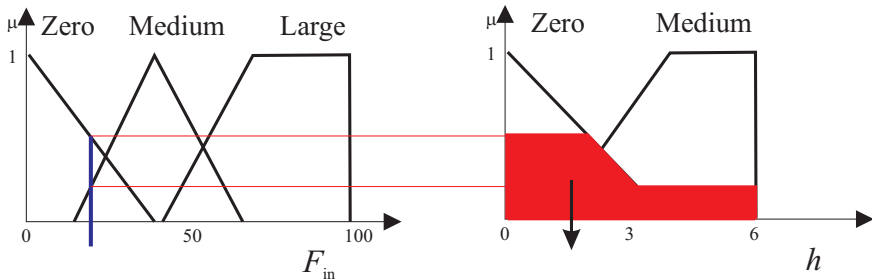


(b) mean of maxima

## Center-of-Gravity Method

$$y_0 = \frac{\sum_{j=1}^F \mu_{B'}(y_j) y_j}{\sum_{j=1}^F \mu_{B'}(y_j)}$$

# Defuzzification



Compute a crisp (numerical) output of the model (center-of-gravity method).