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Fuzzy observers for anaerobic WWTP: Development and implementation

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ABSTRACT

In this paper a set of fuzzy observers for anaerobic digestion is proposed to estimate variables difficult to measure in a completely stirred tank reactor. First, a mathematical model for the process is stated and experimentally validated. Then, a methodology based on principal components analysis is developed to select fuzzy variables, which allow the local observers to be adequately activated. The active local observers are interpolated using the Takagi–Sugeno approach, in order to recover the non-linear behaviour; to ensure their adequate performance, the respective stability analysis is included. The whole estimation scheme is validated via simulations and tested in a real process.

1. Introduction

Anaerobic digestion is currently used for wastewater treatment due to its efficiency for organic wastes transformation. There exists an additional interest: anaerobic micro-organisms transform organic wastes into a biogas mainly composed of carbon dioxide (CO_2) and methane (CH_4), which can be used as an alternative energy source. However, anaerobic digestion is very sensitive to operating conditions. Additionally, some variables are difficult to measure due to technical or economical constraints, i.e.: the substrate consumption measure is expensive, needs 3 h and is done off-line. Biomass measure is still more restrictive because the existing sensors are designed from a biological approach, and are not adequate for automatic control. For this reason, a first step required for control analysis and design is to develop state observers in order to estimate unmeasured variables. The main idea of the control structure proposed in Sanchez, Béteau, and Carlos-Hernandez (2001) is to avoid washout by means of indicator variables supervision such as ODL/X_2 (organic daily load per biomass unit) and CH_4. Since the last one is directly measured and ODL can be determined from input substrate variations and the dilution rate, an adequate estimation of X_2 is very important.

Different kinds of observers for the anaerobic digestion process have been already developed. The asymptotical observer, proposed by Bastin and Dochain (1990), uses a non-linear model; it is robust in presence of parameter uncertainties, but its convergence rate depends on operating conditions. In Bernard and Gouzé (2003), the authors propose closed-loop observers in order to improve the asymptotical one. Another interesting approach is the interval observer (Smith, 1996; Gouze, Rapaport, & Hadj-Sadok, 2000), which takes into account model uncertainties; the idea is to estimate variables over defined intervals. This observer gives good estimations but the interval selection and estimation convergence need to be improved. Other approaches have been tested as shown in Deza, Bossanne, Busvelle, Gauthier, and Rakotopara (1993), Alcaraz-Gonzalez et al. (1999), Lemesle and Gouze (2005) and Chachuat and Bernard (2005). All these observers propose particular solutions; however, some disadvantages are noted, such as difficulties to design, tune and implementation, and numerical instability due to ill conditioning of the process dynamics, estimation errors due to model uncertainties, etc. The interval observer proposed in Alcaraz-Gonzalez et al. (2004) is a very good solution against the process uncertainties (inputs, initial conditions and reaction rates). Nevertheless, the estimation convergence rate cannot be tuned; also an overestimation effect on the estimated intervals can be induced and it could provide unreal behaviours. The observer developed by Theilliol, Ponsart, Harmand, Join, and Gras (2003) is devoted to estimate unmeasured inputs and also unknown state variables and shows good performances. It is designed on the basis of a tangent linearisation which could limit the operation range; moreover, the observer requires on-line measures hard to have with standard methods e.g. substrate, which can be indirectly measured off-line using chemical oxygen demand (COD) values. Other works have been developed focusing on diagnosis and fault detection, such as Lardon, Punal and Steyer (2004), where the authors propose an original approach for a real anaerobic wastewater treatment using

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the evidence theory. Dupuit, Pouet, Thomas, and Bourgois (2007) developed a decision support methodology using rule-based reasoning coupled to non-parametric measurement for industrial wastewater. Also, an interesting work is presented in Wimberger and Verde (2008), which deals with a systematic approach to evaluate fault detectability and isolability in aerobic processes; the methodology is analytical- and signal-based fault detection and isolation and it can be extended for anaerobic wastewater treatment. On the other hand, fuzzy algorithms have been recently used to design observers and controllers for bioprocess (Ascencio, Sbarbaro, & Feyo de Azevedo, 2004; Muller et al., 1997; Polit, Genovesi, & Claudet, 2001). In this paper, for the first time, a Takagi–Sugeno (TS) fuzzy observer (Bergsten & Palm, 2000; Takagi & Sugeno, 1985; Xiao-Jun, Zeng-Qi, & Yan-Yan, 1998), for an anaerobic process in a completely stirred tank reactor (CSTR) with biomass filter behaviour, is proposed. The main idea is to design several local observers and implement a fuzzy interpolation to obtain the global state estimation. The advantages w.r.t. other approaches is that the estimation convergence can be easily tuned since only linear systems techniques are required and a wide operating range can be considered if necessary. Also unreal behaviours are not induced by this approach.

Due to the process complexity, the first challenge is to choose adequate fuzzy variables, which will select the local models as a function of operating conditions. There exist different methods for multivariate data analysis (Hair, Anderson, Tatham, & Black, 1998; Manly, 2004; Murtagh & Heck, 1987), which can be used in order to reach this objective, such as discriminant function analysis (DFA, known in other technical domains as data mining or data exploration) and factor analysis (FA). DFA is used to generate a new representation based on linear combinations (discriminant axes) of original variables, which allows the variables to be classified (discriminated) adequately in different *categories*. A characteristic of this method is that *categories* must be predefined, which is not convenient to determine fuzzy variables which select the active local models in anaerobic digestion; furthermore, it is more complex than other techniques. FA is used to discover if the variables can be explained largely or entirely in terms of a much smaller number of variables called factors; there are two main types of FA: common factor analysis (CFA) and principal component analysis (PCA). Factors are expressed in function of common variance of the variables, excluding unique variance; it implies that CFA is a correlation-focused method which reproduces the intercorrelation among variables. On the other hand, components (linear combinations) include both common and unique variance of the variables; then PCA is a variance-focused method which reproduces the correlations and total variable variance with all components. Furthermore, CFA requires a selection of factors to keep for further analysis; meanwhile, PCA provides a unique solution. Considering all these factors, PCA is selected in this paper.

Recently, the term *soft sensor* has been widely used in automatic control. A *soft sensor* or virtual sensor is a mechanism which associates a model and available data to compute non-measured variables. Data may be qualitative or quantitative and models can be of any type (static, dynamic, linear, non-linear, fuzzy). Sometimes in a soft sensor, variables are estimated because they are hard to measure or because they may be inferred from the available signals (Fortuna, Graziani, Rizzo, & Xibilia, 2007; Gonzalez, Redard, Barrera, & Fernandez, 1994). Then, a state observer is a *soft sensor* but a *soft sensor* is not necessarily a state observer.

2. Anaerobic digestion

In general, four stages can be discerned in substrate transformation by anaerobic digestion: hydrolysis, acidogenesis, acetogenesis and methanogenesis. In Gujer and Zenhder (1983), Mosey (1983), Moletta, Verrier, and Albagnac (1986), and Angelidaki, Ellegaard, and Ahring (1998), the authors describe in detail the process. The methanogenesis is the slowest stage and the most important for process stability; it is very sensitive to variations in the operating conditions such as: temperature, overload on substrate concentration, etc. A description of the operating conditions of anaerobic digestion and the parameter variation effects in different kinds of processes are presented in Van den Berg, Patel, Clark, and Lentz (1976), Parkin & Owen (1986), De Beer, Huisman, Van den Heuvel, and Ottengraf (1992) and Penaud et al., 1997).

2.1. The CSTR

This kind of reactor is currently used in industrial processes because its hydrodynamic behaviour is relatively easy to model and to control. In this paper, two operating modes are considered: batch and continuous. In the first one, the input and output flow rates are equal to zero; the micro-organisms and the substrate stay inside the reactor without human handling; the experiment finishes when the substrate is completely transformed by the bacteria and no more biogas production is obtained. The batch configuration is used here for biomass adaptation to the substrate and for parameter identification. In the continuous mode, the input and output flow rates are equal but different from zero; hence, the influents are treated continuously. Besides, the continuous operating conditions can be modified (the substrate pH, the COD, the input flow rate), depending on the experiments to be performed. The continuous mode is employed in this paper to validate experimentally the fuzzy observer and it can be used to test control strategies. The continuous scheme is shown in Fig. 1. Note that the wasting sludge flow is neglected.

2.2. Modelling anaerobic digestion

Even if the ADM1 (Batstone et al., 2002) is indeed very detailed, it becomes quite complex to be used for the purposes of this work. Then, a reduced model validated experimentally is used. A synthetic substrate similar to effluents of paper mills is considered. The organic components are classified in *equivalent glucose* (S_1) and *equivalent acetate* (S_2). The first one is assumed to model complex molecules and the second one represents molecules, which are transformed directly into acetic acid. On the other hand, biomass is also classified into two types, noted X_1 and X_2. X_1 represent the bacteria populations, which transform *equivalent glucose* substrates. X_2 stands for bacteria degrading *equivalent acetate* substrates. This classification allows keeping only two stages: the methanogenesis, which is the limiting one

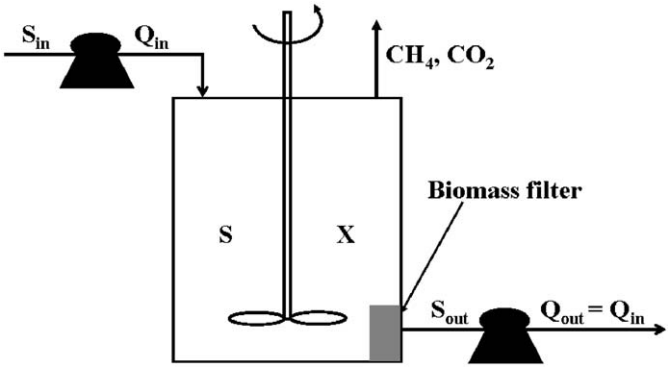

Fig. 1. Continuous configuration for a completely stirred tank reactor.

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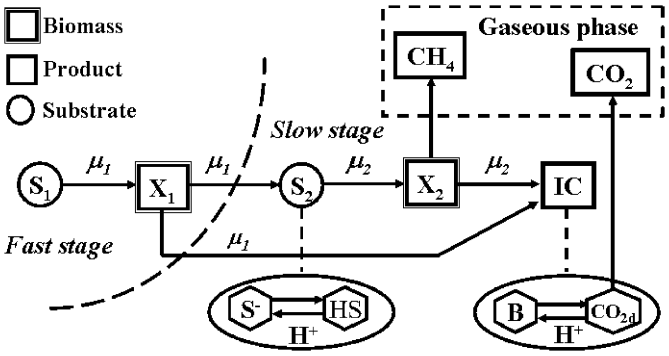

Fig. 2. A functional scheme of the anaerobic digestion process.

(the most interesting for the automatic control and energy approaches), and a preliminary stage composed mainly by the hydrolysis. In fact, acidogenesis and acetogenesis are so fast and can be neglected in face of methanogenesis dynamic; however, hydrolysis is not as fast and could not be neglected. Thus, the simplified process is represented as a fast stage and a slow one; it is shown in Fig. 2 (Beteau, 1992; Rozzi, 1984).

The physico-chemical phenomena (basic–acid equilibria and mass conservation) are modelled by five algebraic equations. Biological phenomena (biomass growth and substrate transformation) are represented by ordinary differential equations, which also include the hydrodynamic phenomena. Finally, the gaseous phase, which is represented by two equations, is considered as the process output. The complete model is as follows:

Algebraic equations:

$$\begin{aligned}
&H^+S^- - K_aHS = 0\\
&HS + S^- - S_2 = 0\\
&H^+B - K_b\mathrm{CO}_{2d} = 0\\
&B + \mathrm{CO}_{2d} - IC = 0\\
&B + S^- - Z = 0
\end{aligned} \tag{1}$$

Differential equations:

$$\begin{aligned}
\frac{\mathrm{d}X_1}{\mathrm{d}t} &= (\mu_1 - k_{d1})X_1\\
\frac{\mathrm{d}S_1}{\mathrm{d}t} &= -R_6\mu_1 + D(S_{1in} - S_1)\\
\frac{\mathrm{d}X_2}{\mathrm{d}t} &= (\mu_2 - k_{d2})X_2\\
\frac{\mathrm{d}S_2}{\mathrm{d}t} &= -R_3\mu_2X_2 + R_4\mu_1X_1 + D(S_{2in} - S_2)\\
\frac{\mathrm{d}IC}{\mathrm{d}t} &= R_2R_3\mu_2X_2 + R_5\mu_1X_1 - \lambda R_1R_3\mu_2X_2 + D(IC_{in} + b_{inc} - IC)\\
\frac{\mathrm{d}Z}{\mathrm{d}t} &= D(Z_{in} + b_{inc} - Z)
\end{aligned} \tag{2}$$

Output equations:

$$\begin{aligned}
Q_{\mathrm{CH}_4} &= R_1R_2\mu_2X_2\\
Q_{\mathrm{CO}_2} &= \lambda R_3R_2\mu_2X_2
\end{aligned} \tag{3}$$

with

$$\mu_1 = \frac{\mu_{1\max}S_1}{K_{s1} + S_1 + \dfrac{S_1HS}{K_{i1}}}, \quad \mu_2 = \frac{\mu_{2\max}HS}{K_{s2} + HS + \dfrac{HS^2}{K_{i2}}}$$

$$\lambda = \frac{P\mathrm{CO}_{2d}}{P_t - P\mathrm{CO}_{2d}}, \quad P\mathrm{CO}_{2d} = \frac{\mathrm{CO}_{2d}}{K_h}$$

where $R_1,\ldots,R_6$ are the yield coefficients, μ_1 and μ_2 are the Haldane growth rates, λ is a coefficient for gaseous carbon dioxide production rate depending on the dissolved carbon dioxide, the reader is referred to Beteau, Otton, Hihn, Delpech, and Cheruy (2004) for further details respecting this coefficient, k_{d1} and k_{d2} are the death rates of X_1 and X_2, respectively, K_{i1} stands for the inhibition constant for substrate 1, K_{i2} stands for the inhibition constant for substrate 2, K_{s1} stands for the dissociation constant for substrate 1 and inorganic carbon, K_{s2} stands for the dissociation constant for substrate 2 and others acids, K_h stands for an equivalent of the Henry constant, $P\mathrm{CO}_{2d}$ stands for the partial pressure for the dissolved carbon dioxide and P_t stands for the atmospheric pressure.

3. Fuzzy logic and PCA preliminaries

3.1. The TS fuzzy systems

The TS system is a special case of the functional fuzzy systems. The premise of fuzzy rules is composed of linguistic terms with an appropriate logical operation (e.g. minimum, maximum or product) and the consequent by dynamic systems. Fuzzy values can be transformed in real values using the centre-average defuzzification. One way to view the TS fuzzy system is as a non-linear interpolator between the linear models, which are defined by the consequents of the rules (Passino & Yurkovich, 1998; Tanaka & Wang, 2001) as follows:

If v_1 is V_1^j and v_2 is V_2^k and,…,and v_k is V_n^l then

$$\begin{aligned}
\dot{x} &= A_ix + B_iu\\
y &= C_ix
\end{aligned}$$

where x, u and y represent, respectively, the vectors of state variables, inputs and outputs for a linearised representation of a non-linear system. A_i, B_i and C_i are Jacobian matrices computed on the i-th operating point of the non-linear system.

Defuzzification can be done by

$$\begin{aligned}
\dot{x} &= \frac{\sum_{i=1}^r \gamma_i\{A_ix + B_iu\}}{\sum_{i=1}^r \gamma_i}\\
y &= \frac{\sum_{i=1}^r \gamma_i\{C_ix\}}{\sum_{i=1}^r \gamma_i}
\end{aligned} \tag{4}$$

γ is known as the membership function and it is calculated as

$$\gamma_i = \prod \gamma[v_j^k] \tag{5}$$

$\gamma[v_j^k]$ is the membership degree of variable v_j on the fuzzy set V_k and $\sum_{i=1}^r \gamma_i = 1$.

In Tanaka and Wang (2001), the authors prove that TS systems are universal interpolators; then, any non-linear system can be represented by this method. In order to obtain the TS representation, different representatives operating points are selected and the non-linear system is linearised around them. Then, at least one variable is selected to detect the linear system which is active. Finally, the active linear systems are interpolated to find back the non-linear dynamics. One main advantage of this representation is the simplification of controllers and observers design for non-linear systems since only linear systems techniques are required. In addition, the control or observation criteria can include some empirical knowledge of the considered process by means of the fuzzy premises.

3.2. Principal components analysis

PCA is a statistical method to ease the representation and the interpretation of a large data set (Jolliffe, 2002; Morineau & Aluja-Banet, 1998; Smith, 2002). The idea is to determinate the principal components, which best characterise the available information. The data set is composed of different behaviours (*variables*) of each member (*values*) in the studied population. In the anaerobic digestion case, for each selected *variable*, different *values* are

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obtained depending on operating conditions. A condition to apply PCA is that *variables* must be interdependent. This dependence allows reducing the number of *variables* to the minimum necessary in order to preserve the essential information (principal components) of the original data set. Hence, the principal components are linear combination of the selected *variables*. The algorithm to find the principal components is resumed as follows:

(1) Initialisation of the data matrix: *Variables* are stocked in columns and *values* in rows as

$$\Psi = \begin{bmatrix} \psi_{11} & \cdots & \psi_{1m} \\ \vdots & \vdots & \vdots \\ \psi_{n1} & \cdots & \psi_{nm} \end{bmatrix} \tag{6}$$

(2) Normalisation of the values: The element ij of matrix Ψ_N is

$$\Psi_N = \begin{bmatrix} \psi_{N11} & \cdots & \psi_{N1m} \\ \vdots & \vdots & \vdots \\ \psi_{Nn1} & \cdots & \psi_{Nnm} \end{bmatrix}, \quad \psi_{Nij} = \frac{\psi_{ij} - \bar{v}_j}{\sigma(\bar{v}_j)} \tag{7}$$

where $\bar{v}_j$ and $\sigma(\bar{v}_j)$ are, respectively, the average and the bias of the variable j.

(3) Computation of the covariance matrix (Ψ_{cov}) for the normalised variables:

$$\Psi_{\text{cov}} = \begin{bmatrix} \psi_{\text{cov }11} & \cdots & \psi_{\text{cov }1m} \\ \vdots & \vdots & \vdots \\ \psi_{\text{cov }n1} & \cdots & \psi_{\text{cov }nm} \end{bmatrix}$$

$$\psi_{\text{cov }ij} = \langle(\psi_{N1i} - \bar{\psi}_{N1i})(\psi_{N1j} - \bar{\psi}_{N1j})\rangle \tag{8}$$

{\hskip 0.16em}$\langle\cdot\rangle$ is the expectation value operator and $\bar{\psi}_{N1j}$ is the arithmetic mean of variable j.

(4) Computation of the correlation matrix (Ψ_ρ) of the covariance one: The elements of that matrix are computed with the next equation:

$$\Psi_{\text{cov}} = \begin{bmatrix} \psi_{\rho 11} & \cdots & \psi_{\rho 1m} \\ \vdots & \vdots & \vdots \\ \psi_{\rho n1} & \cdots & \psi_{\rho nm} \end{bmatrix}$$

$$\psi_{\rho ij} = \frac{\psi_{\text{cov }ij}}{\sqrt{\psi_{\text{cov }ii}\psi_{\text{cov }jj}}} \tag{9}$$

(5) Calculation and decreasing ordering of Ψ_ρ eigenvectors, depending on its associated eigenvalues; these eigenvectors are arranged on the matrix υ.

(6) Finally, computation of principal components matrix as the product of the normalised matrix and the eigenvectors matrix:

$$M_{cp} = \Psi_n \upsilon \tag{10}$$

Matrix (10) contains the coordinates of the *values* in the space formed by the principal components. The data interpretation is done from two graphic representations of principal components: the dispersion diagram and the correlation circle. The first one is used here because it represents a coordinate system (composed by the principal components), which illustrates the influence of *variables* for *values* clustering. Additionally, it allows the similar behaviours and isolated cases to be detected. The second graphic representation is a unitary circle (in function of principal components), and it is employed here because it allows the proposed methodology to point out relations between *variables*.

4. Methodology

4.1. Prototype instrumentation

Fig. 3 portrays the used prototype. The reactor is a tank with a maximal capacity of 7 L. To keep constant the reactor volume, the input and output flow rates are controlled independently by means of pumps. The prototype is equipped with sensors to measure temperature and pH inside the reactor. The temperature is keep at 37 °C by means of a heating silicon belt. A homogeneous environment is assured inside the reactor by a stirring system. The produced biogas (CH_4 and CO_2) is measured by means of an electronic balance. In order to determine CH_4, every 5 h a gas sample is sent to a gaseous phase chromatograph. The instruments are managed by a specialised control system, which is connected to a data acquisition system.

4.2. Analysis of operating conditions

The substrate is a synthetic composition similar to paper mill effluents, which is studied in Barascud, Ehlinger, Pichon, and Ruoger (1992). It is composed of complex macromolecules: glucose, lipids and proteins. In this case, cornstarch is considered as the *equivalent glucose* substrate (S_1). The *equivalent acetate* substrate (S_2) is deduced from the transformation of maltose, glucose and the lactic, acetic and propionic acids into acetate molecules. The substrate time evolution is detected measuring the COD, which is an indicator of the pollution in the substrate. The measure of this variable is easy, but it requires at least 3 h and has to be done off-line. COD is usually measured in anaerobic

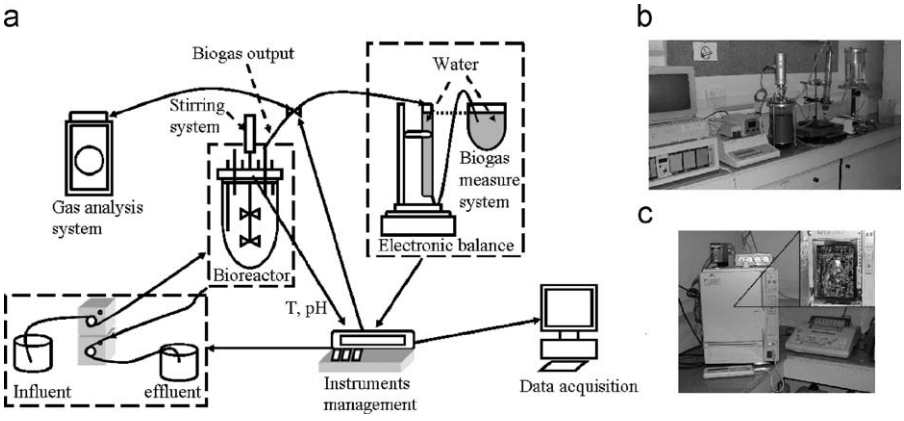

Fig. 3. Anaerobic digestion process prototype: (a) operation scheme, (b) bioreactor and accessories and (c) biogas analysis system.

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wastewater treatment plants; it is associated with S_2 concentration and it is possible to deduce a numerical relation between them (Mosey, 1983):

$$S_{2(g/L)} = 0.66 * COD$$
$$S_{2(mol/L)} = 0.011 * COD \quad (11)$$

Besides, the protonic equilibrium into the reactor is an important factor for the process evolution. It is necessary to have a certain quantity of cations and anions in order to achieve the adequate initial operating conditions. The anions initial quantity is determined as a function of the input COD and the initial pH:

$$S^- = \frac{K_a HS}{H^+} = \frac{H^+ * 0.011 * COD}{K_a + 10^{-pH}} \quad (12)$$

If this initial quantity is not enough to ensure the protonic equilibrium, a base such as sodium bicarbonate must be added. The synthetic substrate resulting from the composition proposed in Barascud et al. (1992) is very acid (pH is around 3.5). In general the pH, which allows the optimal bacteria grow, is around 7. Then, considering the desired value of pH, the desired value of S^-, named $S^-_{desired}$, can be computed. The difference between S^-_{int} and $S^-_{desired}$ corresponds to anions, which must be added to achieve the adequate operating conditions.

Anaerobic bacteria obtained from the wastewater treatment plant of a paper mill are considered in the experimental phase of this work. These bacteria are attached to a solid support named Biolite™ in order to implement biomass filter behaviour. This situation allows the reactor to have a high concentration of biological material; since the support is heavy, it stays in the bottom and only the decayed bacteria leave the reactor by the extraction effect.

4.3. Experimental methodology

First, a batch experiment is performed. The operating conditions considered are 5 L of substrate at 5 g COD/L; initial pH is around 7.4. The reactor is filled with substrate and biomass, and closed hermetically; nitrogen injection can be necessary in order to eliminate the remaining oxygen at the high section inside the reactor. The process parameters (pH, temperature, methane production) are automatically sampled every hour. At the beginning of the experiment, special attention is required: when the acidogenesis phase begins, the pH decreases, and if its value becomes less than 6, the process can be biologically destabilised due to acidification. To avoid this situation, it is advisable to keep pH close to 7. The biogas composition is an indicator of the biological activity inside the reactor; then, the produced biogas is analysed every 4 h. The substrate can be considered as totally transformed when the biogas production becomes negligible; then the biological reaction is over.

At the end of a batch experiment (around ten days) the initial conditions for a continuous experiment are reached. After that the influent and effluent are activated. The input COD is selected by the operator; a value equal or less than the concentration used in the previous phase is recommended in order to avoid an eventual biological instability. When the bacteria are adapted to continuous operating conditions, the input COD can be modified. The biomass adaptation to each different COD level requires 10 or 15 days. This is verified via the pH and methane flow rate variations, which become constant when the biomass is adapted.

4.4. Parameter identification

Parameter identification is used for experimental validation of the process model. The main idea is to obtain the numerical value of each parameter from experimental data and simulations. Different authors mention that parameter identification is an important stage for control and analysis of wastewater treatment plants (Brdys, Grochowski, Gminski, Konarczak, & Drewa, 2008; Simeonov & Queinnec, 2006; Zhang & Hoo, 2008).

The objective of this section is to identify biological and physico-chemical parameters of the model (1–3) using data obtained from the experiments described in Section 4.3. The model method (Lawson & Hauson, 1995; Richalet, Rault, Pouliquen, & Naslin, 1971) is used for parameter identification. This approach is based on the comparison between model simulations and data obtained from experiments. For each measure, the output error is computed (difference between the simulated and the measures) and is used to calculate the next equation, where θ is the parameters vector.

$$J(\theta) = \sum_{experiment} \sqrt{\sum_{sample\_time} \frac{(y_{measured} - y_{simulated})^2}{y^2_{measured}}} \quad (13)$$

After that an optimisation algorithm is implemented in order to find the vector θ, which minimises $J(\theta)$. In this paper, the Matlab™ *minsearch* function is used.

The parameter validation is done comparing the biogas production obtained from the model with the measured one. The resulting values from the parameter identification are shown in Table 1. Fig. 4 illustrates the validation of the identified parameters. The plots correspond to different experiments (different operating conditions). The simulated biogas values are very close to the measured biogas production on each experiment.

5. Observer design

5.1. Linearisation of the process model

Due to model complexity associated with the non-linearity of the biological process, a linearisation analysis around different representative operating points is done. These points are selected considering different values in a valid interval of the input variables, as shown in Table 2.

S_{1in} and Z_{in} are not considered here; the first one because is a variable of the fast stage, which depends directly of the COD concentration (S_{2in}). Besides, the input cations can be calculated from pH; moreover, pH is easy to measure and is a good indicator of the biological activity. Hence, Z_{in} is replaced by pH for this analysis.

One hundred ninety-two input scenarios are possible from the considered values. In Carlos-Hernandez, Mallet and Beteau (2004) a classic stability analysis for the linearised models is presented. This analysis shows that the local models are stable for the considered input intervals; all poles have a real negative component. The biomass hydrodynamic behaviour (biomass filter)

Table 1
Values resulting from parameters identification.

Parameter	Value	Unit	Comment
μ_{2max}	0.012	h^{-1}	Identified
K_{s2}	3.7e−3	mol/L	Identified
K_{i2}	8.22e−4	mol/L	Fixed
R_1	0.54	mol/mol	Identified
R_2	0.6	mol/mol	Identified
R_3	350	mol/AU	Fixed
K_a	7.69e−7	mol	Identified
K_b	1.5e−7	mol	Identified
K_h	0.059	mol/bar	Identified

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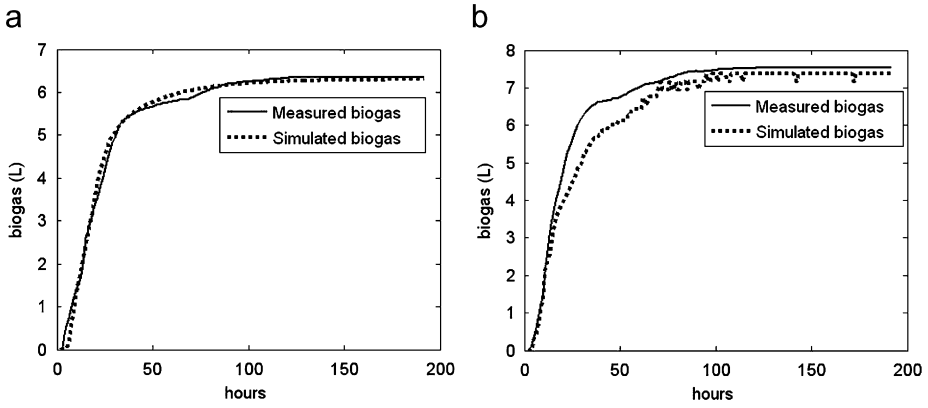

Fig. 4. Identified parameters validation.

Table 2
Intervals of the input variables.

Input variable	Considered values	Unit
Dilution rate (D)	[0.1, 0.25, 0.37, 0.5]	h^{-1}
Substrate 2 (S_{2in})	[0.05, 0.1, 0.17, 0.2]	mol/L
pH	[6.5, 6.7, 7, 7.3, 7.5]	
Inorganic carbon (IC_{in})	[0.03, 0.05, 0.08]	mol/L

induces only one zero, which is slower than the slowest pole; then the global system dynamic is imposed by the biomass. The observability analysis is done using the Rosenbrock theorem (Petkov, Kristov, & Konstantonov, 1991) considering biogas measures. The linear systems do not verify the observability condition because of cations, which are the non-observable state. This situation is not relevant since cations are a biologically inert state and the respective dynamic depends only on the input substrate.

However, the previous analysis is not enough to determine the importance of the different input scenarios on the process evolution. Hence, the following additional analysis is required.

5.2. Methodology for fuzzy variables selection

The different operating conditions generated by the 192 input scenarios are analysed using PCA. A careful study of the inputs variation interval is required to select the set of the operation conditions, and consequently the set of local models, which represent adequately the anaerobic process. Applying the algorithm described in Section 3.2, it is possible to determine that three principal components represent 90% of the whole data set. After that, the dispersion diagram and the correlation circle between the three principal components must be studied.

Fig. 5 shows the dispersion diagram and the correlations circle considering the first and the second principal components. The dispersion diagram illustrates the linear models grouping from dilution rate (“vertical grouping”) and pH (“horizontal grouping”). Dispersion diagrams for pair first–third components and pair second–third are similar to the diagram shown in Fig. 5. This analysis determines the relevance of D and pH in the process behaviour.

Concerning the correlation circle, the variables are classified in three groups.

Group 1: D, CH_4, biomass X_2 at equilibrium, and poles corresponding to S_1, S_2, IC and Z. Group 2: Substrate S_2 at equilibrium and poles corresponding to X_1 and X_2. Group 3: IC_{in} and pH.

Variables, which compose a group, are strongly correlated, i.e. the variations in a specific variable affect the others. In the first group, D is the only input variable; then this is a representative variable for group 1. In the second group there are not input variables but the group is anticorrelated with group 3; then, the variables of group 2 affect the group 3 ones. Since pH is an indicator of the biological activity, it is selected as a representative variable for groups 2 and 3. This information confirms the relevance of D and pH in the process behaviour. The first–third and second–third circles do not supply relevant complementary information concerning correlations between variables, and then that circles are not further considered.

From this analysis, it is possible to conclude that pH and D are relevant variables for anaerobic digestion. Then, they are selected as fuzzy variables in order to activate the local linear observers.

5.3. Fuzzy observer development

From the analysis done above, two fuzzy variables are selected: pH and D. The fuzzification is done as shown in Fig. 6. Since there are five sets for pH and four sets for D, 20 local models are selected in order to design 20 local observers (see Appendix A).

There exist two alternatives to solve the unobservability problem due to cations: (a) to separate the observable from the non-observable states and (b) to eliminate cations (non-observable state) from the observer model (since Z is biologically inert and its dynamic depends only on hydrodynamic behaviour) and to consider the equilibrium value for this variable. Additionally, S_1 and X_1 can be eliminated from the observer model because they are related to the fast stage and the main interest is on the slow one. Furthermore, preserving only the slow stage, it is possible to avoid the ill numerical conditioning of the state matrix. Then, 20 linear observers are designed as follows:

$$\dot{\hat{x}} = A_i\hat{x} + B_i u + K_i(y - \hat{y})$$
$$y = C_i\hat{x} \qquad (14)$$

where $i = 1,\ldots, 20$, $A \in \mathbf{R}^{3\times 3}$ is the state matrix, $B \in \mathbf{R}^{3\times 3}$ is the input matrix, $C \in \mathbf{R}^{2\times 3}$ is the output matrix and $K \in \mathbf{R}^{3\times 2}$ is the observer vector gains. The fuzzy observer has three inputs (S_{2in}, IC_{in} and D) and three outputs (X_2, S_2 and IC). It is important to remind that, in this paper, the most important observed variable for future control purposes is X_2. The dual property of the pair controllability—observability is considered in order to apply the LQR approach to obtain K for the local observers. This method computes the optimal K which allows the feedback control $u = -Kx$ to minimise the performance index $J(u) = \int_0^\infty x'Qx + u'Ru$.

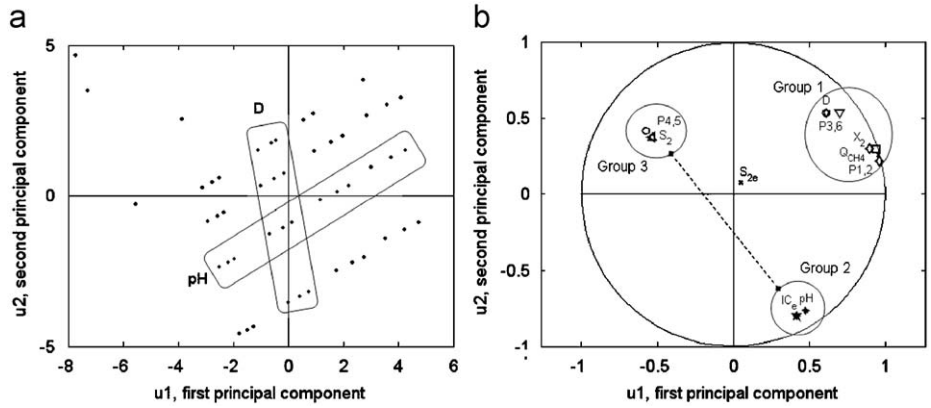

Fig. 5. Graphics of first-second principal components: (a) dispersion diagram and (b) correlation circle.

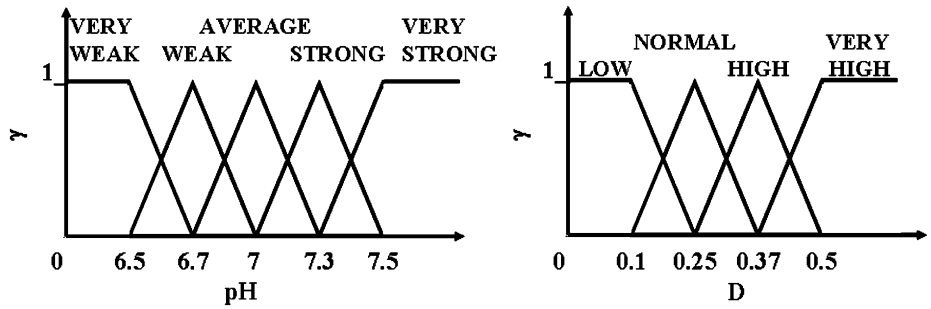

Fig. 6. Fuzzy input variables.

From the fuzzy variables and the local observers, 20 inference rules are implemented. Hence, the fuzzy rules have the structure:

IF pH is $pH(\iota)$ AND D is $D(\varphi)$ THEN the observed state is

$$\dot{\hat{x}} = A_i\hat{x} + B_i u + K_i(y - \hat{y})$$
$$y = C_i\hat{x}$$

where ι stands for VERY WEAK, WEAK, AVERAGE, STRONG or VERY STRONG for pH fuzzy sets and φ stands for LOW, NORMAL, HIGH or VERY HIGH for D fuzzy sets.

To recover the non-linear dynamic, the estimated states supplied by the local observers are interpolated using the defuzzification algorithm described in Section 3.1.

$$\dot{\hat{x}} = \frac{\sum_{i=1}^{r}\gamma_i\{A_i x + B_i u + K_i(y - \hat{y})\}}{\sum_{i=1}^{r}\gamma_i}$$
$$\dot{\hat{y}} = \frac{\sum_{i=1}^{r}\gamma_i\{C_i x\}}{\sum_{i=1}^{r}\gamma_i} \qquad (15)$$

This fuzzy observer works as follows:

Suppose pH = 7.3 and $D = 0.125$. In this case the pH belongs to AVERAGE and STRONG; meanwhile D belongs to: NORMAL and HIGH. Only four inference rules are truth, and then four local observers are active:

R.10. If pH is AVERAGE and D is NORMAL then

$$\dot{\hat{x}} = A_{10}\hat{x} + B_{10}u + K_{10}(y - \hat{y})$$
$$y = C_{10}\hat{x}$$

R.11. If pH is AVERGAE and D is HIGH then

$$\dot{\hat{x}} = A_{11}\hat{x} + B_{11}u + K_{11}(y - \hat{y})$$
$$y = C_{11}\hat{x}$$

R.12. If pH is STRONG and D is NORMAL then

$$\dot{\hat{x}} = A_{14}\hat{x} + B_{14}u + K_{14}(y - \hat{y})$$
$$y = C_{14}\hat{x}$$

R.13. If pH is STRONG and D is HIGH then

$$\dot{\hat{x}} = A_{15}\hat{x} + B_{15}u + K_{15}(y - \hat{y})$$
$$y = C_{15}\hat{x}$$

For the defuzzification step, the observed variables are computed with Eq. (4). The membership degree is $\gamma_{10} = \gamma_{pH(AVERAGE)} * \gamma_{D(NORMAL)} = 0.25$, $\gamma_{11} = \gamma_{pH(AVERAGE)} * \gamma_{D(HIGH)} = 0.25$, $\gamma_{14} = \gamma_{pH(STRONG)} * \gamma_{D(NORMAL)} = 0.25$ and $\gamma_{15} = \gamma_{pH(STRONG)} * \gamma_{D(HIGH)} = 0.25$, any other combination (inactive inference rules) is zero. Then, $\sum_{i=1}^{r}\gamma_i = 1$.

For this specific case, it is easy to see that each linear fuzzy observer contributes with 25 percent of the non-linear behaviour of the observed variables.

The fuzzy observer stability is analysed by means of the theorem further detailed and proven in (Ma, Sun & He, 1998; Tanaka & Wang, 2001), which states the following: the equilibrium of the continuous fuzzy observation system described by (15) is globally asymptotically stable if there exists a common positive definite matrix P such that

$$G_{ii}^{T}P + PG_{ii} < 0$$
$$\left(\frac{Gij + Gji}{2}\right)^{T}P + P\left(\frac{Gij + Gji}{2}\right) \leqslant 0 \qquad (16)$$

where

$$Gii = A_i - K_i C_i$$
$$Gij = A_i - K_j C_i$$

For $i<j$ and $\gamma_i \cap \gamma_j \neq \phi$, note that $\gamma_i \cap \gamma_j = \phi$ if and only if the i-th and j-th rules have not overlap.

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Considering the fuzzy sets and the process dynamic, a maximum of four rules are fired simultaneously. The possible combinations of overlapping rules are shown in Table 3. Using the Matlab LMI toolbox, it is ready to validate that the matrix

$$P = \begin{bmatrix} 73.3252 & 0.0980 & 0.0189 \\ 0.0980 & 0.5343 & -0.0135 \\ 0.0189 & -0.0135 & 0.0925 \end{bmatrix}$$

Table 3
Possible combination of active rules.

Rule	Fired simultaneously
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

which is definite positive, verifies the two stability conditions for all the overlapping rules. Then the fuzzy observer proposed in this paper is globally asymptotically stable.

6. Results

6.1. Validation via simulation

Figs. 7 and 8 illustrate the fuzzy observer performances considering simulations close to experimental conditions. This observer is initialised arbitrarily to test the convergence, which is clearly shown at the beginning of the simulation (Fig. 7). After convergence, the biomass is well estimated. On the other hand, when the input substrate changes due to this disturbance, a transitory error in the estimation of S_2 and IC is noted. However, this error does not represent an important problem since it is eliminated in steady state; furthermore, the substrate S_2 and the inorganic carbon IC can be calculated off-line from chemical analysis of substrate samples.

Disturbances such as model errors and noise in the outputs are considered (Fig. 8). For most cases, the biomass is well estimated, but in presence of model errors where a bias is noted. The substrate and the inorganic carbon are always estimated with a bias.

6.2. Experimental validation

For the experimental validation, the measures of pH, dilution rate and biogas production are required. The pH and the dilution rate are used to select the active local observers. The fuzzy observer is tested using different experiments. The results shown in Fig. 9 correspond to the experiment with the largest experimentation time. This experiment is developed with a volume of 5 L inside the reactor. The batch configuration is used as start-up stage during 192 h with 5 L of substrate at 5 g COD/L. A first continuous configuration stage is developed during 237 h

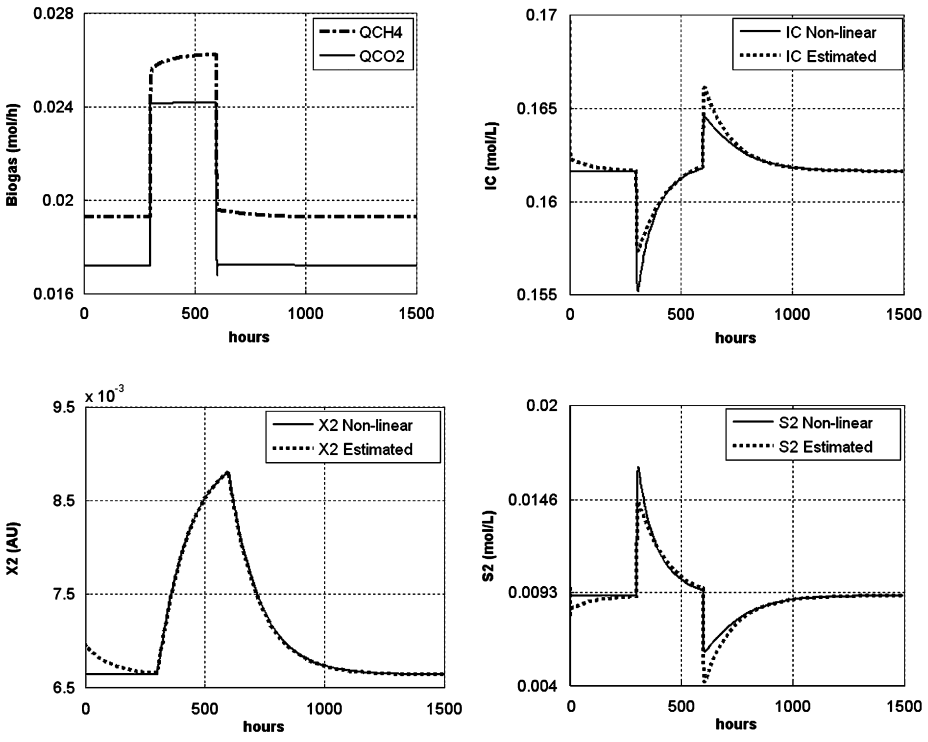

Fig. 7. Observer performances considering a square wave (amplitude: 30 percent of S_{2in}, 200 h).

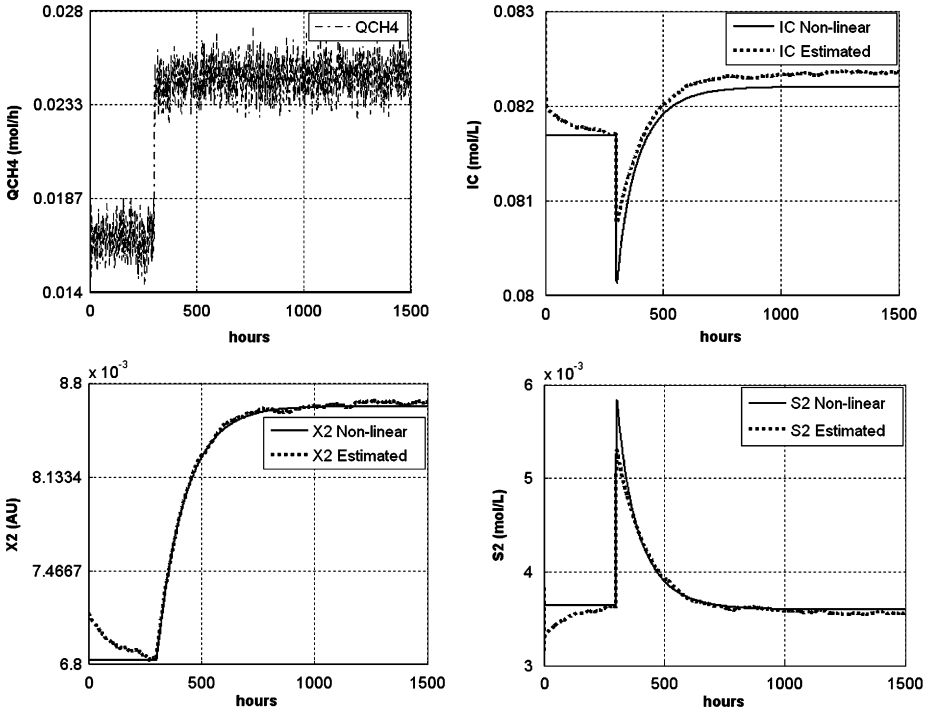

Fig. 8. Observer performances considering noise on the measures.

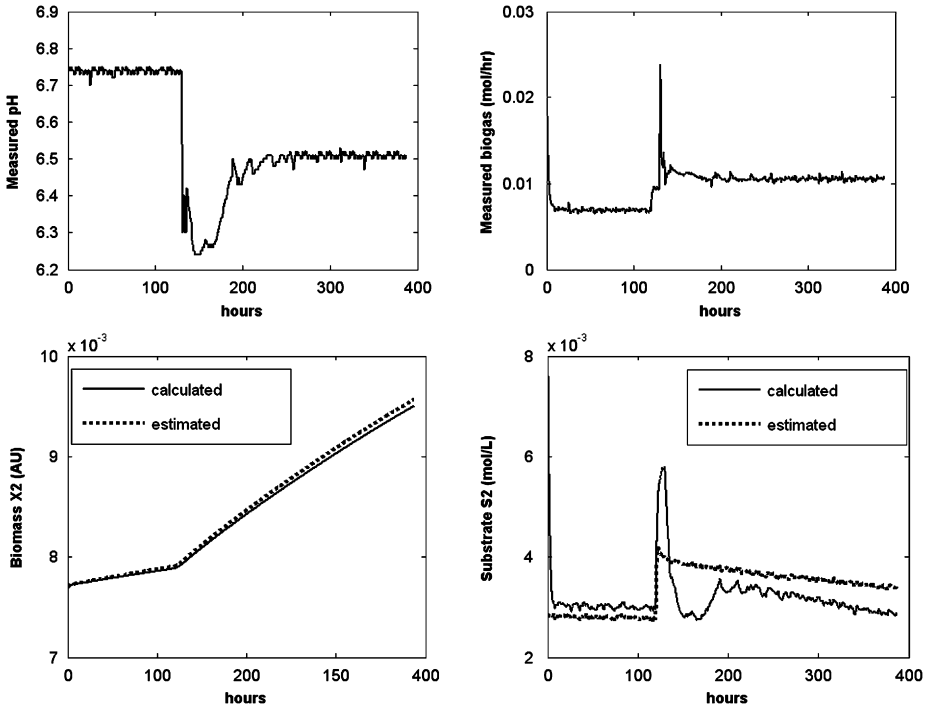

Fig. 9. Experimental validation of the fuzzy observer.

with an input flow rate of 0.4 L/h at 5 g COD/L, and a corresponding dilution rate $D = 0.08\,\mathrm{h}^{-1}$. Finally, a second continuous configuration stage (259 h) with an input flow rate of 0.4 L/h at 7.5 g COD/L is performed to simulate a step on $S_{2\mathrm{in}}$.

At the end of the batch experiment, values of biomass and substrate determine the initial conditions for the subsequent continuous stage. During the transition from batch to continuous operation, the process presents an oscillatory behaviour. This transitory stage is neglected. The observer is validated using the measures from time equals to 295 h to the end of the experiment.

Since the biomass cannot be measured, X_2 is calculated using the model validated experimentally and the on-line pH measures, is compared with the values obtained from the observer. Fig. 9 displays such comparison.

The biomass X_2 continuously grows to a steady value. There are two possible reasons to explain such behaviour: (a) in previous experiments, the biomass was under high temperature producing

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selection bacteria and modifying the X_2 dynamics and (b) the step in S_{2in} is a big one (50 percent of its initial value). These situations may require a longest response time to reach steady state than usual. On the other hand, the substrate estimation presents a transitory error and a bias as in Section 6.1. This error could be due to the model linearisation: it is possible that the number of local models is not sufficient enough to represent the non-linear dynamics, or the local models are not able to follow immediately the fast change of operating conditions. To solve this new problem, the number of local models could be increased; it is also advisable to adapt the gain K as a function of operation conditions.

On the other hand, the fuzzy observer is designed only for state variable estimation and not for fault detection. However, combining different approaches, such as interval estimation or unknown inputs, with the methodology proposed here, a fault detection methodology could be developed.

7. Conclusions

A fuzzy observer development methodology based on the TS and the PCA approaches has been proposed to estimate the state variables, mainly biomass, of an anaerobic digestion process in a CSTR. The variables are estimated from methane and carbon dioxide flow rates, which are typically measured for this kind of process. The fuzzy observer is designed using local linear observers interpolated with the TS algorithm. Additionally, the PCA is used to determine the variables, which activate the local linear observers in function of operating conditions. Twenty local observers selected by two fuzzy variables (D and pH) compose the fuzzy observer. Simulation results show satisfying performances of the observer in presence of input disturbances and measurement noise. The biomass is well estimated and the substrate and inorganic carbon are estimated with a transitory and a bias. However, these errors are not an essential problem since the substrate evolution and the inorganic carbon can be measured off-line. Besides, experimental validation illustrates the observer performances: the biomass is well estimated considering disturbances in the input variables, the substrate is estimated with a bias and a transitory error as in simulations. This fuzzy observer offers a good compromise between the quality of the estimation and the difficulty of implementation. For future works a possible solution to improve the observer performances, concerning the model errors and the substrate estimation, could be to design a fuzzy algorithm, which will allow the gain observer to be modified in function of operating conditions, similar to the high gain approach.

Acknowledgements

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Appendix A. Local observers

Local observer no. 1

$$A_1 = \begin{bmatrix} 0 & 0.0265 & 0.0009 \\ -2.9050 & -9.3493 & -0.3052 \\ 0.4039 & 0.2390 & -1.1253 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0.0688 & 0 & 0.1 \\ -0.0289 & 0.1 & 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 2.9050 & 2.5011 \\ 9.9625 & 9.0236 \\ 0.3184 & 1.3438 \end{bmatrix}^{\mathrm{T}}, \quad K_1 = \begin{bmatrix} 0.0010 & 0.0001 \\ -0.0001 & 0.0002 \\ 0.0003 & 0.0002 \end{bmatrix}$$

Local observer no. 2

$$A_2 = \begin{bmatrix} 0 & 0.0662 & 0.0023 \\ -2.9050 & -23.373 & -0.7630 \\ 0.4039 & 0.5976 & -2.8132 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0.0688 & 0 & 0.25 \\ -0.0289 & 0.25 & 0 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 2.9050 & 2.5011 \\ 23.156 & 22.559 \\ 0.7960 & 3.3595 \end{bmatrix}^{\mathrm{T}}, \quad K_2 = \begin{bmatrix} 0.9936 & 0.0826 \\ 0.1245 & 0.2278 \\ 0.1252 & 0.2505 \end{bmatrix} \times 10^{-3}$$

Local observer no. 3

$$A_3 = \begin{bmatrix} 0 & 0.0979 & 0.0034 \\ -2.9050 & -34.593 & -1.1292 \\ 0.4039 & 0.8845 & -4.1636 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0.0688 & 0 & 0.37 \\ -0.0289 & 0.37 & 0 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} 2.9050 & 2.5011 \\ 34.2713 & 33.3872 \\ 1.1781 & 4.9720 \end{bmatrix}^{\mathrm{T}}, \quad K_3 = \begin{bmatrix} 0.9833 & 0.0810 \\ 0.1644 & 0.2296 \\ 0.0906 & 0.2491 \end{bmatrix} \times 10^{-3}$$

Local observer no. 4

$$A_4 = \begin{bmatrix} 0 & 0.1323 & 0.0045 \\ -2.9050 & -46.745 & -1.5259 \\ 0.4039 & 1.1952 & -5.6265 \end{bmatrix}$$

$$B_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0.0688 & 0 & 0.5 \\ -0.0289 & 0.5 & 0 \end{bmatrix}$$

$$C_4 = \begin{bmatrix} 2.9050 & 2.5011 \\ 46.313 & 45.118 \\ 1.5920 & 6.7189 \end{bmatrix}^{\mathrm{T}}, \quad K_4 = \begin{bmatrix} 0.9778 & 0.0802 \\ 0.1860 & 0.2308 \\ 0.0715 & 0.2481 \end{bmatrix} \times 10^{-3}$$

Local observer no. 5

$$A_5 = \begin{bmatrix} 0 & 0.0171 & 0.0009 \\ -2.9050 & -6.0706 & -0.3059 \\ 0.3962 & -0.2396 & -1.1255 \end{bmatrix}$$

$$B_5 = \begin{bmatrix} 0 & 0 & 0 \\ 0.0682 & 0 & 0.1 \\ -0.0283 & 0.1 & 0 \end{bmatrix}$$

$$C_5 = \begin{bmatrix} 2.9050 & 2.5088 \\ 5.9838 & 6.2236 \\ 0.3191 & 1.3447 \end{bmatrix}^{\mathrm{T}}, \quad K_5 = \begin{bmatrix} 0.0016 & 0.0001 \\ -0.0005 & 0.0002 \\ 0.0007 & 0.0002 \end{bmatrix}$$

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Local observer no. 6

$$A_6 = \begin{bmatrix} 0 & 0.0427 & 0.0023 \\ -2.9050 & -15.177 & -0.7648 \\ 0.3962 & -0.5990 & -2.8137 \end{bmatrix}$$

$$B_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0.0682 & 0 & 0.25 \\ -0.0283 & 0.25 & 0 \end{bmatrix}$$

$$C_6 = \begin{bmatrix} 2.9050 & 2.5088 \\ 14.960 & 15.559 \\ 0.7978 & 3.3617 \end{bmatrix}^{\mathrm{T}}, \quad K_6 = \begin{bmatrix} 0.0016 & 0.0001 \\ -0.0001 & 0.0002 \\ 0.0003 & 0.0002 \end{bmatrix}$$

Local observer no. 7

$$A_7 = \begin{bmatrix} 0 & 0.0633 & 0.0034 \\ -2.9050 & -22.461 & -1.1319 \\ 0.3962 & -0.8866 & -4.1643 \end{bmatrix}$$

$$B_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0.0682 & 0 & 0.37 \\ -0.0283 & 0.37 & 0 \end{bmatrix}$$

$$C_7 = \begin{bmatrix} 2.9050 & 2.5088 \\ 22.140 & 23.027 \\ 1.1807 & 4.9754 \end{bmatrix}^{\mathrm{T}}, \quad K_7 = \begin{bmatrix} 0.0015 & 0.0001 \\ 0.0001 & 0.0002 \\ 0.0002 & 0.0002 \end{bmatrix}$$

Local observer no. 8

$$A_8 = \begin{bmatrix} 0 & 0.0855 & 0.0046 \\ -2.9050 & -30.353 & -1.5295 \\ 0.3962 & -1.1981 & -5.16274 \end{bmatrix}$$

$$B_8 = \begin{bmatrix} 0 & 0 & 0 \\ 0.0682 & 0 & 0.5 \\ -0.0283 & 0.5 & 0 \end{bmatrix}$$

$$C_8 = \begin{bmatrix} 2.9050 & 2.5088 \\ 29.919 & 31.118 \\ 1.5956 & 6.7235 \end{bmatrix}^{\mathrm{T}}, \quad K_8 = \begin{bmatrix} 0.0015 & 0.0001 \\ 0.0001 & 0.0002 \\ 0.0001 & 0.0002 \end{bmatrix}$$

Local observer no. 9

$$A_9 = \begin{bmatrix} 0 & 0.0090 & 0.0009 \\ -2.9050 & -3.2337 & -0.3036 \\ 0.3753 & -0.6543 & -1.1266 \end{bmatrix}$$

$$B_9 = \begin{bmatrix} 0 & 0 & 0 \\ 0.0664 & 0 & 0.1 \\ -0.0266 & 0.1 & 0 \end{bmatrix}$$

$$C_9 = \begin{bmatrix} 2.9050 & 2.5297 \\ 3.1469 & 3.8013 \\ 0.3168 & 1.3435 \end{bmatrix}^{\mathrm{T}}, \quad K_9 = \begin{bmatrix} 0.0032 & 0.0002 \\ -0.0027 & 0.0002 \\ 0.0025 & 0.0003 \end{bmatrix}$$

Local observer no. 10

$$A_{10} = \begin{bmatrix} 0 & 0.0225 & 0.0023 \\ -2.9050 & -8.0843 & -0.7590 \\ 0.3753 & -1.6358 & -2.8165 \end{bmatrix}$$

$$B_{10} = \begin{bmatrix} 0 & 0 & 0 \\ 0.0664 & 0 & 0.25 \\ -0.0266 & 0.25 & 0 \end{bmatrix}$$

$$C_{10} = \begin{bmatrix} 2.9050 & 2.5297 \\ 7.8673 & 9.5034 \\ 0.7920 & 3.3588 \end{bmatrix}^{\mathrm{T}}, \quad K_{10} = \begin{bmatrix} 0.0031 & 0.0002 \\ -0.0009 & 0.0002 \\ 0.0010 & 0.0003 \end{bmatrix}$$

Local observer no. 11

$$A_{11} = \begin{bmatrix} 0 & 0.0333 & 0.0033 \\ -2.9050 & -11.965 & -1.1233 \\ 0.3753 & -2.4209 & -4.1685 \end{bmatrix}$$

$$B_{11} = \begin{bmatrix} 0 & 0 & 0 \\ 0.0664 & 0 & 0.37 \\ -0.0266 & 0.37 & 0 \end{bmatrix}$$

$$C_{11} = \begin{bmatrix} 2.9050 & 2.5297 \\ 11.643 & 14.065 \\ 1.1721 & 4.9710 \end{bmatrix}^{\mathrm{T}}, \quad K_{11} = \begin{bmatrix} 0.0030 & 0.0002 \\ -0.0005 & 0.0002 \\ 0.0007 & 0.0003 \end{bmatrix}$$

Local observer no. 12

$$A_{12} = \begin{bmatrix} 0 & 0.0450 & 0.0045 \\ -2.9050 & -16.169 & -1.5179 \\ 0.3753 & -3.2715 & -5.6331 \end{bmatrix}$$

$$B_{12} = \begin{bmatrix} 0 & 0 & 0 \\ 0.0664 & 0 & 0.5 \\ -0.0266 & 0.5 & 0 \end{bmatrix}$$

$$C_{12} = \begin{bmatrix} 2.9050 & 2.5297 \\ 15.735 & 19.007 \\ 1.5839 & 6.7175 \end{bmatrix}^{\mathrm{T}}, \quad K_{12} = \begin{bmatrix} 0.0030 & 0.0002 \\ -0.0003 & 0.0002 \\ 0.0003 & 0.0002 \end{bmatrix}$$

Local observer no. 13

$$A_{13} = \begin{bmatrix} 0 & 0.0049 & 0.0009 \\ -2.9050 & -1.7922 & -0.2960 \\ 0.3325 & -0.8662 & -1.1293 \end{bmatrix}$$

$$B_{13} = \begin{bmatrix} 0 & 0 & 0 \\ 0.0627 & 0 & 0.1 \\ -0.0233 & 0.1 & 0 \end{bmatrix}$$

$$C_{13} = \begin{bmatrix} 2.9050 & 2.5725 \\ 1.7054 & 2.5717 \\ 0.3092 & 1.3386 \end{bmatrix}^{\mathrm{T}}, \quad K_{13} = \begin{bmatrix} 0.0061 & 0.0005 \\ -0.0106 & -0.0001 \\ 0.0095 & 0.0005 \end{bmatrix}$$

Local observer no. 14

$$A_{14} = \begin{bmatrix} 0 & 0.0122 & 0.0022 \\ -2.9050 & -4.4805 & -0.7401 \\ 0.3325 & -2.1654 & -2.8233 \end{bmatrix}$$

$$B_{14} = \begin{bmatrix} 0 & 0 & 0 \\ 0.0627 & 0 & 0.25 \\ -0.0233 & 0.25 & 0 \end{bmatrix}$$

$$C_{14} = \begin{bmatrix} 2.9050 & 2.5725 \\ 4.2636 & 6.4292 \\ 0.7731 & 3.3466 \end{bmatrix}^{\mathrm{T}}, \quad K_{14} = \begin{bmatrix} 0.0059 & 0.0004 \\ -0.0041 & 0.0001 \\ 0.0039 & 0.0004 \end{bmatrix}$$

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Local observer no. 15

$$A_{15}=\begin{bmatrix}0 & 0.0180 & 0.0033\\ -2.9050 & -6.6312 & -1.0953\\ 0.3325 & -3.2048 & -4.1785\end{bmatrix}$$

$$B_{15}=\begin{bmatrix}0 & 0 & 0\\ 0.0627 & 0 & 0.37\\ -0.0233 & 0.37 & 0\end{bmatrix}$$

$$C_{15}=\begin{bmatrix}2.9050 & 2.5725\\ 6.3101 & 9.5152\\ 1.1441 & 4.9530\end{bmatrix}^{\mathrm{T}},\quad K_{15}=\begin{bmatrix}0.0058 & 0.0004\\ -0.0027 & 0.0001\\ 0.0026 & 0.0003\end{bmatrix}$$

Local observer no. 16

$$A_{16}=\begin{bmatrix}0 & 0.0244 & 0.0044\\ -2.9050 & -8.9611 & -1.4801\\ 0.3325 & -4.308 & -5.6466\end{bmatrix}$$

$$B_{16}=\begin{bmatrix}0 & 0 & 0\\ 0.0627 & 0 & 0.5\\ -0.0233 & 0.5 & 0\end{bmatrix}$$

$$C_{16}=\begin{bmatrix}2.9050 & 2.5725\\ 8.5271 & 12.8584\\ 1.5461 & 6.6932\end{bmatrix}^{\mathrm{T}},\quad K_{16}=\begin{bmatrix}0.0058 & 0.0004\\ -0.0019 & 0.0002\\ 0.0019 & 0.0003\end{bmatrix}$$

Local observer no. 17

$$A_{17}=\begin{bmatrix}0 & 0.0033 & 0.0044\\ -2.9050 & -1.2487 & -0.2865\\ 0.2803 & -0.9473 & -1.1326\end{bmatrix}$$

$$B_{17}=\begin{bmatrix}0 & 0 & 0\\ 0.0585 & 0 & 0.1\\ -0.0194 & 0.1 & 0\end{bmatrix}$$

$$C_{17}=\begin{bmatrix}2.9050 & 2.6247\\ 1.1619 & 2.1093\\ 0.2997 & 1.3324\end{bmatrix}^{\mathrm{T}},\quad K_{17}=\begin{bmatrix}0.0090 & 0.0004\\ -0.0241 & -0.0007\\ 0.0215 & 0.0010\end{bmatrix}$$

Local observer no. 18

$$A_{18}=\begin{bmatrix}0 & 0.0083 & 0.0021\\ -2.9050 & -3.1217 & -0.7162\\ 0.2803 & -2.3682 & -2.8315\end{bmatrix}$$

$$B_{18}=\begin{bmatrix}0 & 0 & 0\\ 0.0585 & 0 & 0.25\\ -0.0194 & 0.25 & 0\end{bmatrix}$$

$$C_{18}=\begin{bmatrix}2.9050 & 2.6247\\ 2.9047 & 5.2732\\ 0.7492 & 3.3309\end{bmatrix}^{\mathrm{T}},\quad K_{18}=\begin{bmatrix}0.0086 & 0.0006\\ -0.0096 & -0.0003\\ 0.0089 & 0.0007\end{bmatrix}$$

Local observer no. 19

$$A_{19}=\begin{bmatrix}0 & 0.0123 & 0.0032\\ -2.9050 & -4.6201 & -1.0599\\ 0.2803 & -3.5050 & -4.1906\end{bmatrix}$$

$$B_{19}=\begin{bmatrix}0 & 0 & 0\\ 0.0585 & 0 & 0.37\\ -0.0194 & 0.37 & 0\end{bmatrix}$$

$$C_{19}=\begin{bmatrix}2.9050 & 2.6247\\ 4.2990 & 7.8044\\ 1.1088 & 4.9298\end{bmatrix}^{\mathrm{T}},\quad K_{19}=\begin{bmatrix}0.0086 & 0.0006\\ -0.0064 & -0.0001\\ 0.0060 & 0.0005\end{bmatrix}$$

Local observer no. 20

$$A_{20}=\begin{bmatrix}0 & 0.0166 & 0.0043\\ -2.9050 & -6.2434 & -1.4323\\ 0.2803 & -4.7364 & -5.6630\end{bmatrix}$$

$$B_{20}=\begin{bmatrix}0 & 0 & 0\\ 0.0585 & 0 & 0.5\\ -0.0194 & 0.5 & 0\end{bmatrix}$$

$$C_{20}=\begin{bmatrix}2.9050 & 2.6247\\ 5.8095 & 10.5464\\ 1.4984 & 6.6619\end{bmatrix}^{\mathrm{T}},\quad K_{20}=\begin{bmatrix}0.0085 & 0.0006\\ -0.0047 & -0.0001\\ 0.0045 & 0.0005\end{bmatrix}$$

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