Identification of Resonance Waves in Open Water Channels

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Abstract

This article describes a way to determine the properties of resonance (reflecting) waves in open water channels. For channels that are sensitive to resonances, information about the first resonance mode is required for controller and filter design. This research applies standard System Identification techniques and is tested on an actual channel at the Central Arizona Irrigation and Drainage District, Eloy, AZ. The identification experiment results in good estimations of the frequency and magnitude of the first resonance peak of the open water channel.
Irrigation systems in the Western part of the United States consist of multiple open water channels in series, delivering water to remote areas. To maximize the yield of these agricultural areas, it is important that the delivery of water to these sites is accurate and flexible. This can be seen as the main function of an irrigation canal. The operational management of the water system infrastructure hence, plays a vital role in the performance of an irrigation canal.

Automated irrigation systems can meet the goals of being accurate and flexible, if the controllers that operate the structures are correctly tuned. The most common control loop for an irrigation open water channel consists of the following elements: a sensor that measures the water level, a low-pass anti-aliasing filter, a Programmable Logic Controller or microprocessor that calculates the required control actions based on the differences between measurement and setpoint and a structure, such as an undershot or overshot gate, that implements the control action. This loop is repeated at a fixed interval and is referred to as feedback control. A control algorithm that is simple, has a good performance over the full range of flow conditions and keeps the water levels at a predefined target level is the Proportional Integral (PI) feedback controller. Multiple successful implementations of this type of controller have been reported in the Western part of the US, Europe and Australia (Burt & Piao (2002), Malaterre & Baume (1998a), Ooi & Weyer (2008)). Often, the water level measurement is located at the downstream end of the channel and this measurement signal is communicated to the upstream structure (see Figure 1). This setup is referred to as distant downstream control. It has the advantage that the offtakes, which are generally located at the downstream end of the open water channel, have an upstream water level that is (close to being) constant. This makes flow delivery to these offtakes more accurate. Also, the embankment of the channel can remain more or less parallel to the bed slope, which is less costly to construct.

The dynamic behavior of open water channels can be described with the well-known Saint-Venant equations (Chow (1959)). These are partial differential equations that consist of a mass balance and momentum equations. Some channels have structures, such as culverts, which can be described by another type of momentum equation. The momentum equations are non-linear. As control theory is mainly based on linear systems and ordinary differential equations, simplification of the dynamic behavior is required. In Schuurmans (1997), a division into two classes of open water channels is made. One class is defined as channels that are long and shallow. The dynamic behavior is mainly determined by the long delay times of the water that has to travel from the upstream side to the downstream end of the channel. In Schuurmans et al. (1999) and later in Litrico & Fromion (2006), research has been conducted to find stable and well-performing controllers for these channels without an extensive trial-and-error tuning procedure. Their research has resulted in controller tuning rules for this class of open water channels based on the delay time $\tau_d$ and the storage area $A_s$ of the back water part. The other class of channels can be characterized by being short and deep and is dominated by reflecting waves. After a change in the boundaries of the system, for example the opening of a gate or a pump that is turned off, a wave travels up and down the channel a number of times before the water surface settles again. Also in Litrico & Fromion (2004a), Overloop (2006) and Ooi & Weyer (2009), these so-called resonance waves are described and analyzed. The main reason for the badly damped behavior in short and deep channels is the low bed friction force along the open water channel. Figure 1 shows a representation of the properties of the two classes of open water channels. For the resonance class open water channel, only the resonance mode with the lowest
frequency is shown (first peak in frequency spectrum). It is important to consider exactly this frequency, as the measurement location for distant downstream control is in counter-phase (phase shift of -180°) with the control actions. According to the Nyquist stability criterion (Vandevegte (1990)), this control system will, in closed loop, be unstable, when the magnitude of this resonance peak, in open loop, is larger than 1. Schuurmans (1997) proposed to filter the resonance wave with a first-order low-pass filter and developed a tuning rule for the filter. This rule requires input of the frequency $\omega_p$ and the magnitude of the first peak $M_p$ of the resonance wave in the frequency spectrum.

At present, stable and well-performing PI controllers for all classes of open water channels can be directly determined when the properties delay time, storage area, frequency and peak magnitude of the lowest resonance mode can be estimated accurately at low and high flow (Schuurmans (1997)). Other implementations of feedback controllers (Weyer 2003) require this first resonance frequency for selection of the cut-off frequency of the applied high-order low-pass filter.

![Figure 1. Delay-time-dominated open water channel (left) and resonance-dominated open water channel (right) including control loop](image)

This article describes a system identification experiment that was executed on an actual open water channel that was specifically designed to estimate the important first resonance mode. First, the dynamic behavior of open channels is described in a simplified manner, focused on the first resonance mode. In the following section, system identification methods for estimating the properties of an open water channel are discussed. Next, the applied system identification experiment on an actual canal is described. This section includes the results. The last sections are the discussions section leading to the conclusions of the article.

*Open water channel dynamics*
The flows and water levels in an open channel can be described by the Saint-Venant equations (Chow (1959)). These nonlinear hyperbolic partial differential equations consist of a mass balance and a momentum balance. The momentum balance is a summation of the descriptions for the inertia (1), advection (2), gravitational force (3) and friction force (4):

\[
\frac{\partial Q}{\partial x} + \frac{\partial A_f}{\partial t} = q_{lat} \tag{Eq. 1}
\]

\[
\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A_f}\right) + g \cdot A_f \frac{\partial h}{\partial x} + \frac{g \cdot Q |Q|}{C^2 \cdot R_f \cdot A_f} = 0 \tag{Eq. 2}
\]

where \(Q\) represents the flow \((m^3/s)\), \(t\) the time \((s)\), \(x\) the distance \((m)\), \(A_f\) the wetted area of the flow \((m^2)\), \(q_{lat}\) the lateral inflow per unit length \((m^2/s)\), \(g\) the gravitational acceleration \((\approx 9.81 \text{ m/s}^2)\), \(h\) the water level \((\text{mMeanSeaLevel})\), \(C\) the Chézy friction coefficient \((\text{m}^{1/2}/\text{s})\) and \(R_f\) the hydraulic radius \((m)\). \(R_f\) is calculated by \(A_f\) over \(P_f\), where \(P_f\) represents the wetted perimeter \((m)\). Figure 2 gives a schematic representation of a typical open channel with its variables and parameters.

To use the formulas in a numerical model of an open water channel, the partial differential equations are discretized in time \((\Delta t)\) and space \((\Delta x)\). In case these discretized formulas are simulated, the model results in time series solutions of water levels and flows at discrete locations along the channel. Also, the time series are discrete solutions in time.

In order to clarify the resonance behavior of an open water channel, a horizontal channel with a rectangular cross section is discretized in two elements (see Figure 3).
When the water level changes are small compared to the depth in the channel, the advection can be neglected and the depth $d$, wetted area $A_f$ and the wetted radius $R_f$ can be considered constant. A constant positive mean flow $Q_m$ is present and the lateral inflow $q_{lat}$ is zero. After linearization and transformation to the Laplace-domain, Eq. 2 and 3 (in time-domain) and Eq. 5 to 7 (in Laplace-domain) can be derived.

$$\frac{\Delta Q(t)}{\Delta x} + W \cdot \frac{dh(t)}{dt} = 0 \quad \text{(Eq. 3)}$$

$$\frac{dQ(t)}{dt} + \frac{g \cdot W \cdot d \cdot (h_2(t) - h_1(t))}{\Delta x} + 2 \cdot \frac{g \cdot Q_m}{C^2 \cdot R_f \cdot W \cdot d} \cdot Q(t) - \frac{g \cdot Q_m^2}{C^2 \cdot R_f \cdot W \cdot d} = 0 \quad \text{(Eq. 4)}$$

$$h_1(s) = \frac{Q_1(s) - Q(s)}{\frac{1}{2} \cdot L \cdot W \cdot s} \quad \text{(Eq. 5)}$$

$$h_2(s) = \frac{Q(s) - Q_2(s)}{\frac{1}{2} \cdot L \cdot W \cdot s} \quad \text{(Eq. 6)}$$

$$Q(s) + \frac{g \cdot W \cdot d \cdot (h_2(s) - h_1(s))}{\frac{1}{2} \cdot L \cdot s} + 2 \cdot \frac{g \cdot Q_m}{C^2 \cdot R_f \cdot W \cdot d \cdot s} \cdot Q(s) = \frac{g \cdot Q_m^2}{C^2 \cdot R_f \cdot W \cdot d \cdot s} \quad \text{(Eq. 7)}$$

where $s$ represents the Laplace operator, $L$ the length of the channel (m), $\Delta x$ the discretization length ($=\frac{1}{2} \cdot L$) (m), $h_1$ the water level in the upstream element, $h_2$ the water level in the downstream element, $Q_1$ and $Q_2$ the (positive) in- and outflows ($m^3/s$), $Q_m$ the (positive) mean flow through the channel ($m^3/s$), $W$ the width of the channel (m) and $d$ the depth (m).

Substitution of Eq. 5 and 6 in Eq. 7 and rearranging the terms gives:
Eq. 6 can be used to arrive at the total transfer function of all inputs in which the transfer function from the upstream inflow $Q_1$ to the distant downstream water level $h_2$ is of order three:

$$Q(s) = \frac{4 \cdot g \cdot d}{s^2 + \frac{2 \cdot g \cdot Q_m}{C^2 \cdot R_f \cdot W \cdot d} \cdot s + \frac{8 \cdot g \cdot d}{L^2}} \cdot (Q_1(s) + Q_2(s)) + \frac{g \cdot Q_m^2}{s^2 + \frac{2 \cdot g \cdot Q_m}{C^2 \cdot R_f \cdot W \cdot d} \cdot s + \frac{8 \cdot g \cdot d}{L^2}}$$

(Eq. 8)

To quantify the damping in a second order resonance system, commonly, the damping ratio $\zeta$ is used (Vandevegte (1990)). For the second order resonance system, in the transfer function from $Q_1$ to $h_2$, the damping ratio is:

$$h_2(s) = \frac{8 \cdot g \cdot d}{s \left( s^2 + \frac{2 \cdot g \cdot Q_m}{C^2 \cdot R_f \cdot W \cdot d} \cdot s + \frac{8 \cdot g \cdot d}{L^2} \right)} \cdot Q_1(s)$$

$$+ \frac{2}{L \cdot W \cdot s} \left( \frac{4 \cdot g \cdot d}{L^2} \right) \cdot Q_2(s)$$

$$+ \frac{2 \cdot g \cdot Q_m^2}{L \cdot C^2 \cdot R_f \cdot W^2 \cdot d}$$

$$+ \frac{2 \cdot g \cdot Q_m}{C^2 \cdot R_f \cdot W \cdot d} \cdot s + \frac{8 \cdot g \cdot d}{L^2}$$

(Eq. 9)

Values of $\zeta$ smaller than 1 indicate that a resonance wave is present. The natural frequency $\omega_0$ (rad/s) of this system is:

$$\omega_0 = \sqrt{\frac{8 \cdot g \cdot d}{L^2}}$$

(Eq. 11)

As badly damped systems are considered in this research ($\zeta << 1$), the damped natural frequency $\omega_d$ as given in Eq. 12 is considered equal to the natural frequency.

$$\omega_d = \omega_0 \sqrt{1 - \zeta^2} \approx \omega_0$$

(Eq. 12)
When taking a hydro-dynamic starting point, the resonance frequency can also be estimated as 
\((2\pi\times)\) the reciprocal of the time \(T_{\omega_0}\) it takes a wave to travel forth and back through an open water channel of length \(L\). The velocity of this wave can be estimated using the celerity \(c\) (m/s), defined as the velocity by which a wave front travels (Chow (1959)). The celerity is commonly estimated as:

\[ c = \sqrt{g \cdot d} \]  
(Eq. 13)

\[ \omega_{0,\text{est}} = \frac{2 \cdot \pi \cdot c}{2 \cdot L} \approx \sqrt{8 \cdot \frac{c}{L}} = \frac{\sqrt{8 \cdot g \cdot d}}{L} = \omega_0 \]  
(Eq. 14)

It is remarkable to find that the natural frequency of the simplified model results in a close approximation of the resonance frequency as estimated from a wave travelling down and up the actual channel. Apparently, the lowest resonance frequency in a channel is mainly depending on the depth and the length and hardly depending on the other parameters and variables, such as width, flow, hydraulic radius and even friction. This is in line with results found in Overloop (2006).

The magnitude of the resonance peak of the two element model can be calculated by substituting the Laplace operator by \(j.\omega_0\) for analysis in the frequency domain (\(j\) is the complex number for which is defined: \(j^2=-1\)):

\[ M(\omega_0) = \begin{vmatrix} \frac{8 \cdot g \cdot d}{W \cdot L^3} \\ -j \cdot \omega_0^3 \end{vmatrix} = \begin{vmatrix} C^2 \cdot R_f \cdot d \\ 2 \cdot g \cdot Q_m \cdot L \end{vmatrix} \]  
(Eq. 15)

Larger depths and smaller length of open water channel lead to higher values of the magnitude of the resonance wave. The same holds for lower flows, higher Chézy friction coefficients (lower friction forces) and higher hydraulic radius.

**Identification methods applied to open water channels**

Deriving the properties from theoretical or numerical models of the dynamic behavior of open water channels can be risky, as there can be mismatches between actual channel and model. To identify the actual properties, field experiments can be executed. Below, various identification methods are described.

A step test is commonly used to find the delay time \(\tau_d\) and the storage area \(A_s\). In this type of tests, starting from an initial steady state in the channel, a positive step change in flow is applied at the upstream structure, while the downstream flow is kept constant. The measured water level trajectory typically shows a delay time before the water level start to rise with a more or less constant slope that is determined by the size of the storage area. Figure 4 presents the result of a step test on an open water channel.
Another field experiment to identify the delay time and the storage area is applied in the Auto Tune Variation proposed by Litrico et al. (2007). This method applies a switching pattern of upward and downwards steps. The switches are made when the water level reaches the allowed minimum or maximum of the band around the target level.

The resonance properties, especially the resonance peak magnitude $M_p$, are harder to identify. In Figure 4, a resonance wave can be distinguished, super-positioned on the sloped trajectory. The resonance frequency can be roughly estimated according to (the first part of) Eq. 14. An experiment to identify the resonance peak magnitude can be executed by applying a Proportional feedback controller. The experiment starts with a low value of the proportional gain $K_p$ of the controller and at each interval the gain is turned up with a small increment. After some time, an oscillation with a frequency $\omega_p$ an increasing amplitude will occur indicating that the controlled open water channel becomes unstable (the amplitude of the oscillating water level error continues to rise with time). The critical value $K_{p,cr}$ at which the system becomes unstable can be used to calculate the resonance peak magnitude (Vandevegte (1990)):

$$M_p = \frac{1}{K_{p,cr}}$$

(Eq. 16)
Obviously, applying this experiment on a channel is risky, as oscillating water levels with increasing amplitude are generated. Another, more controlled, experiment is to apply a switching flow pattern at the upstream structure (alternating positive and negative steps). The experiment, known as Chirp-test, starts with a switch-frequency slightly lower than the a-priori estimated resonance frequency and, gradually, the frequency is increased and exceeds the resonance frequency. For resonance-dominated open water channels, the oscillating water level signal will show an increase in amplitude at the exact resonance frequency.

The previously described experiments aim at finding one or a subset of the properties that are required to tune a controller for resonance-dominated channels. Potentially, the engineering field of System Identification (Ljung (1999)), in which models with low order are estimated from input and output signals, can produce the full subset of required properties. Silvis et al. (1998) apply SI-techniques to identify the Integrator Delay model (Schuurmans (1997)) from experimental data. In Eurén & Weyer (2007) the same model of an integrator and a delay in series is used. The difference between the two approaches is that the first article applies flows at the control structures as input variables, while the second article uses the positions of the undershot and overshot control gates. Rivas Perez et al. (2007) identify a second order model in series with a delay time. The second order model is suggested, as a step test often shows a fast upwards step at the beginning and a slower sloped behavior later on (also see Figure 4). The authors claim that this first step is due to motor and gate dynamics, but this seems unrealistic, as these dynamics would more likely slow down the water level increase at the beginning due to inertia and hysteresis in the equipment. In Litrico & Fromion (2004b) the first step upwards is linked to the high frequency behavior of resonance-dominated water channels that consists of the first resonance and its higher harmonics. In that article, a low order model is proposed that contains a zero in the transfer function in order to model this high-frequency resonance behavior.

Weyer (2001) applies a delay time in series with a third order model. The reasoning behind this is that this model captures the storage area behavior (first order) in parallel with the first resonance wave (second order oscillating mode).

In this article, based on Miltenburg (2008), various model structures are tested on actual measurement data in order to find the structure that best captures the frequency response of the system around the first resonance mode. System Identification techniques are applied to estimate the parameters of the models.

**System Identification applied to a resonance-dominated open water channel**

Experiments are conducted at the irrigation district office of the Central Arizona Irrigation and Drainage District in Arizona (www.caidd.com). The water level and flow measurements are sent to the central computer at a sample rate of once per 10 seconds (= T_s). The flow input signal is sent to the gates at a rate of once every 20 seconds. Communication is realized by means of radio-signals. As the open water channels are part of an operational delivery system, the experiments took place at night.

Multiple experiments on various open water channels of the irrigation district are performed. In this article, the experiment on open water channel NB12 is presented, which is supposed to be
resonance-dominated. The dimensions of the channel are presented in Table 1. Figure 5 represents the longitudinal profile of the open water channel NB12. The first two resonance modes (shifted in level) are schematized by dashed lines.

<table>
<thead>
<tr>
<th>Length (m)</th>
<th>Bed slope (-)</th>
<th>Bottom width (m)</th>
<th>Side slope (-)</th>
<th>Average depth (m)</th>
<th>Base Flow (m$^3$/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>786</td>
<td>2.0e-4</td>
<td>0.61</td>
<td>1.5</td>
<td>0.80</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 1. Dimensions of open water channel NB12

Figure 5. Backwater curve of open water channel NB12 with two resonance modes

Figure 6 presents the Bode diagram of a high-order linearized Saint-Venant model ($\Delta x=10$) of the resonance-dominated channel NB12 with a customary Mannings-friction coefficient of 0.02 s/m$^{1/3}$ (≈ Chézy 43 m$^{1/2}$/s). In this Bode diagram, it can be observed that the frequency of the first resonance peak is $\omega_{0,est} \approx 0.0085$ rad/s and that the magnitude at that frequency has an magnitude of approximately 0.24. Note that the estimate for the frequency without using a Saint-Venant model, according to Eq. 14 is 0.011 rad/s. In the sequel, prediction error identification will be used to refine the model estimates of the first resonance frequency and of its magnitude. The estimates of these quantities deduced from Figure 6 will be used in the design of the identification experiment.
Figure 6. Bode diagram of high-order linearized transfer function from upstream flow to downstream water level

Experiment setup

In order to excite the system, a Random Binary Signal (RBS) is super-positioned on the upstream flow. The amplitude of this RBS is here approximately equal to 0.075 m$^3$/s. In general, the amplitude should be of at least 25% of the base flow in order to guarantee a sufficient signal-to-noise ratio.

The upstream flow perturbed in this manner is applied at the upstream gate, while keeping the outflow constant. The initial condition in the channel has to be steady and the water level has to be close to target level. In order to contain enough information, the duration of the experiment is chosen to be equal to approximately 25 times the fundamental period $2\pi/\omega_{1,est}$ of the estimated first resonance frequency. The duration of the experiment was therefore of approximately 5 hours and 20 minutes which corresponds to 1937 data points with the available sampling time of 10 seconds.

Preprocessing of the data
Both the applied input signal (i.e. the measured upstream structure flow) and the measured output signal (i.e. the measured water level at the downstream end of the channel) are cleared of outliers. Their means are removed and they are subsequently also de-trended with a linear interpolation method. Finally, they are divided into two data sets: one for the identification itself and one for the validation step. 75% of the data length (i.e. 1453 data points) was used for identification purpose and the rest (i.e. 484 data points) is reserved for the validation of the identified model.

The to-be-identified system contains an integrator (see also low-frequency behavior in Figure 6) and it is well known that identification of a system with an integrator is troublesome as it is on the edge of being unstable. Consequently, the output data have to be differentiated using a tamed differentiator $D$. A tamed differentiator only differentiates the signal up to a certain frequency $\omega_d$ in order to avoid any amplification of high frequency noise. The frequency $\omega_d$ is here chosen equal to 0.002 rad/s. Since this frequency is four times smaller than the expected resonance frequency $\omega_{0,est}$, the resonance peaks are not affected by the differentiator.

With the original sampling time $T_s = 10$ seconds, the frequency response of the system could be described up to 0.31 rad/s (the Nyquist frequency $\pi/T_s$). In this frequency range, the frequency response of the system is expected to present a large number of resonance peaks (see Figure 6). In order to avoid a model of too large order and to avoid the classical problems arising when identifying dynamics in distant frequency ranges, the input-output data are decimated by a factor 10 (after the application of an anti-aliasing filter). The new sampling time is therefore equal to $T_s^* = 100$ seconds. This choice for the decimation factor has been done by trial and error.

**Identification of the model using prediction error identification**

Prediction error identification (Ljung (1999)) will now be used in order to deduce a model between the input signal and the (differentiated) output signal. A Box-Jenkins structure is used for this purpose. This model structure models the process transfer function as well as the noise perturbing the output. Even though a description of the noise is not required here, it was decided to model the noise as well, since it can be proven that this is more optimal statistically (see Ljung (1999)). In order to find a model of a too large order and to avoid the classical problems arising when identifying dynamics in distant frequency ranges, the input-output data are decimated by a factor 10 (after the application of an anti-aliasing filter). The new sampling time is therefore equal to $T_s^* = 100$ seconds. This choice for the decimation factor has been done by trial and error.

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\[
\hat{G} = z^{-2} \cdot \frac{-0.03874 z^4 + 0.1817 z^3 - 0.1426 z^2 + 0.09231 z}{z^4 - 1.064 z^3 + 1.121 z^2 - 0.4134 z + 0.2917}
\]

\(^1\)The number of delays in the model has been determined by trial and error and by observing the dimensions of the considered open water channel.
and the following noise model:

\[
\hat{H} = \frac{z^5 + 0.2249z^4 - 0.9687z^3 + 0.5155z^2 + 0.7963z - 0.05723}{z^5 - 0.3568z^4 - 0.6657z^3 + 0.5195z^2 + 0.6471z - 0.4518}
\]

(Eq. 18)

where \(z^{-1}\) is the discrete backwards step of \(T_s^*\) seconds.

Figure 7 presents the residual tests corresponding to the model \(\hat{G}, \hat{H}\) and shows that this model is indeed validated.

The model \(\hat{G}\) is a model relating the applied flow input to the differentiated water level. In order to determine a model relating effectively the flow and the water level, the identified model \(\hat{G}\) above must be multiplied by the inverse of \(D^*\). \(D^*\) is the tamed differentiator \(D\) re-sampled to produce a transfer function with sampling time \(T_s^* = 100\) seconds. The Matlab function \(d2d\) can be used for this purpose. The model \(\hat{W}\) obtained in this way is \((T_s^* = 100)\):
\[
\hat{W} = \frac{z^{-2} -0.03874z^{-5} + 0.2135z^{-4} -0.2914z^{-3} + 0.2091z^{-2} -0.07558z^{-1}}{z^{-5} -2.064z^{-4} + 2.185z^{-3} -1.534z^{-2} + 0.705z - 0.2917}
\]  \hspace{1cm} (Eq. 19)

The Bode diagram of this fifth order model is represented in Figure 8, where the integrator behavior in low frequencies and the first two resonance peaks can be observed.

![Bode diagram of fifth order transfer function of open water channel](image)

Figure 8. Bode diagram of fifth order transfer function of open water channel \((M_p = 0.223)\)

**Validation of the model**

Now, the validation data set to verify the accuracy of model \(\hat{W}\) is used. The input signal in this data set is simulated with model \(\hat{W}\) and this simulated output \(y_{sim}\) is compared with the actual output signal \(y_{val}\) in the validation data set (after de-trending). This delivers Figure 9. The fit defined as:
Fit\%(k) = 1 - \frac{1}{N} \sqrt{\sum_{k=1}^{N} (y_{val}(k) - y_{sim}(k))^2} \sqrt{\frac{1}{N} \sum_{k=1}^{N} y_{val}(k)^2} \\
\text{(Eq. 20)}

is here equal to 78\%.

Figure 9. Validation of estimated model (solid) and measurements (dashed) of open water channel (top-graph is input signal flow around base flow at upstream side of the channel, while bottom-graph is output signal water level around average depth at downstream side).

Comparison of the followed procedure and the procedures followed in literature

The main difference between the followed procedure and the procedure proposed in the literature e.g. Weyer (2001), is that the order of the to-be-identified transfer function is not fix a-priori. Indeed, the order of the model will depend on the number of resonance peaks present in the frequency range up to the chosen Nyquist frequency. With the Nyquist frequency chosen in this paper (i.e. \( \pi/100 \approx 0.03 \text{ rad/s} \)), choosing a model of order three seems not appropriate since this
would only allow you to identify the integrator and only one resonance peak. The model will therefore be biased and this bias can have a negative influence on the accuracy of the model. In order to illustrate this, the same data is used and a third order model \( \hat{W}^* \) is identified. For this purpose, instead of using a fourth order model to identify \( \hat{G} \), a second order model \( \hat{G}^* \) is used. In Figure 10, it can be seen that this model is not validated. A second order model structure is chosen, because this indeed delivers a third order model \( \hat{W}^* \) after the multiplication with the inverse of \( D^* \). The Bode plot of this third order model \( \hat{W}^* \) is given in Figure 12 and the results of this model with the validation data set is given in Figure 11. The fit is here equal to 69%.

![Figure 10. Model residuals for the identified second order BJ-model](image-url)
Figure 11. Simulation results of third order model over validation period
Figure 12. Bode diagram of third order model ($M_p = 0.193$)

**Resulting properties**

Finally, in this results section, the properties are derived from the identified fifth order model. Table 2 presents the determined properties.

<table>
<thead>
<tr>
<th>Estimated delay time estimated $\tau_d$ (s)</th>
<th>Estimated storage area $A_s$ (m$^2$)</th>
<th>Estimated resonance frequency $\omega_p$ (rad/s)</th>
<th>Estimated resonance peak magnitude $M_p$ (s/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>5217</td>
<td>0.0088</td>
<td>0.223</td>
</tr>
</tbody>
</table>

Table 2. Properties of open water channel found from identification experiment

**Discussion**
This discussion section is mainly based on the experiment described in this paper, but also on other identification experiments run (not presented here, but available in Miltenburg (2008)) in the Central Arizona Irrigation and Drainage District where similar results were found.

To begin with, the properties that are not related to the resonance are not consistent with expectations. The estimated storage area of the open water channel $A_s$ is more than twice the top width multiplied by the length of the channel. This seems unrealistic and most probably due to the fact that the data had to be differentiated to get rid of the integrator action. This differentiation could lead to a less accurate model in low frequencies. Also, the delay time $\tau_d$ is identified in a course manner due to the large step size in the procedure. Apparently, for these properties, other identification methods are required that have a more low-frequency focus. As opposed to this, it can be observed that the first resonance frequency of the identified model is equal to 0.0088 rad/s which is very close to the frequency that has been estimated via the St Venant model in Figure 6 and the one estimated via Eq. 14.

When trying to identify the first resonance peak, the fifth order model is an improvement compared to other models that are described in literature. The Integrator Delay-model and the Integrator Delay Zero-model are by nature not capable of estimating resonances, but the third order model is. However, the resulting fit is significantly lower and the model is not validated (no white noise residual). The resulting first resonance peak has a magnitude $M_p$ of 0.193, which is 10% lower than the magnitude estimated by the fifth order model ($M_p = 0.223$). Probably this is due to the inability of the third order model to generate a horizontal tendency at frequencies higher than the first resonance frequency. This horizontal tendency as described in the Integrator Delay Zero model, can be captured to some extend by the fifth order model by allowing it to include the second resonance peak. The result found here is mainly influenced by the chosen Nyquist frequency. By changing this value, more peaks come into the spectrum or are cut-off.

Conclusions

Resonance waves are present in open water channels. The sensitivity for these reflecting waves mainly depends on dimensions of the channel that cause low friction forces opposing the waves travelling up and down the channel. Open water channels that are deep, short, smooth and have low flows are expected to be dominated by resonance behavior.

A field experiment to identify the first resonance mode of an open water channel required for controller design is successfully tested on an actual open water channel in Arizona. In this case, a fifth order model was required to capture the relevant dynamics.

In the experiment, the resonance frequency and peak magnitude of the first resonance could be accurately estimated. For estimating the storage area and delay time, other (low frequency) experiments, such as a step test or the Auto Tune Variation test should be applied.

References


