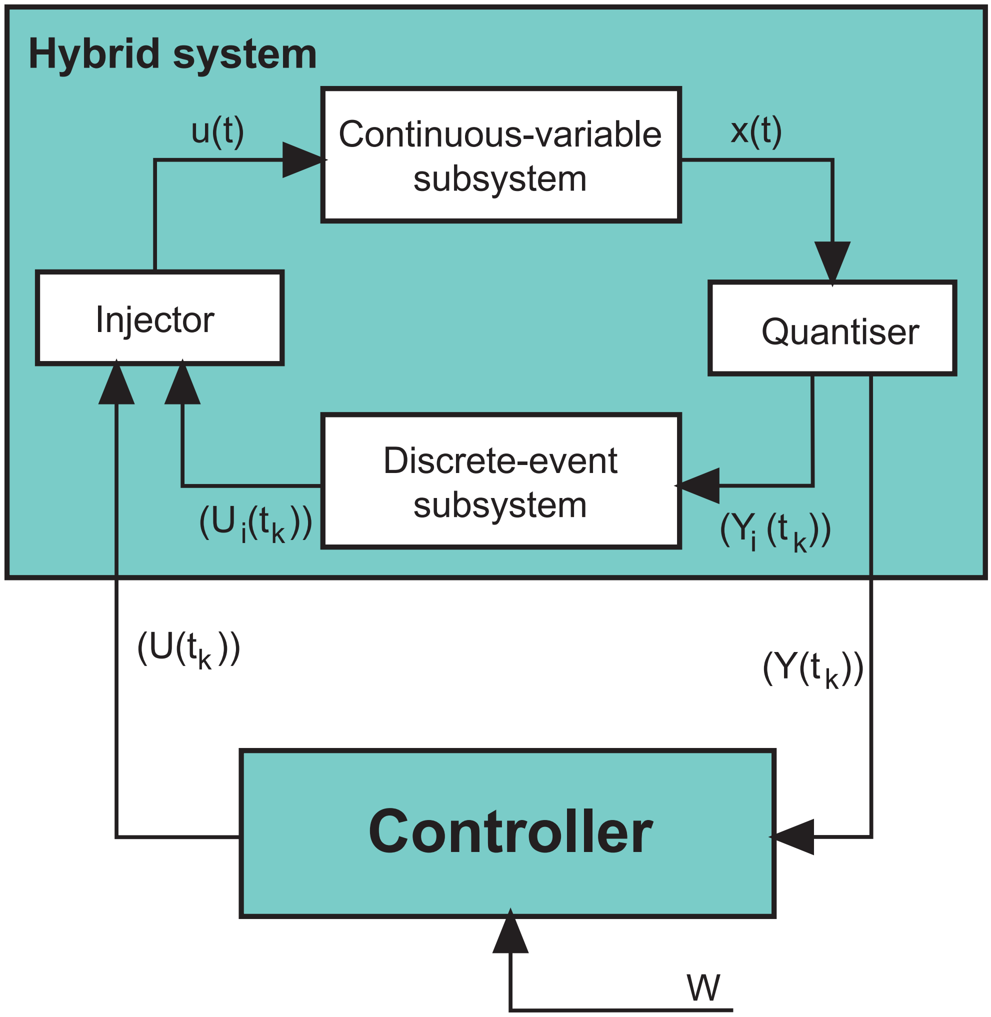
Discrete-event modelling and diagnosis of quantised systems

Jan Lunze

Ruhr-University Bochum
Institute of Automation and Computer Control
email: Lunze@esr.ruhr-uni-bochum.de

1. Introduction to discrete-event modelling of hybrid systems
2. Properties of quantised systems
3. Some basics of automata theory
4. Discrete-event modelling of quantised systems by abstraction
5. Diagnosis of automata
6. Diagnosis of quantised systems
7. Application examples
8. Conclusions

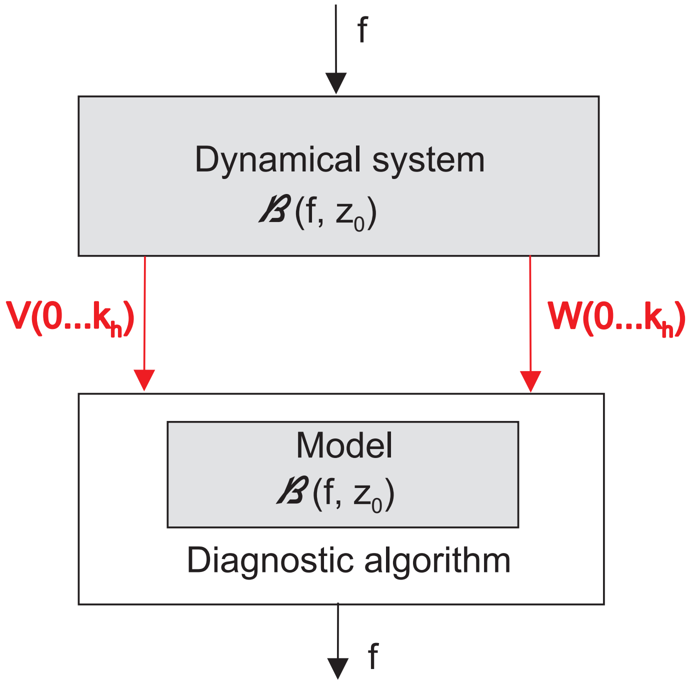
Supervisory control loop



Ways for dealing with hybrid systems

- Combine methods elaborated in continuous and discrete–event systems theories
- Abstract a discrete–event representation of the hybrid system and apply discrete–event systems theory

Model–based diagnosis



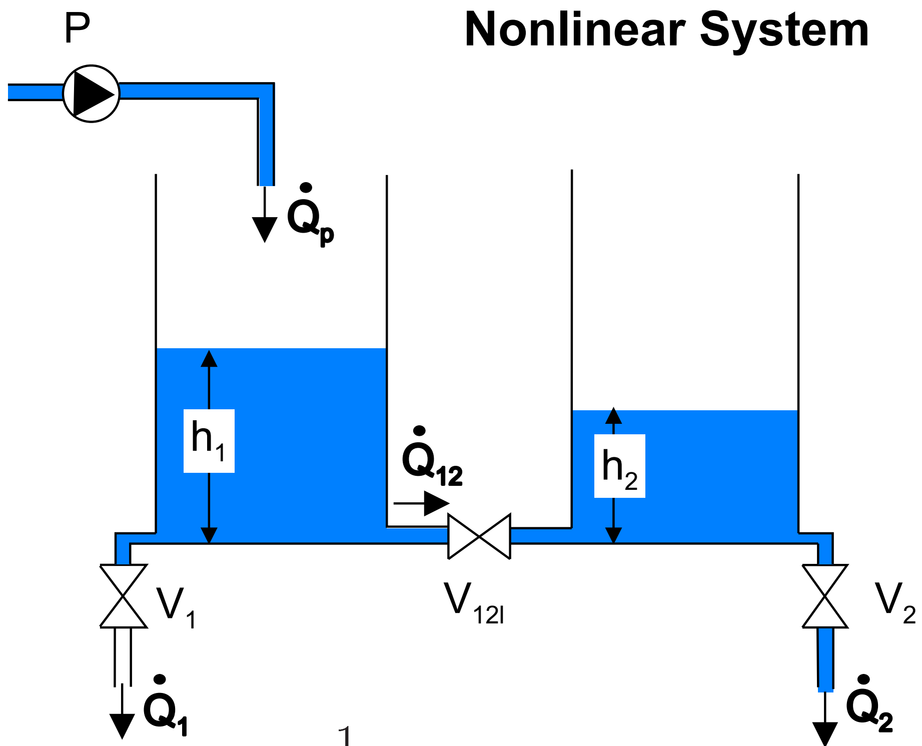
Diagnostic problem

Given: Model depending on f and z_0
Measured I/O pair (V, W)

Consistency-based diagnosis:
Can the system subject to fault f generate the output W if it obtains the input V?

- Diagnostic problems include observation problems.
- **Fault detection:**
  Inconsistency with the faultless system
- **Fault identification:**
  Consistency with the system subject to fault f
  $\rightarrow$ f is a fault candidate

Example: A batch process



$$
\begin{aligned}
\dot{h}_1 &= \frac{1}{A_1}\left(\dot{Q}_p - \dot{Q}_1 - \dot{Q}_{12}\right) \\
\dot{h}_2 &= \frac{1}{A_2}\left(\dot{Q}_{12} - \dot{Q}_2\right) \\
\dot{Q}_1 &= Pos(V_1)\ S_v\sqrt{2gh_1} \\
\dot{Q}_{12} &= Pos(V_{12l})\ S_v\, \mathrm{sgn}(h_1 - h_2)\ \sqrt{2g|h_1 - h_2|} \\
\dot{Q}_2 &= Pos(V_2)\ S_v\ \sqrt{2gh_2} \\
\dot{Q}_p &= p(t)\dot{Q}_{p0}
\end{aligned}
$$

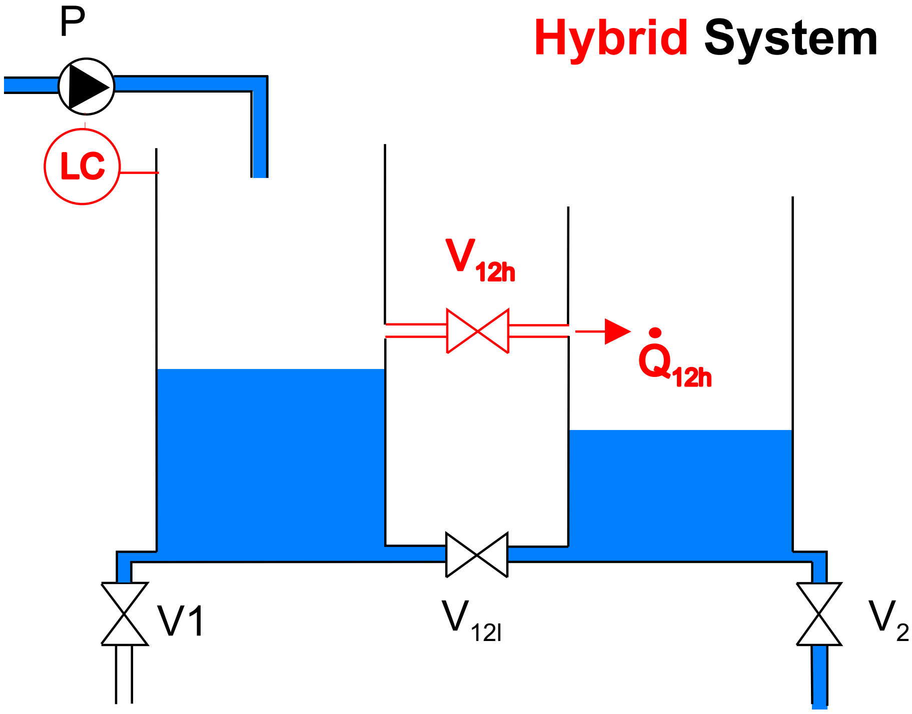
with

$$
\begin{aligned}
0 &\leq h_1(t), h_2(t) \leq h_{max} \\
0 &\leq p(t) \leq 1
\end{aligned}
$$

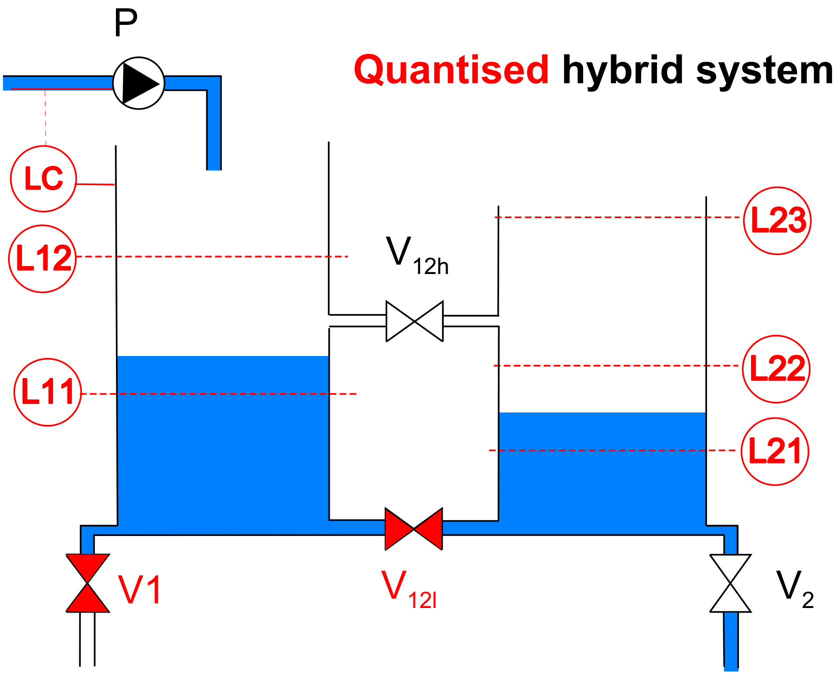
$$\Downarrow$$

$$
\begin{aligned}
\dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t)) \\
\boldsymbol{y}(t) &= \boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{u}(t))
\end{aligned}
$$

Hybrid System



$$
\begin{aligned}
\dot{h}_1 &= \frac{1}{A_1}\left(\dot{Q}_p - \dot{Q}_1 - \dot{Q}_{12} - \dot{Q}_{12h}\right) \\
\dot{h}_2 &= \frac{1}{A_2}\left(\dot{Q}_{12} + \dot{Q}_{12h} - \dot{Q}_2\right) \\
\dot{Q}_1 &= \begin{cases} Pos(V_1)\ S_v\sqrt{2gh_1} & \text{if} \ \ h_1 > 0 \\ 0 & \text{else} \end{cases} \\
\dot{Q}_{12} &= Pos(V_{12l})\ S_v\,\mathrm{sgn}(h_1 - h_2)\sqrt{2g|h_1 - h_2|} \\
\dot{Q}_{12h} &= \begin{cases} Pos(V_{12h})\ S_v\,\mathrm{sgn}(h_1 - h_2)\sqrt{2g|h_1 - h_2|} & \text{if} \ \ h_1, h_2 > h_v \\ Pos(V_{12h})\ S_v\sqrt{2g|h_1 - h_v|} & \text{if} \ \ h_1 > h_v, h_2 \leq h_v \\ -Pos(V_{12h})\ S_v\sqrt{2g|h_2 - h_v|} & \text{if} \ \ h_2 > h_v, h_1 \leq h_v \\ 0 & \text{if} \ \ h_1, h_2 \leq h_v \end{cases} \\
\dot{Q}_2 &= \begin{cases} Pos(V_2)\ S_v\sqrt{2gh_2} & \text{if} \ \ h_2 > 0 \\ 0 & \text{else} \end{cases} \\
\dot{Q}_p &= \begin{cases} p(t)\dot{Q}_{p0} & \text{if} \ \ h_1 < h_{1max} \\ 0 & \text{if} \ \ h_1 \geq h_{1max} \end{cases}
\end{aligned}
$$



$$
\begin{aligned}
\dot h_1 &= \frac{1}{A_1}\left(\dot Q_p - \dot Q_1 - \dot Q_{12} - \dot Q_{12h}\right)\\
\dot h_2 &= \frac{1}{A_2}\left(\dot Q_{12} + \dot Q_{12h} - \dot Q_2\right)\\
\dot Q_1 &= \begin{cases} Pos(V_1)\ S_v\sqrt{2gh_1} & \text{if } \ h_1 > 0\\ 0 & \text{else}\end{cases}\\
\dot Q_{12} &= Pos(V_{12l})\ S_v\,\mathrm{sgn}(h_1-h_2)\ \sqrt{2g|h_1-h_2|}\\
\dot Q_{12h} &= \begin{cases} Pos(V_{12h})\ S_v\,\mathrm{sgn}(h_1-h_2)\ \sqrt{2g|h_1-h_2|} & \text{if } \ h_1,h_2 > h_v\\ Pos(V_{12h})\ S_v\ \sqrt{2g|h_1-h_v|} & \text{if } \ h_1 > h_v, h_2 \le h_v\\ -Pos(V_{12h})\ S_v\ \sqrt{2g|h_2-h_v|} & \text{if } \ h_2 > h_v, h_1 \le h_v\\ 0 & \text{if } \ h_1,h_2 \le h_v\end{cases}\\
\dot Q_2 &= \begin{cases} Pos(V_2)\ S_v\ \sqrt{2gh_2} & \text{if } \ h_2 > 0\\ 0 & \text{else}\end{cases}\\
\dot Q_p &= 0 \qquad \text{if } \ h_1 \ge h_{1max}
\end{aligned}
$$

$$\dot Q_p \in [0.8\,\dot Q_{p0}, \dot Q_{p0}] \quad \text{if } \ h_1 < h_{1max}$$

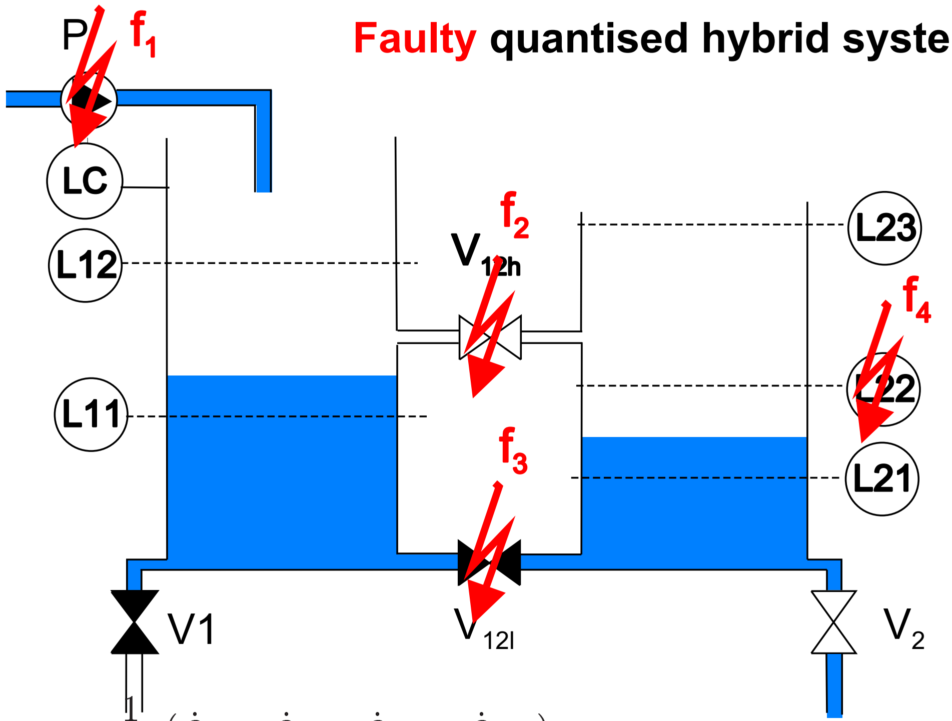
$$Pos(V_1(t)), Pos(V_{12l}(t)) \in \{0,1\}$$

$$L_{11}(t) = \begin{cases} 1 & \text{if } \ h_1(t) > l_{11}\\ 0 & \text{else}\end{cases} \qquad L_{12}(t) = \begin{cases} 1 & \text{if } \ h_1(t) > l_{12}\\ 0 & \text{else}\end{cases}$$

$$L_{21}(t) = \begin{cases} 1 & \text{if } \ h_1(t) > l_{21}\\ 0 & \text{else}\end{cases} \qquad L_{22}(t) = \begin{cases} 1 & \text{if } \ h_1(t) > l_{22}\\ 0 & \text{else}\end{cases}$$

$$L_{23}(t) = \begin{cases} 1 & \text{if } \ h_1(t) > l_{23}\\ 0 & \text{else}\end{cases}$$

Faulty quantised hybrid system



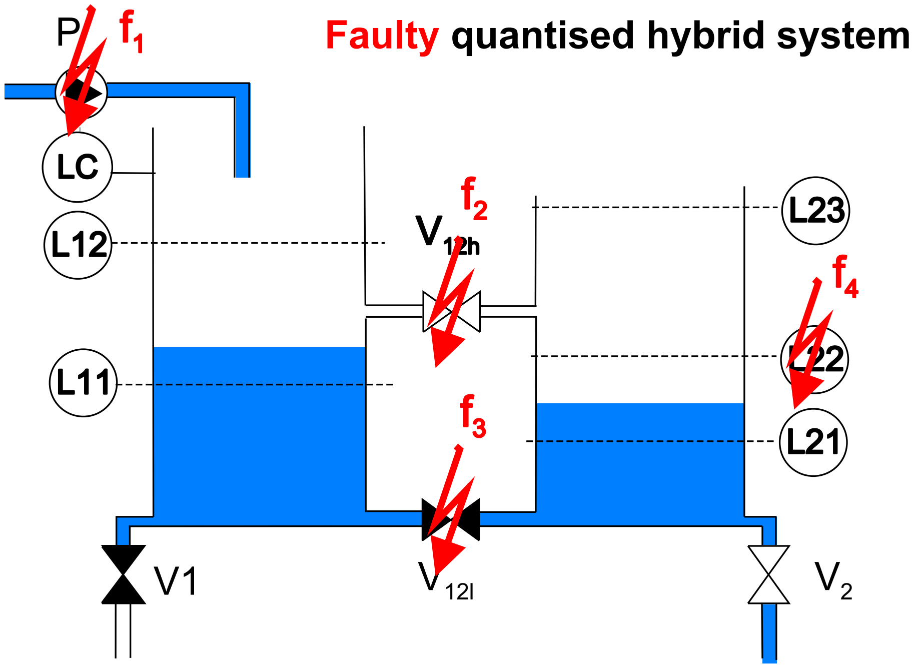
$$
\begin{aligned}
\dot{h}_1 &= \frac{1}{A_1}\left(\dot{Q}_p - \dot{Q}_1 - \dot{Q}_{12} - \dot{Q}_{12h}\right) \\
\dot{h}_2 &= \frac{1}{A_2}\left(\dot{Q}_{12} + \dot{Q}_{12h} - \dot{Q}_2\right) \\
\dot{Q}_1 &= \begin{cases} Pos(V_1)\ S_v\sqrt{2gh_1} & \text{if } \ h_1 > 0 \\ 0 & \text{else} \end{cases} \\
\dot{Q}_{12} &= (1-f_3)Pos(V_{12l})\ S_v \operatorname{sgn}(h_1-h_2)\sqrt{2g|h_1-h_2|} \\
\dot{Q}_{12h} &= \begin{cases} (1-f_2)Pos(V_{12h})\ S_v \operatorname{sgn}(h_1-h_2)\sqrt{2g|h_1-h_2|} & \text{if } \ h_1, h_2 > h_v \\ (1-f_2)Pos(V_{12h})\ S_v\sqrt{2g|h_1-h_v|} & \text{if } \ h_1 > h_v, h_2 \leq h_v \\ -(1-f_2)Pos(V_{12h})\ S_v\sqrt{2g|h_2-h_v|} & \text{if } \ h_2 > h_v, h_1 \leq h_v \\ 0 & \text{if } \ h_1, h_2 \leq h_v \end{cases} \\
\dot{Q}_2 &= \begin{cases} Pos(V_2)\ S_v\sqrt{2gh_2} & \text{if } \ h_2 > 0 \\ 0 & \text{else} \end{cases} \\
\dot{Q}_p &= \begin{cases} (1-f_1)p(t)\dot{Q}_{p0} & \text{if } \ h_1 < h_{1max} \\ 0 & \text{if } \ h_1 \geq h_{1max} \end{cases} \\
Pos(V_1(t)), Pos(V_{12l}(t)) &\in \{0,1\} \\
f_1, f_2, f_3, f_4 &\in \{0,1\}
\end{aligned}
$$

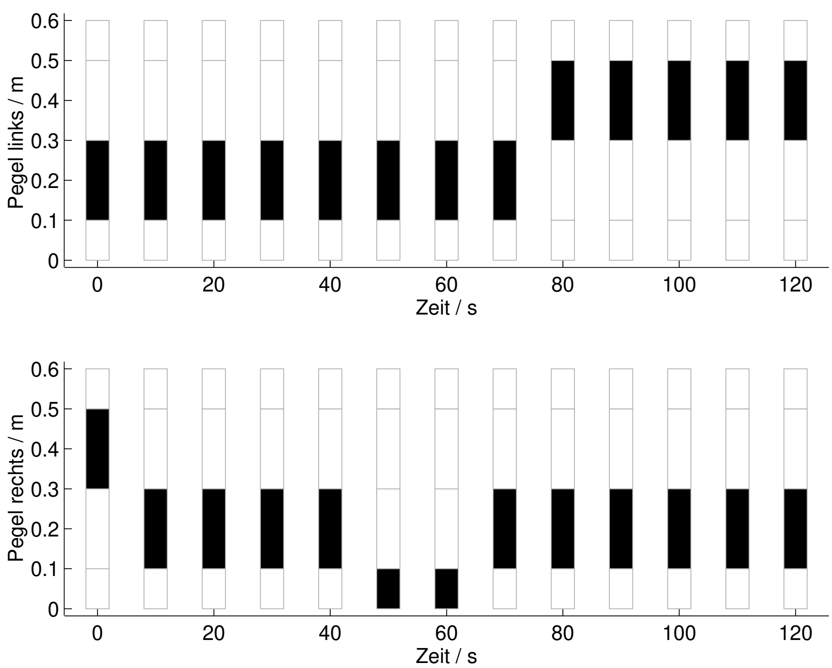
$$
L_{11}(t) = \begin{cases} 1 & \text{if } \ h_1(t) > l_{11} \\ 0 & \text{else} \end{cases} \qquad L_{12}(t) = \begin{cases} 1 & \text{if } \ h_1(t) > l_{12} \\ 0 & \text{else} \end{cases}
$$

$$
L_{21}(t) = \begin{cases} 1 & \text{if } \ h_1(t) > l_{21} \\ 0 & \text{else} \end{cases} \qquad L_{22}(t) = \begin{cases} (1-f_4) & \text{if } \ h_1(t) > l_{22} \\ 0 & \text{else} \end{cases}
$$

$$
L_{23}(t) = \begin{cases} 1 & \text{if } \ h_1(t) > l_{23} \\ 0 & \text{else} \end{cases}
$$

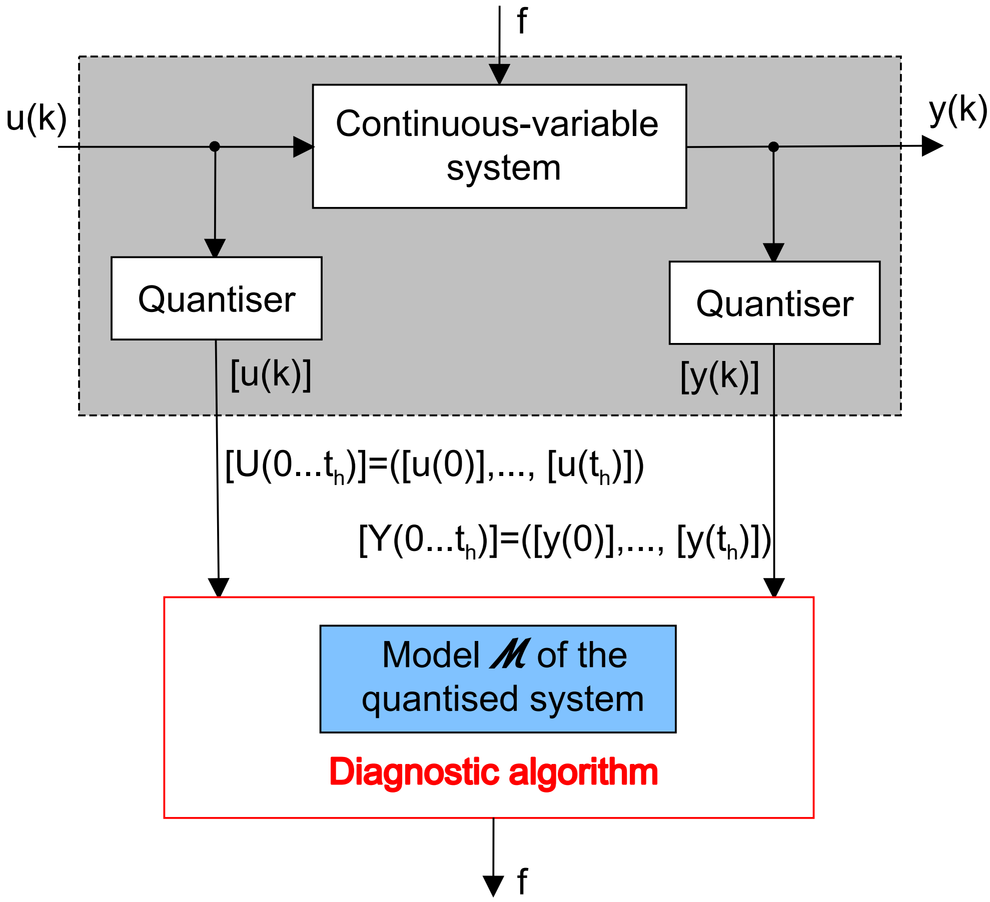
Diagnosis of the tank system





Is the tank system faulty?

Diagnosis of quantised systems

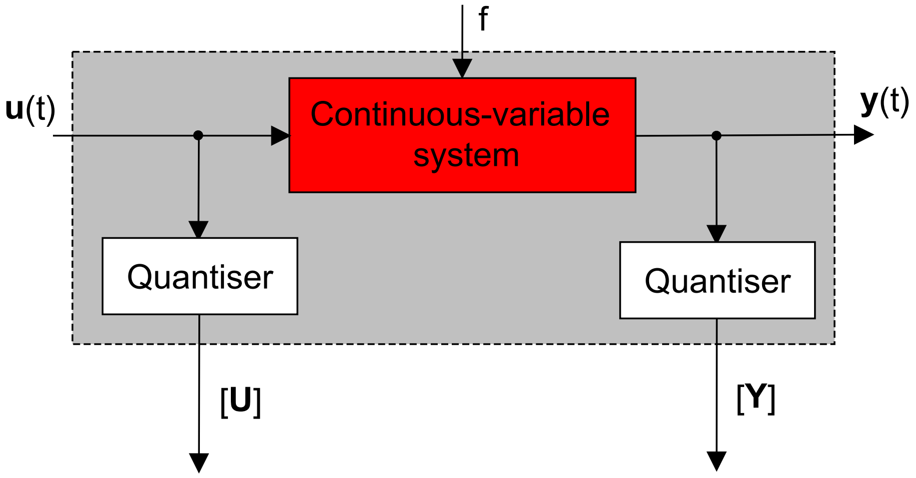


Solution steps

1. **Modelling**
   Find a discrete-event representation of the quantised system

2. **Diagnosis**
   Find a method to decide whether the quantised system behaves like the discrete-event model

Quantised systems



Continuous–variable system:

$$
\begin{aligned}
\boldsymbol{x}(k+1) &= \boldsymbol{f}(\boldsymbol{x}(k), \boldsymbol{u}(k)), \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0 && (*) \\
\boldsymbol{y}(k) &= \boldsymbol{g}(\boldsymbol{x}(k), \boldsymbol{u}(k)) && (**)
\end{aligned}
$$

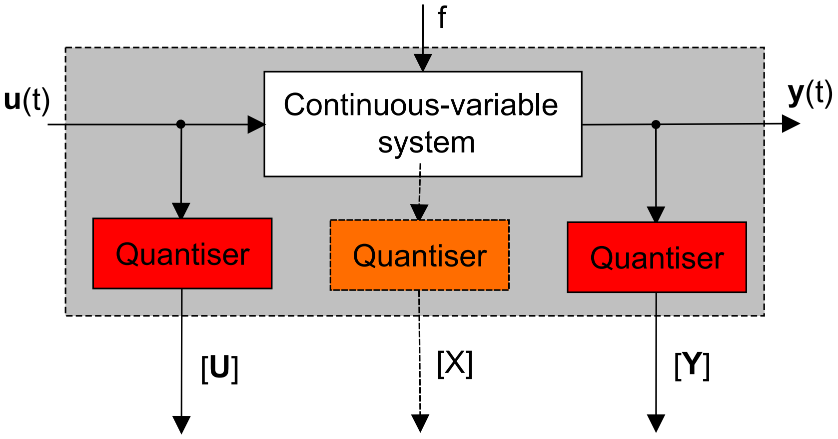
For given $\boldsymbol{x}_0$ and

$$\boldsymbol{U}(0...t_h) \;=\; (\boldsymbol{u}(0), \boldsymbol{u}(1), \boldsymbol{u}(2), ..., \boldsymbol{u}(t_h))$$

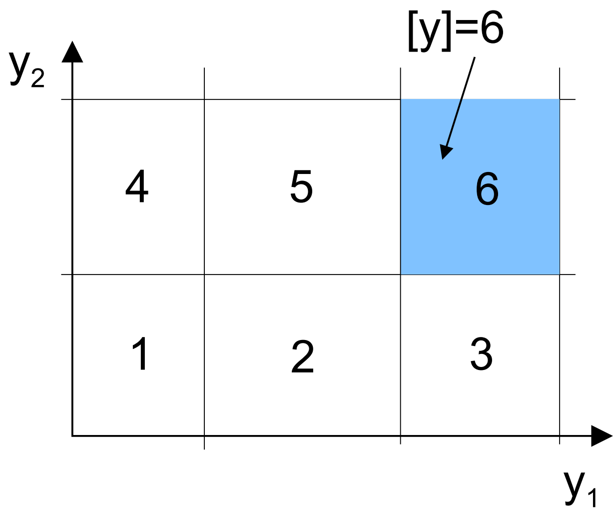
the system generates

$$
\begin{aligned}
\boldsymbol{X}(0...t_h) &= (\boldsymbol{x}(0), \boldsymbol{x}(1), \boldsymbol{x}(2), ..., \boldsymbol{x}(t_h)) \\
\boldsymbol{Y}(0...t_h) &= (\boldsymbol{y}(0), \boldsymbol{y}(1), \boldsymbol{y}(2), ..., \boldsymbol{y}(t_h))
\end{aligned}
$$

Quantised systems



Output quantisation:



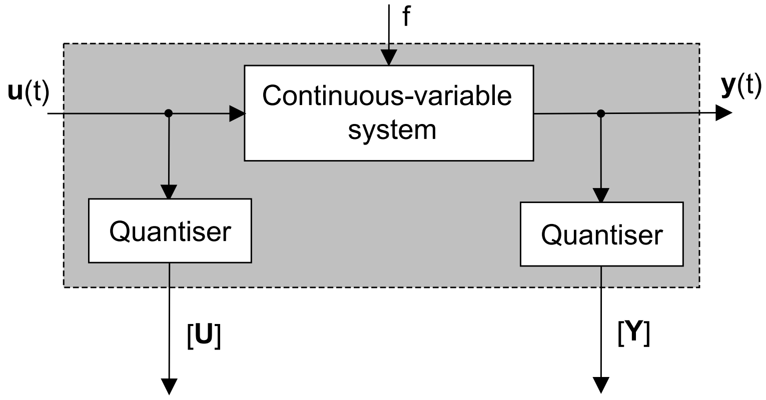
$$[\boldsymbol{y}(k)] = w \qquad \text{if} \qquad \boldsymbol{y}(k) \in \mathcal{Q}_y(w) \qquad \mathcal{N}_w = \{0, 1, 2, ..., R\}$$

Input and state quantisation

$$[\boldsymbol{u}(k)] = v \qquad \text{if} \qquad \boldsymbol{u}(k) \in \mathcal{Q}_u(v) \qquad \mathcal{N}_v = \{0, 1, 2, ..., M\}$$

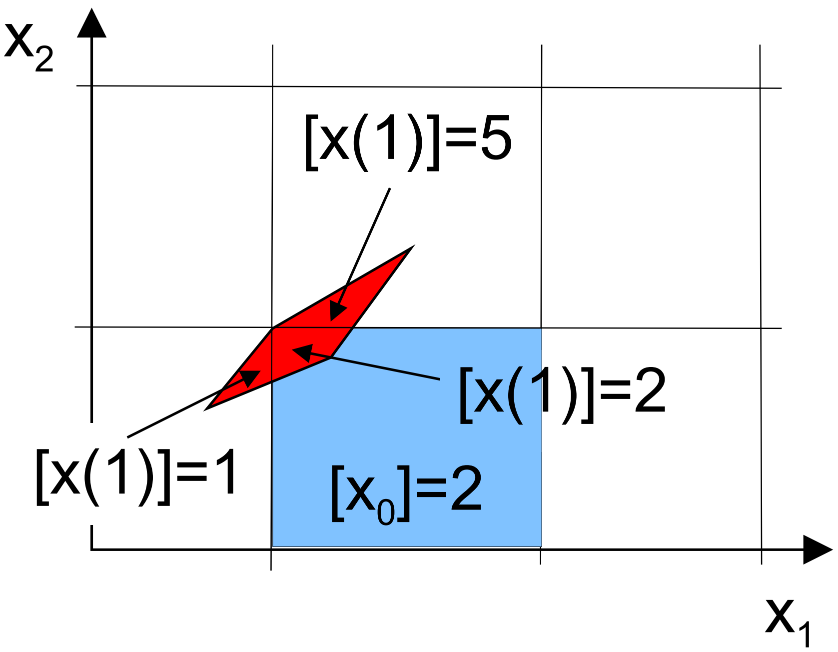
$$[\boldsymbol{x}(k)] = z \qquad \text{if} \qquad \boldsymbol{x}(k) \in \mathcal{Q}_x(z) \qquad \mathcal{N}_z = \{0, 1, 2, ..., N\}$$

Nondeterminism of the quantised system behaviour



For given quantised initial state $[\boldsymbol{x}(0)]$ and quantised input $[\boldsymbol{u}(0)]$ the system may generate more than one quantised successor state $[\boldsymbol{x}(1)]$

$$\boldsymbol{x}(1) = \boldsymbol{f}(\boldsymbol{x}(0), \boldsymbol{u}(0)),$$

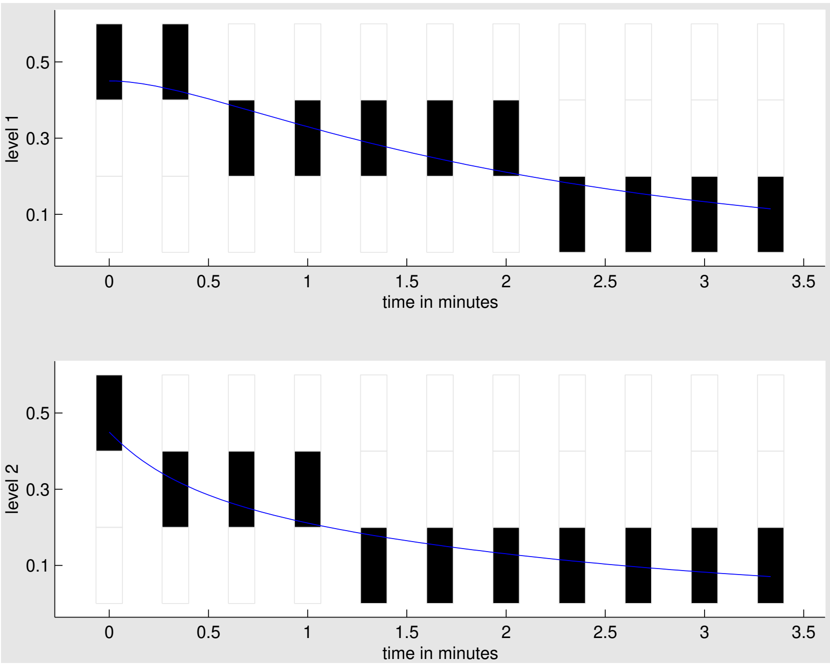


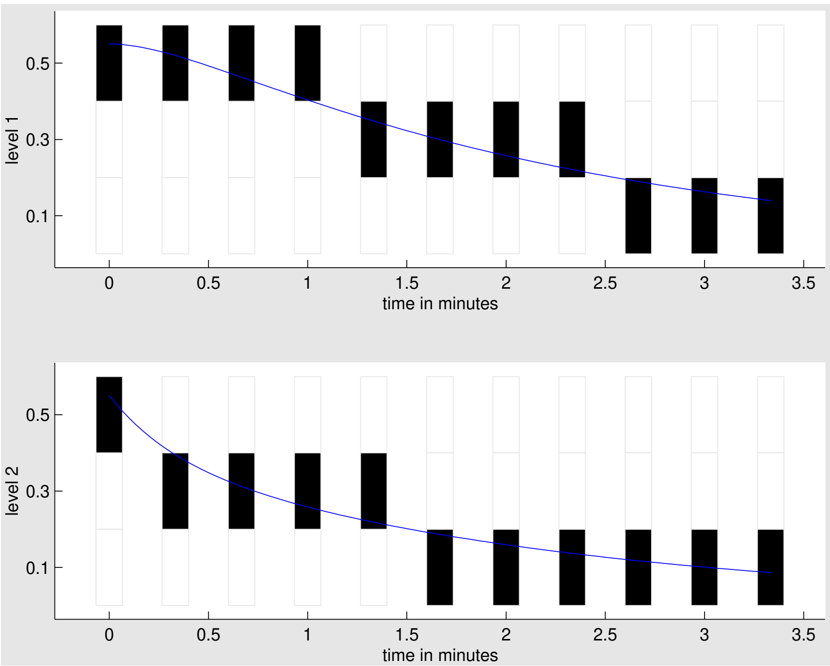
Consequence:

Discrete-event models of quantised systems have to be nondeterministic.

Nondeterminism of the quantised system behaviour

Behaviour of the quantised tank system

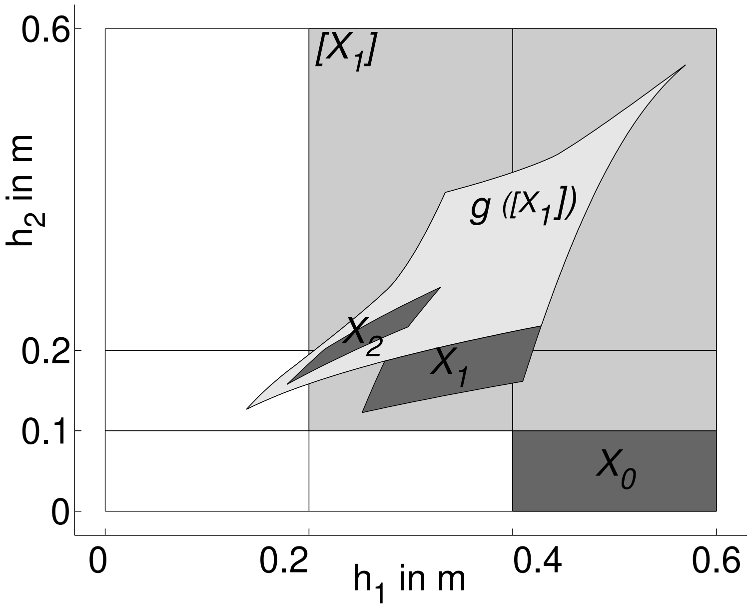




Nondeterminism of the quantised system behaviour

Quantised systems do **not** possess the Markov property

$$\mathrm{Prob}\left([\boldsymbol{x}(k+1)] \,|\, [\boldsymbol{x}(k)], [\boldsymbol{x}(k-1)], ..., [\boldsymbol{x}(0)]\right) \\ = \mathrm{Prob}\left([\boldsymbol{x}(k+1)] \,|\, [\boldsymbol{x}(k)]\right)$$



Exception: (Lunze 1994)

- linear autonomous system $\boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k)$
- equidistant partioning with resolution q_{xi}
- $\boldsymbol{A} = \mathrm{diag}\; q_{xi}\; \mathrm{diag}\; (2n_i+1)^{-1}\; \boldsymbol{P}\; \mathrm{diag}\; q_{xi}^{-1}$

Consequence:

No representation form, which possesses the Markov property, can precisely describe a quantised system

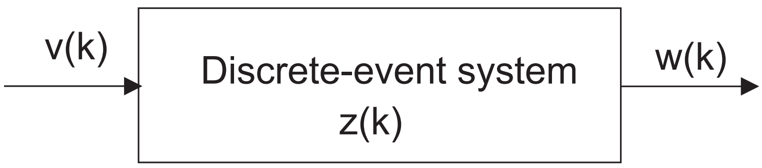
Modelling problem

Given: Quantised system
Find: Automaton with the following property:

$$\text{Set of model trajectories} \;\supseteq\; \text{Set of system trajectories}$$
$$\mathbf{Z}([\boldsymbol{x}(0)], [\boldsymbol{U}]) \;\supseteq\; [\tilde{\boldsymbol{X}}([\boldsymbol{x}(0)], [\boldsymbol{U}])]$$

- Such a model is called complete.
- Spurious solutions = Model trajectories that the quantised system cannot follow

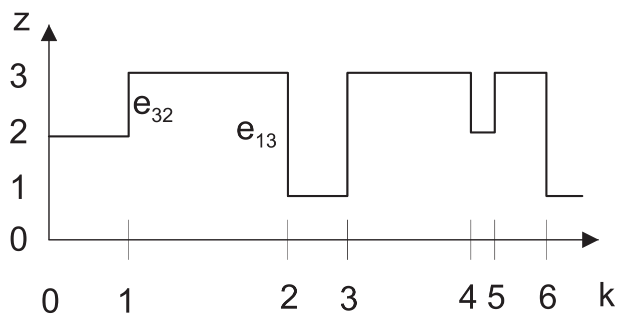
Basics of automata theory



Discrete signal spaces

$$
\begin{aligned}
v &\in \mathcal{N}_v = \{1, 2, ..., M\} \\
z &\in \mathcal{N}_z = \{1, 2, ..., N\} \\
w &\in \mathcal{N}_w = \{1, 2, ..., R\}
\end{aligned}
$$

Event = change of the input, state or output



State sequence

$$Z(0...6) = (2, 3, 1, 3, 2, 3, 1)$$

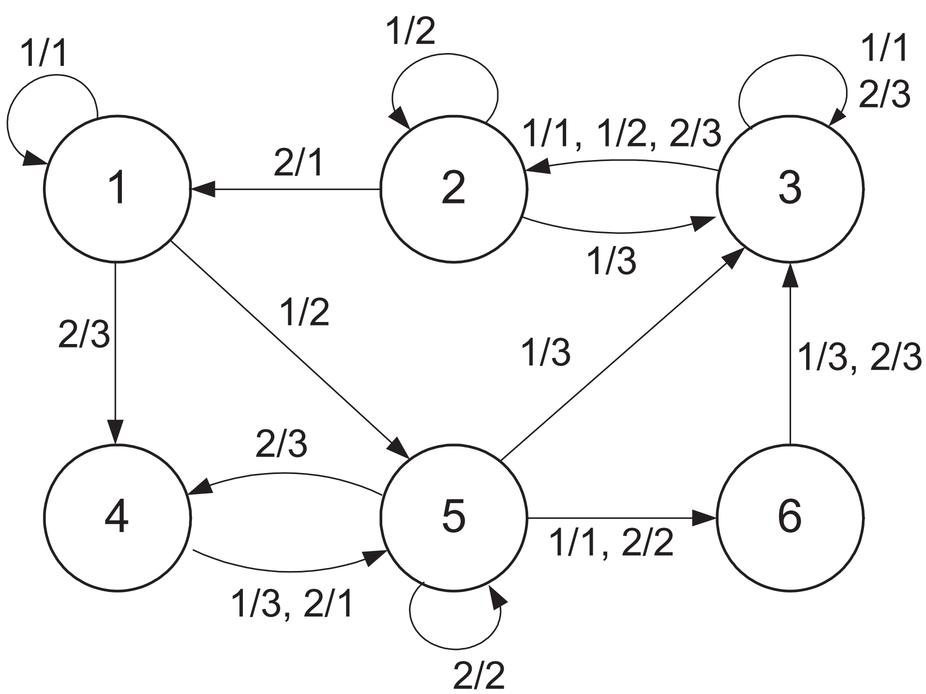
Nondeterministic automaton

$$\mathcal{N}(\mathcal{N}_z, \mathcal{N}_v, \mathcal{N}_w, L, z(0))$$

State transition relation

$$L : \mathcal{N}_z \times \mathcal{N}_w \times \mathcal{N}_z \times \mathcal{N}_v \to \{0, 1\}$$

$L(z', w, z, v) = 1 \Rightarrow$ automaton may jump
from $z(k) = z$ towards $z(k+1) = z'$
while producing the output $w(k) = w$ for input $v(k) = v$



Stochastic automaton

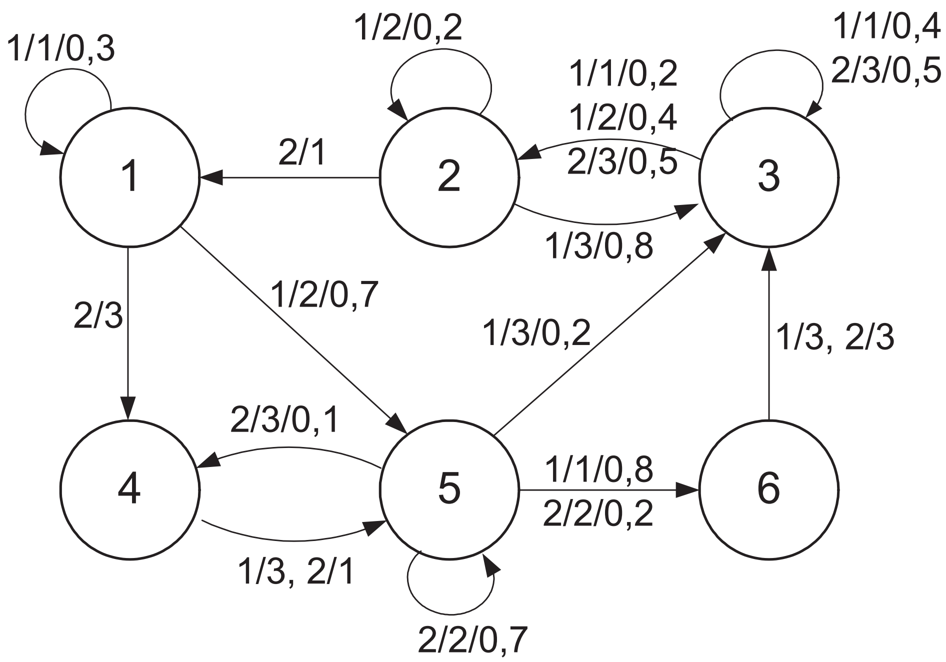
$$\mathcal{S}(\mathcal{N}_z, \mathcal{N}_v, \mathcal{N}_w, L, \mathrm{Prob}\,(z(0)))$$

State transition probability distribution

$$L : \mathcal{N}_z \times \mathcal{N}_w \times \mathcal{N}_z \times \mathcal{N}_v \;\longrightarrow\; [0, 1]$$

$$L(z', w \mid z, v) =$$

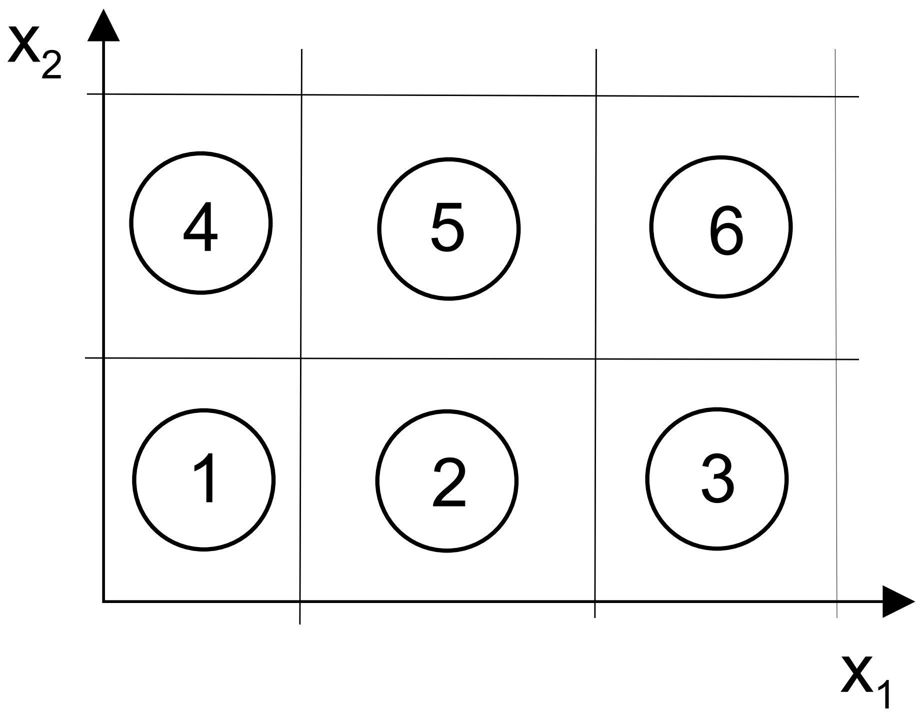
$$\mathrm{Prob}(z_p(1) = z', w_p(0) = w \mid z_p(0) = z, v_p(0) = v)$$



Modelling of quantised systems by stochastic automata

Stochastic automaton $\mathcal{S}(\mathcal{N}_z, \mathcal{N}_v, \mathcal{N}_w, L, \mathrm{Prob}\,(z_0))$

- $\mathcal{N}_z$ - set of quantised state symbols
- $\mathcal{N}_v$ - set of quantised input symbols
- $\mathcal{N}_w$ - set of quantised output symbols

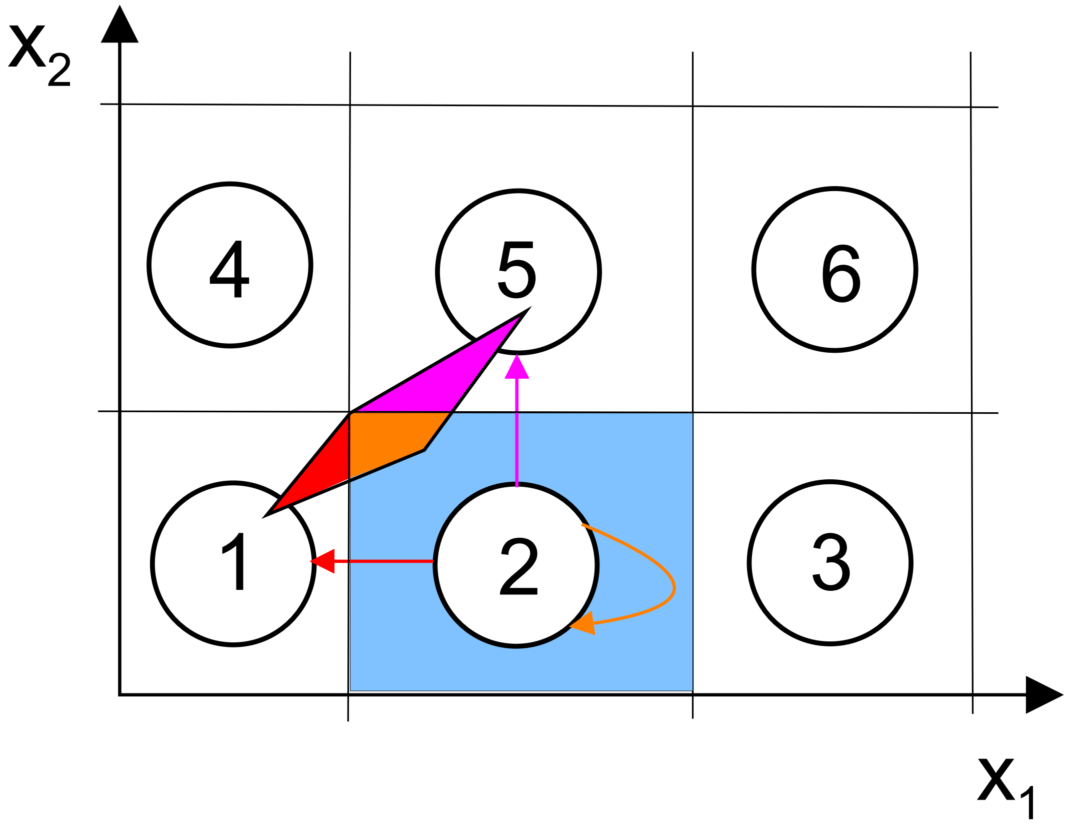


Modelling of quantised systems by stochastic automata

Abstraction

$$L(z', w \mid z, v) =$$
$$\text{Prob}\left([\boldsymbol{x}(1)] = z', [\boldsymbol{y}(0)] = w \mid [\boldsymbol{x}(0)] = z, [\boldsymbol{u}(0)] = v\right)$$

$$\text{Prob}\,(z_0) > 0 \quad \text{for} \quad z_0 = [\boldsymbol{x}_0]$$

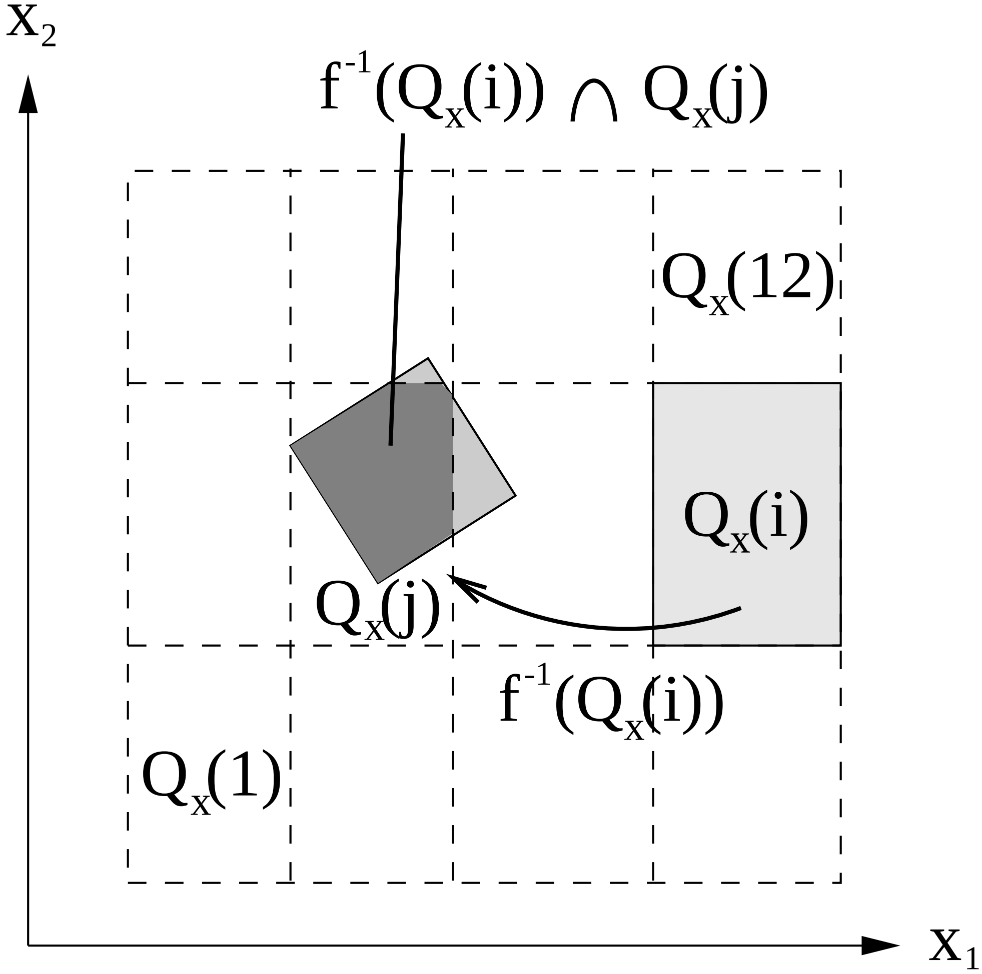


Modelling of quantised systems by stochastic automata

Abstraction

$$\boldsymbol{x}(k+1) = \boldsymbol{f}(\boldsymbol{x}(k))$$

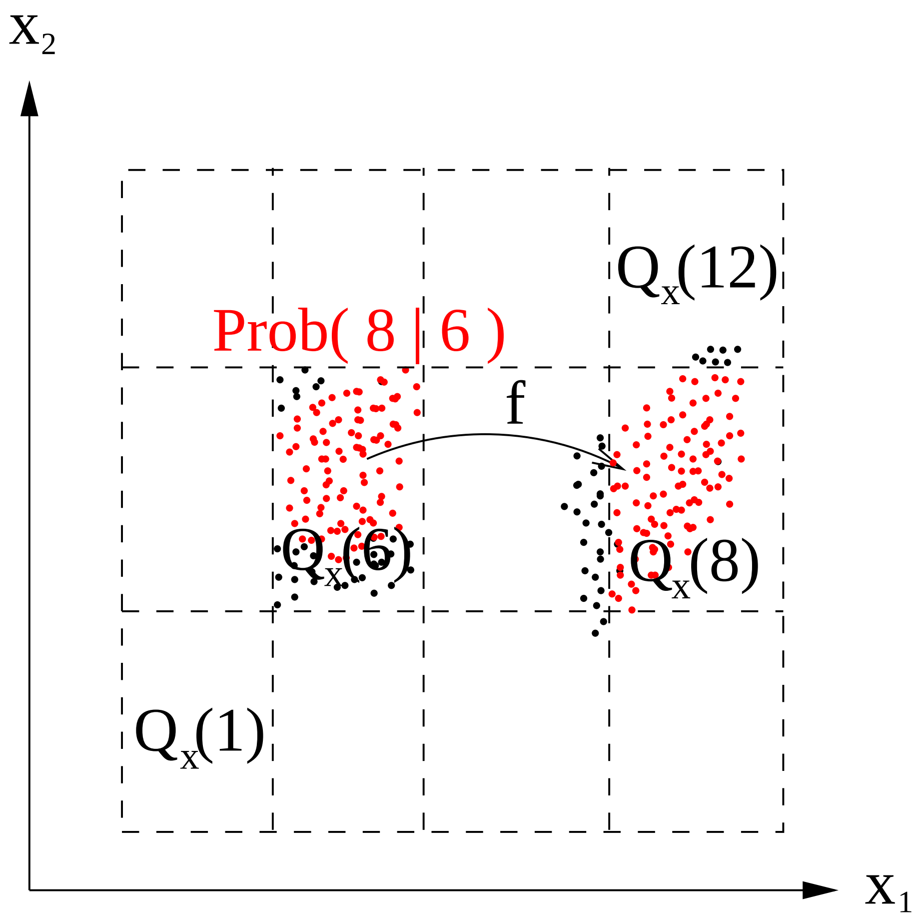
$$L(i \mid j) = \frac{\lambda\left(\boldsymbol{f}^{-1}(\mathcal{Q}_x(i)) \cap \mathcal{Q}_x(j)\right)}{\lambda(\mathcal{Q}_x(j))}$$



Modelling of quantised systems by stochastic automata

Abstraction

Point-based cell-to-cell mapping



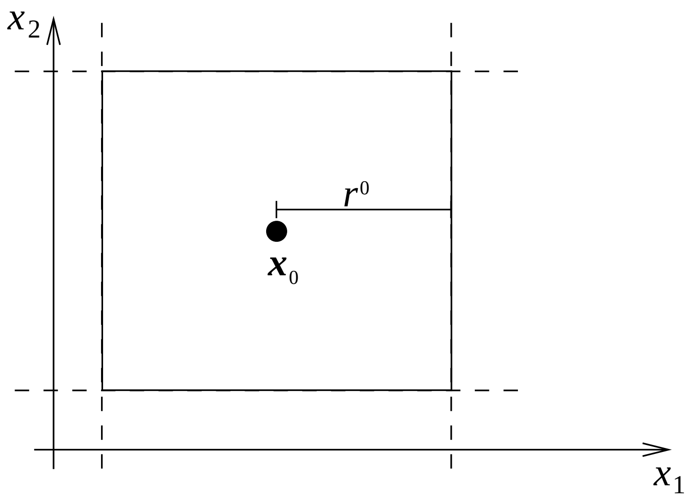
The automaton obtained is, in general, **incomplete**.

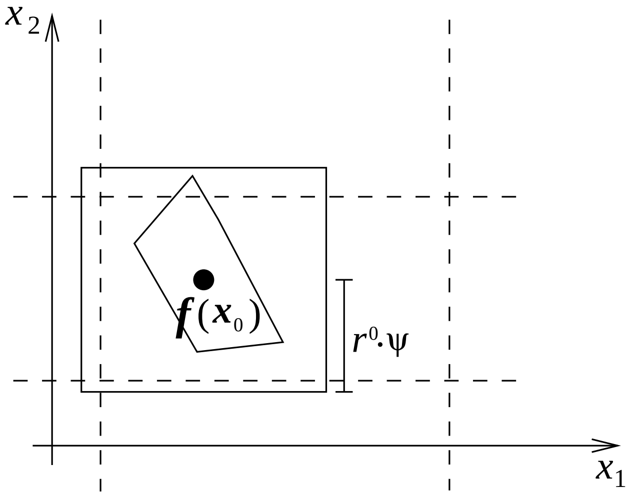
Modelling of quantised systems by stochastic automata

Hyperbox mapping
(Lunze, Schröder, ECC 2001)

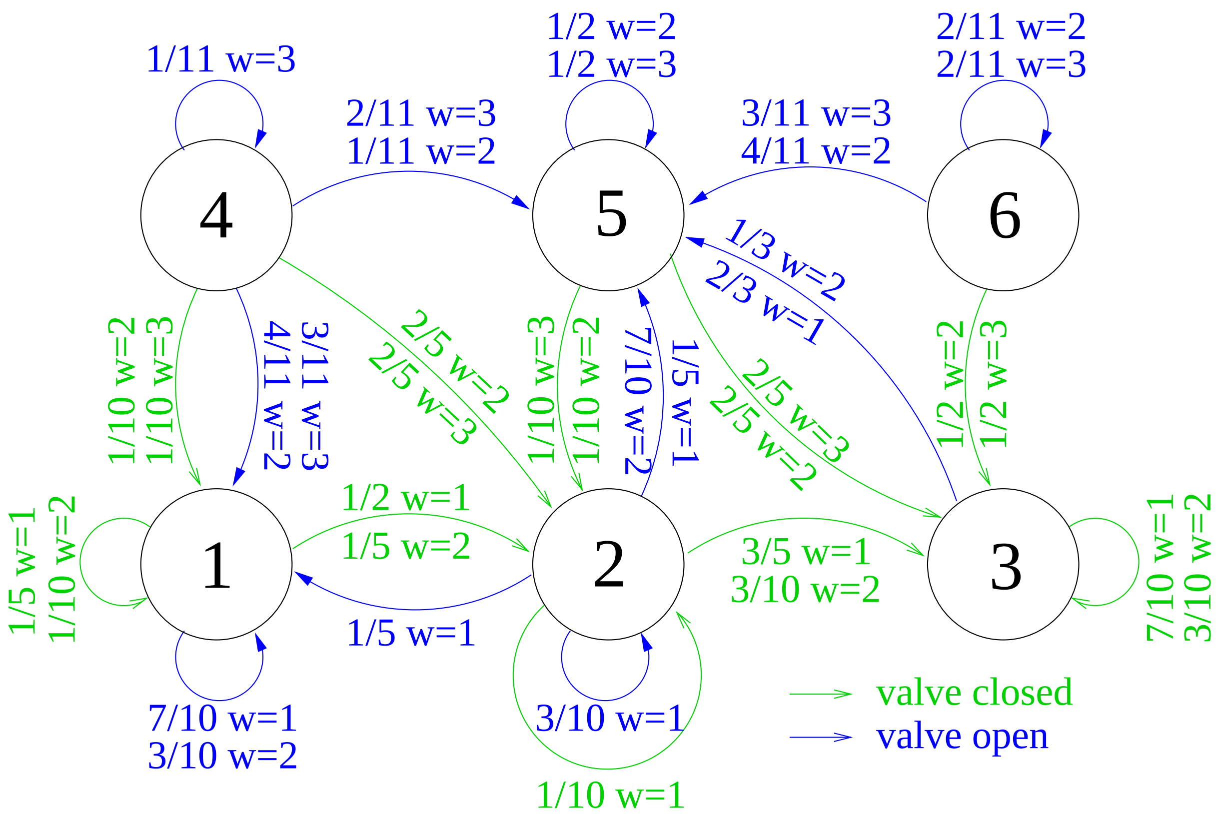
Assumption:
System satisfies a Lipschitz condition:

$$||(\boldsymbol{f}(\boldsymbol{x}_1) - \boldsymbol{f}(\boldsymbol{x}_2)||_\infty \leq \psi \cdot ||\boldsymbol{x}_1 - \boldsymbol{x}_2||_\infty$$





The automaton is complete.



$$\mathbf{Z}([\boldsymbol{x}(0)],[\boldsymbol{U}]) \supseteq [\tilde{\mathbf{X}}([\boldsymbol{x}(0)],[\boldsymbol{U}])]$$

$$\begin{aligned}
\dot{h}_1 &= \frac{1}{A_1}\left(\dot{Q}_p - \dot{Q}_1 - \dot{Q}_{12} - \dot{Q}_{12h}\right)\\
\dot{h}_2 &= \frac{1}{A_2}\left(\dot{Q}_{12} + \dot{Q}_{12h} - \dot{Q}_2\right)\\
\dot{Q}_1 &= \begin{cases} Pos(V_1)\ S_v\sqrt{2gh_1} & \text{if } h_1 > 0\\ 0 & \text{else}\end{cases}\\
\dot{Q}_{12} &= (1-f_3)Pos(V_{12l})\ S_v\,\mathrm{sgn}(h_1-h_2)\ \sqrt{2g|h_1-h_2|}\\
\dot{Q}_{12h} &= \begin{cases} (1-f_2)Pos(V_{12h})\ S_v\,\mathrm{sgn}(h_1-h_2)\ \sqrt{2g|h_1-h_2|} & \text{if } h_1,h_2 > h_v\\ (1-f_2)Pos(V_{12h})\ S_v\ \sqrt{2g|h_1-h_v|} & \text{if } h_1 > h_v, h_2 \le h_v\\ -(1-f_2)Pos(V_{12h})\ S_v\ \sqrt{2g|h_2-h_v|} & \text{if } h_2 > h_v, h_1 \le h_v\\ 0 & \text{if } h_1,h_2 \le h_v\end{cases}\\
\dot{Q}_2 &= \begin{cases} Pos(V_2)\ S_v\ \sqrt{2gh_2} & \text{if } h_2 > 0\\ 0 & \text{else}\end{cases}\\
\dot{Q}_p &= \begin{cases} (1-f_1)p(t)\dot{Q}_{p0} & \text{if } h_1 < h_{1max}\\ 0 & \text{if } h_1 \ge h_{1max}\end{cases}
\end{aligned}$$

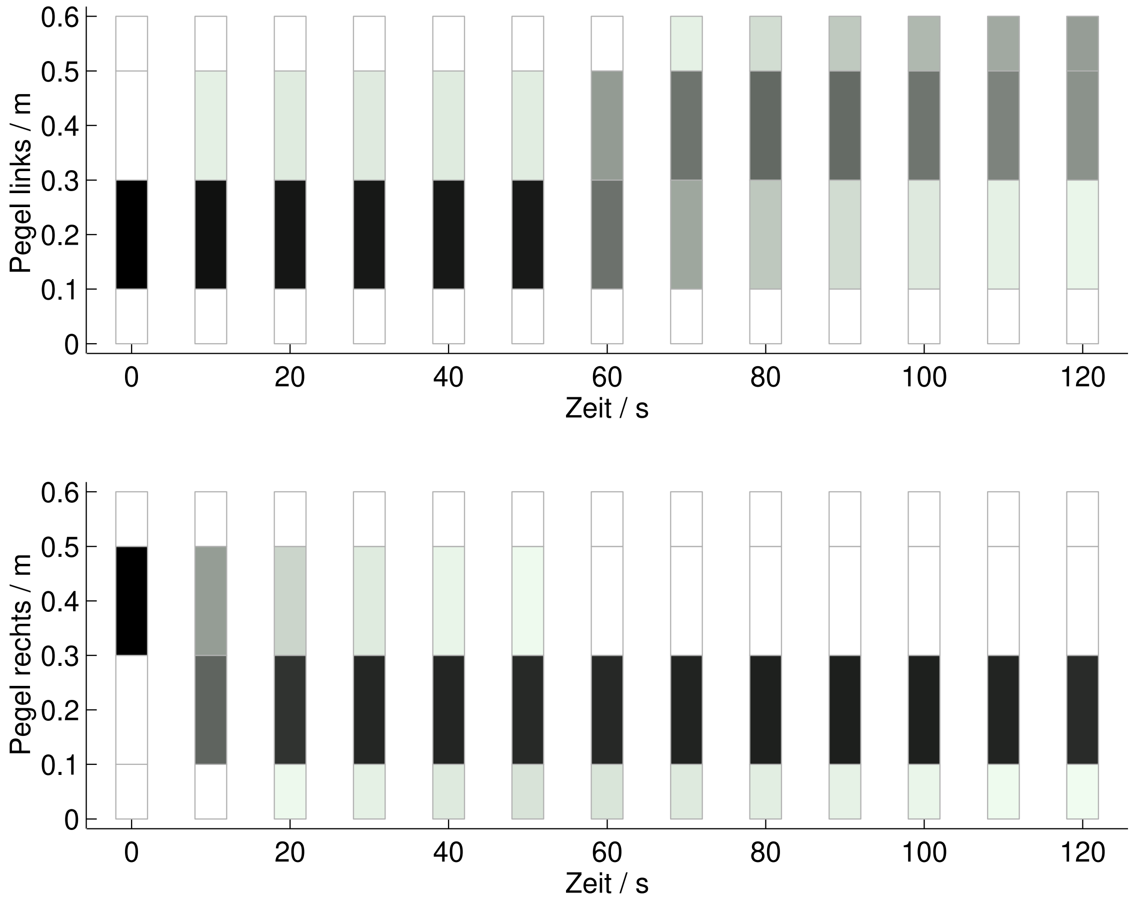
$$Pos(V_1(t)), Pos(V_{12l}(t)) \in \{0,1\} \qquad f_1,f_2,f_3,f_4 \in \{0,1\}$$

$$L_{11}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{11}\\ 0 & \text{else}\end{cases} \quad L_{12}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{12}\\ 0 & \text{else}\end{cases} \quad L_{21}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{21}\\ 0 & \text{else}\end{cases}$$

$$L_{22}(t) = \begin{cases} (1-f_4) & \text{if } h_1(t) > l_{22}\\ 0 & \text{else}\end{cases} \quad L_{23}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{23}\\ 0 & \text{else}\end{cases}$$

Modelling of quantised systems by stochastic automata

Simulation:



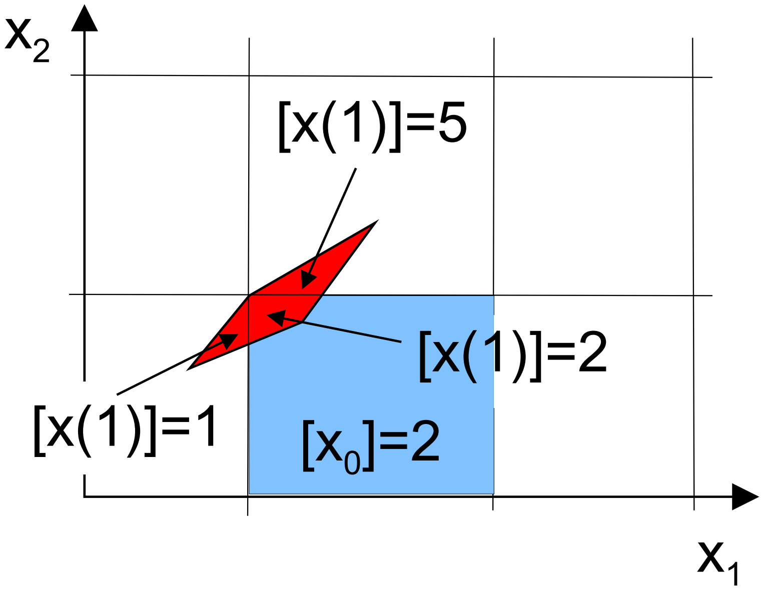
The stochastic automaton is a complete model.

The state partitioning problem

to appear in Blondel/Megretski: 60 open problems..., Princeton Univ. Press 2003

Under what conditions is the discrete-event behaviour deterministic?

$$\forall i \in \mathcal{N}_z \;\; \exists j : \boldsymbol{f}(\boldsymbol{x}) \in \mathcal{Q}(j) \text{ for all } \boldsymbol{x} \in \mathcal{Q}(i)$$



Problem A

Given: $\boldsymbol{x}(k+1) = \boldsymbol{f}(\boldsymbol{x}(k))$

Partition $\mathcal{Q}_x$

Find: Conditions on $\boldsymbol{f}$, $\mathcal{Q}_x$ such that

the discrete-event behaviour is deterministic

Problem B

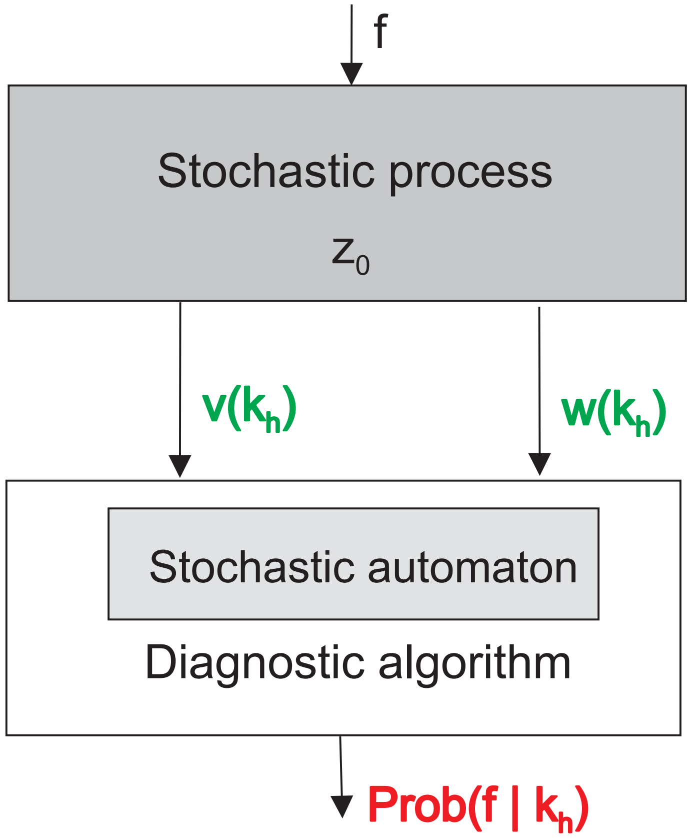
Given: $\boldsymbol{x}(k+1) = \boldsymbol{f}(\boldsymbol{x}(k))$

Find: Partition $\mathcal{Q}_x$ such that

the discrete-event behaviour is deterministic

Diagnosis of automata

Assumption: The fault f is time-invariant



Diagnostic problem

Given: Stochastic automaton $\mathcal{S}$
$V(0...k_h), W(0...k_h)$

Find: Fault f

Restriction to the fault detection problem: test the consistency with the model of the faultless system

State observation
of nondeterministic automata

Consistency check:
Is the I/O pair consistent with the automaton?

Given: I/O pair

$$\begin{aligned} V(0...k_h) &= (v(0), v(1), ..., v(k_h)) \\ W(0...k_h) &= (w(0), w(1), ..., w(k_h)) \end{aligned}$$

The I/O pair is consistent with the automaton if there exists a state sequence $Z(0...k_h)$ such that

$$L(z(k+1), w(k), z(k), v(k)) = 1 \quad \text{for all } k$$

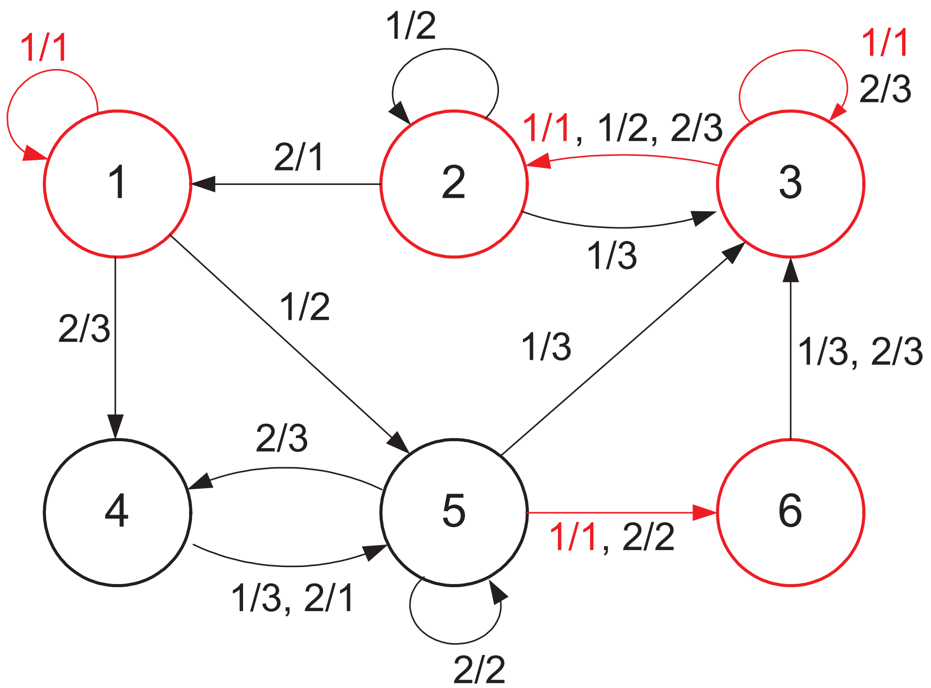
The consistency check includes a state observation problem.

State observation of nondeterministic automata

A-priori information:

$$\mathcal{Z}(0\,|\,-1) = \{1,2,3,4,5,6\}$$

Measurement: $v(0) = 1, w(0) = 1$



Information about the state obtained by the measurement:

$$\begin{aligned}\mathcal{Z}(0\,|\,0) &= \{1,3,5\}\\ \mathcal{Z}(1\,|\,0) &= \{1,2,3,6\}\end{aligned}$$

State observation of nondeterministic automata

$$\mathcal{Z}(k_h\,|\,k_h) = \{z \in \mathcal{Z}(k_h\,|\,k_h - 1)\ :\ \exists z' : L(z', w, z, v) = 1\}$$

$$\begin{aligned}\mathcal{Z}(k_h + 1\,|\,k_h)\ =\ \{z'\ :\ \exists z : L(z', w, z, v) = 1 \\ \text{for a}\ \ z \in \mathcal{Z}(k_h\,|\,k_h)\}\end{aligned}$$

Consistency check:

The I/O pair is consistent with the automaton if and only if

$$\mathcal{Z}(k_h\,|\,k_h) \neq \emptyset \quad \text{for all} \quad k_h$$

Algorithm *Observation of non–deterministic automata*

Given: Non–deterministic automaton $\mathcal{N}$
Initial state set $\mathcal{Z}(0 \,|\, -1)$

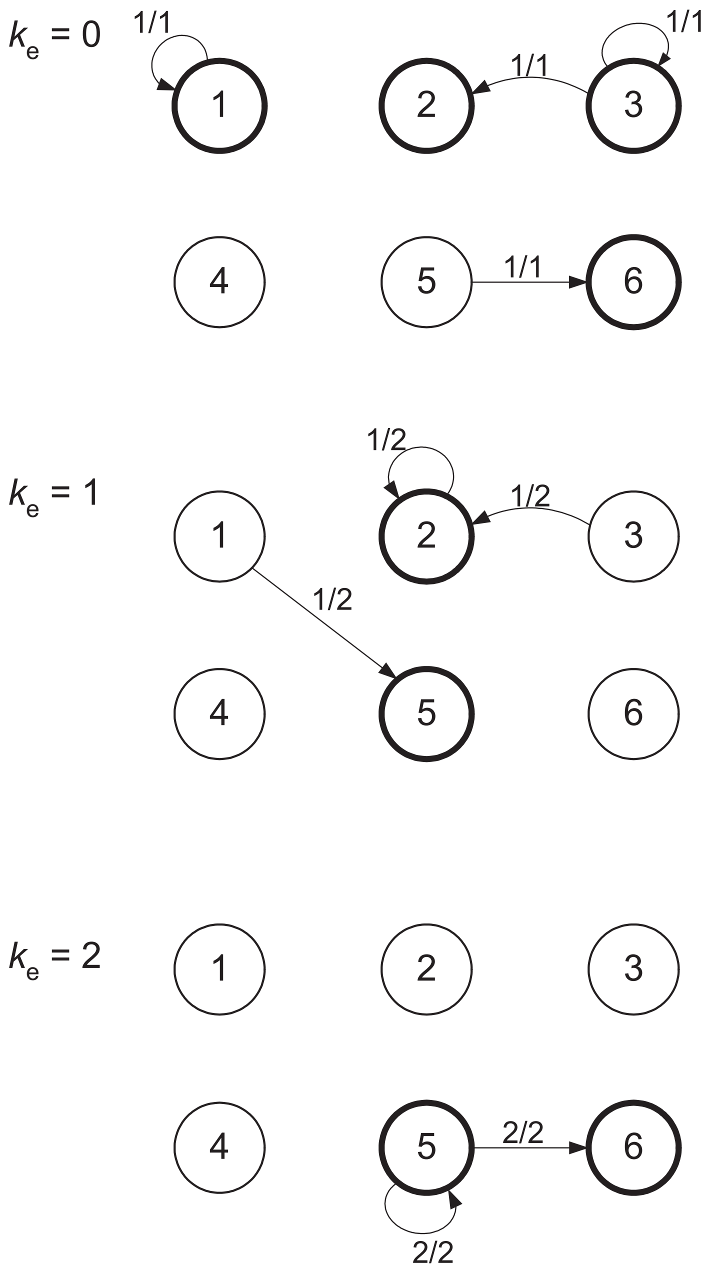
Init.: $\mathcal{Z}_r = \mathcal{Z}(0 \,|\, -1)$
$k_h = 0$

Do:
1. Measure the I/O pair (v, w)
2. Determine
   $\mathcal{Z}_k := \{z \in \mathcal{Z}_r \;:\; L(z', w, z, v) = 1 \text{ for a } z' \in \mathcal{N}_z\}$
3. **Consistency check:**
   If $\mathcal{Z}_k = \emptyset$, stop the algorithm
   (inconsistent I/O pair or wrong initial state set)
4. Determine
   $\mathcal{Z}_r := \{z' \;:\; L(z', w, z, v) = 1 \text{ for a} z \in \mathcal{Z}_k\}$
5. $k_h := k_h + 1$
   Continue with Step 1

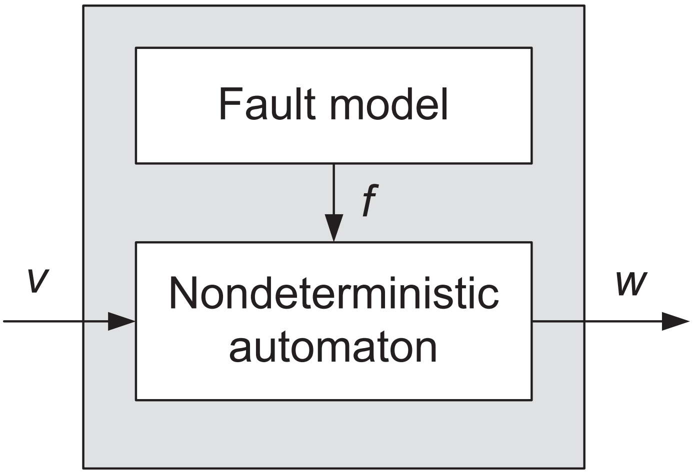
Result: $\mathcal{Z}_k = \mathcal{Z}(k_h \,|\, k_h)$ for increasing time horizon k_h

State observation of nondeterministic automata

I/O pair: $(V, W) = ((1,1,2,2),\ (1,2,2,3))$



Diagnosis of nondeterministic automata



Fault model: $\mathcal{N}_{\mathrm{F}}(\mathcal{N}_f, G_{\mathrm{F}}, z_{\mathrm{F}0})$

Nondeterministic automaton including the fault model:

State:

$$\tilde{z} = \begin{pmatrix} z \\ f \end{pmatrix}$$

State transition relation:

$$\tilde{L}\left(\begin{pmatrix} z' \\ f' \end{pmatrix}, w, \begin{pmatrix} z \\ f \end{pmatrix}, v\right) = L(z', w, z, v, f) \cdot G(f', f),$$

Diagnosis is based on a consistency check or the I/O pair for this automaton.

Algorithm *Diagnosis of non–deterministic automata*

Given: Non–deterministic automaton $\mathcal{N}$
Fault model $\mathcal{N}_{\mathrm{F}}$
Initial state set $\mathcal{Z}(0\,|\,-1)$
Initial fault set $\mathcal{F}(0\,|\,-1)$

Init.: $\mathcal{Z}_r = \mathcal{Z}(0\,|\,-1) \times \mathcal{F}(0\,|\,-1)$
$k_h = 0$

Do: 1. Measure the I/O pair (v, w)

2. Determine
$\tilde{\mathcal{Z}}_k := \{(z, f) \in \tilde{\mathcal{Z}}_r \ :$
$$L(\tilde{z}', w, z, v, f) \cdot G(f', f) = 1 \text{ for a } \tilde{z}' \in \mathcal{N}_z\}$$

3. **Consistency check:**
If $\mathcal{Z}_k = \emptyset$, stop the algorithm
(wrong initial state set or initial fault set)

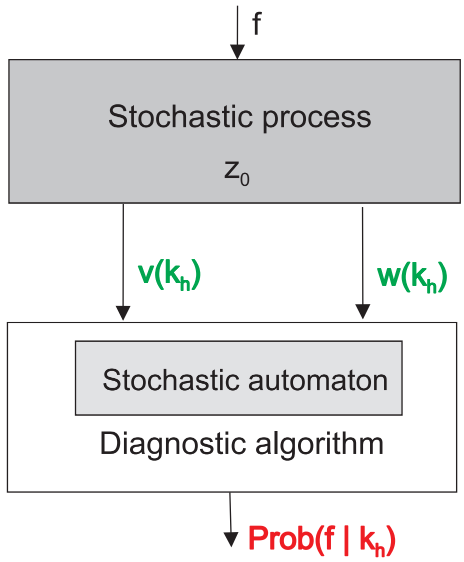
4. Determine
$\mathcal{Z}_r := \{(z', f') \ : \ L(z', w, z, v, f) \cdot G(f', f) = 1$
for a $(z, f) \in \tilde{\mathcal{Z}}_k\}$

5. Determine $\mathcal{F}_k = \{f \ : \ (z, f) \in \tilde{\mathcal{Z}}_k\}$

6. $k_h := k_h + 1$
Continue with Step 1

Result: $\mathcal{F}(k_h\,|\,k_h)$ for increasing time horizon k_h

Diagnosis of stochastic automata



Diagnostic problem

Given: Stochastic automaton $\mathcal{S}$
$V(0 \dots k_h), W(0 \dots k_h)$

Find: Fault f

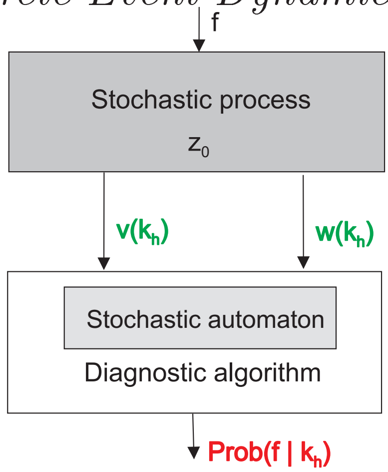
Do there exist some f, z_0 such that (V, W) is consistent with the automaton?

Result: $\text{Prob}\,(f \mid V(0 \dots k_h), W(0 \dots k_h)) =: \text{Prob}\,(f \mid k_h)$

$$\mathcal{F}(k_h) = \{f \;:\; \text{Prob}\,(f \mid k_h) > 0\}$$

Diagnosis of stochastic automata

(Lunze, Schröder, *Discrete Event Dynamic Systems*, 2001)



Initialisation: $\text{Prob}\,(f, z(0)\,|\,-1) = \text{Prob}\,(f, z(0))$

Iteration: $k_h := k_h + 1$

1. Measure $v(k_h)$, $w(k_h)$

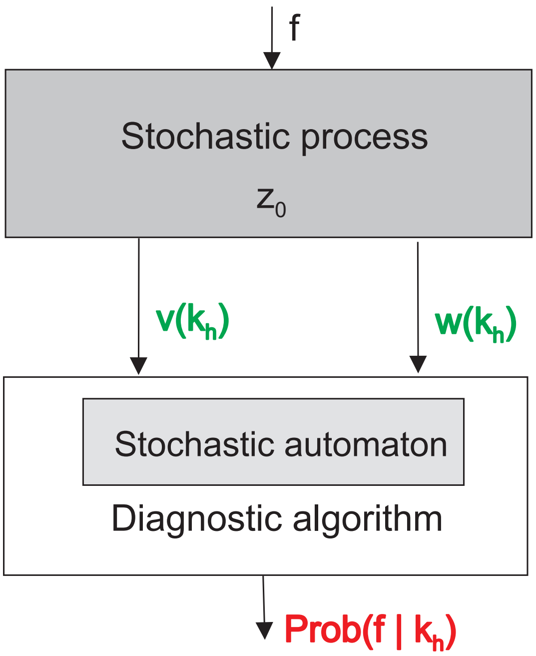
2. Determine $L(k_h) = L(z(k_h{+}1), w(k_h)\,|\,z(k_h), v(k_h), f)$

3. $$\text{Prob}\,(f|k_h) = \frac{\sum\limits_{\substack{z(k_h+1)\\ z(k_h)}} L(k_h)\cdot \text{Prob}\,(f, z(k_h)\,|\,k_h-1)}{\sum\limits_{\substack{z(k_h+1)\\ z(k_h), f}} L(k_h)\cdot \text{Prob}\,(f, z(k_h)\,|\,k_h-1)}$$

4. $\mathcal{F}(k_h) = \{f \mid \text{Prob}\,(f\,|\,k_h) > 0\}$

5. $$\text{Prob}\,(f, z(k_h+1)\,|\,k_h) = \frac{\sum\limits_{z(k_h)} L(k_h)\cdot \text{Prob}\,(f, z(k_h)\,|\,k_h-1)}{\sum\limits_{\substack{z(k_h+1)\\ z(k_h), f}} L(k_h)\cdot \text{Prob}\,(f, z(k_h)\,|\,k_h-1)}$$

Diagnosis of stochastic automata

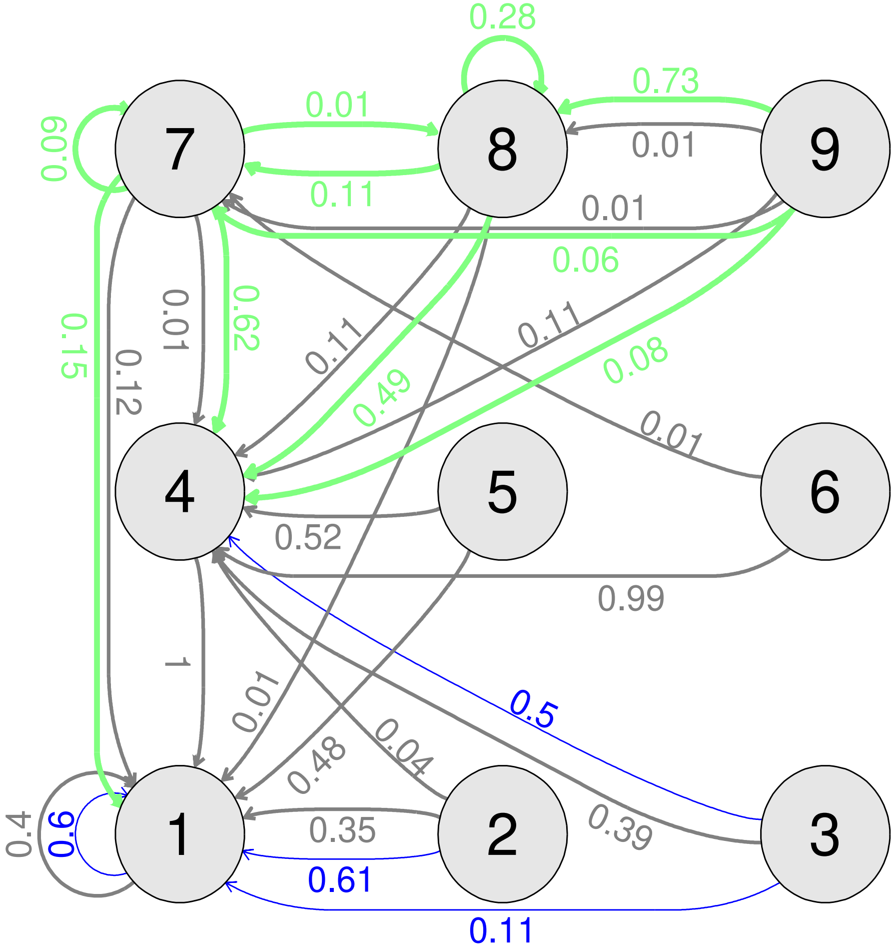


Result: $\text{Prob}\,(f \mid k_h)$

$$\mathcal{F}(k_h) = \{f \;:\; \text{Prob}\,(f \mid k_h) > 0\}$$

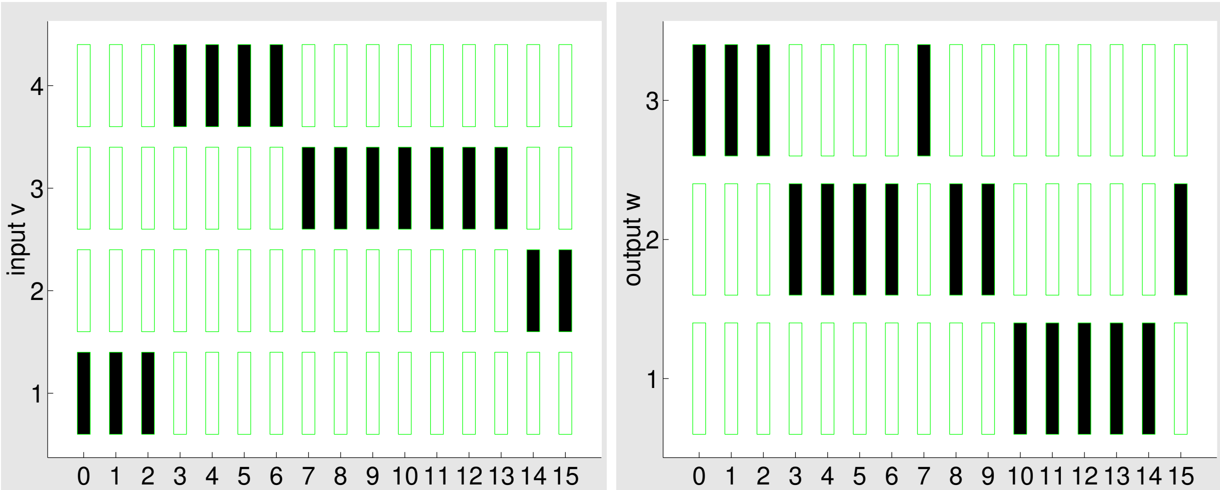
- The system is subject to some fault $f \in \mathcal{F}(k_h)$.
- **Fault detection:**
  If $f_0 \notin \mathcal{F}(k_h)$ holds, the system is known to be subject to some fault.
- **Fault identification:**
  If $\mathcal{F}(k_h) = \{f_i\}$ is a singleton, the system is known to be subject to fault f_i.

BRIDGE Benchmark problem
Scenario 2

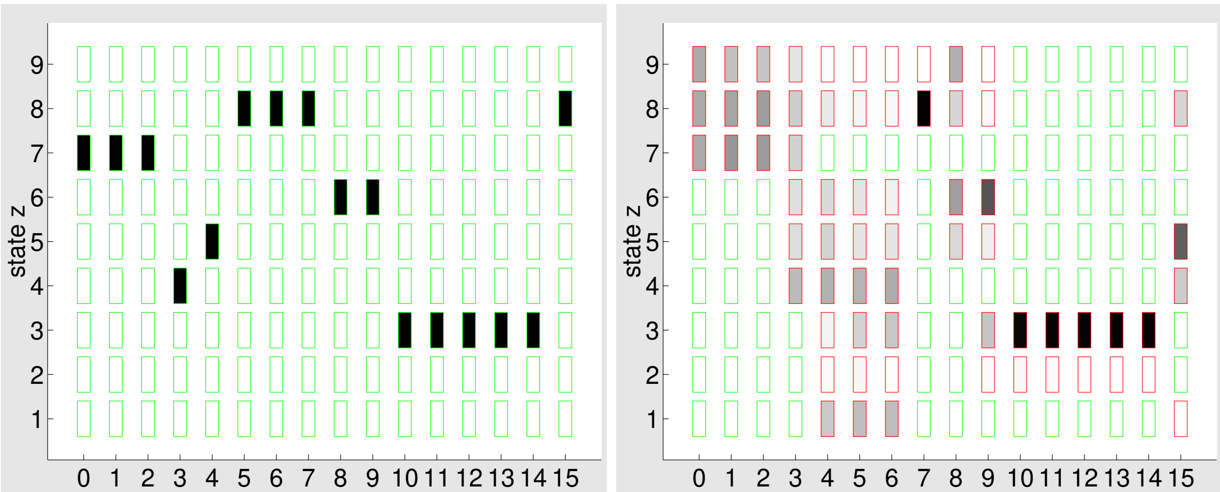


BRIDGE Benchmark problem
Scenario 2

I/O pair:

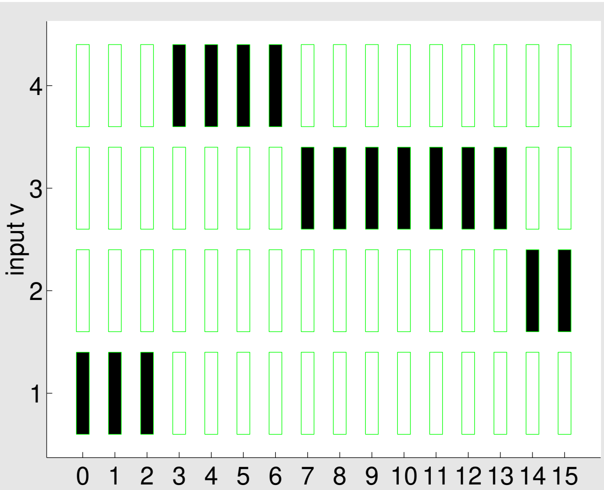


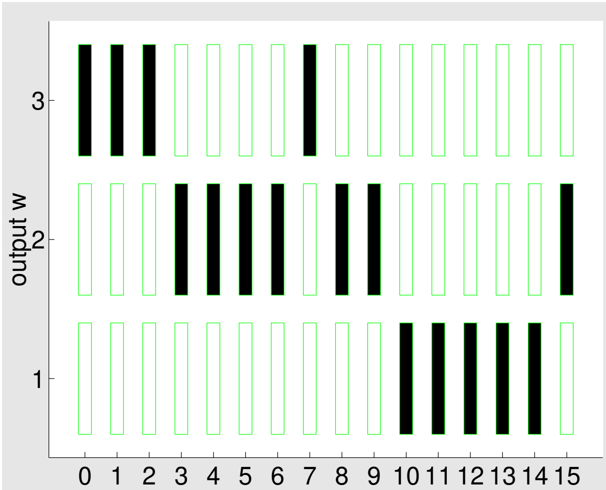
State observation:



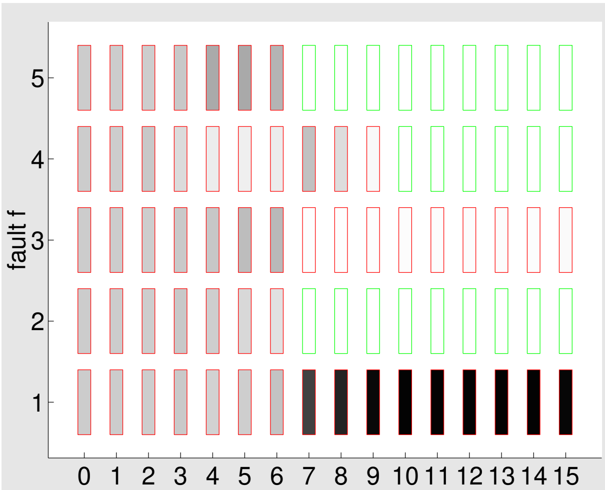
BRIDGE Benchmark problem
Scenario 2

I/O pair:

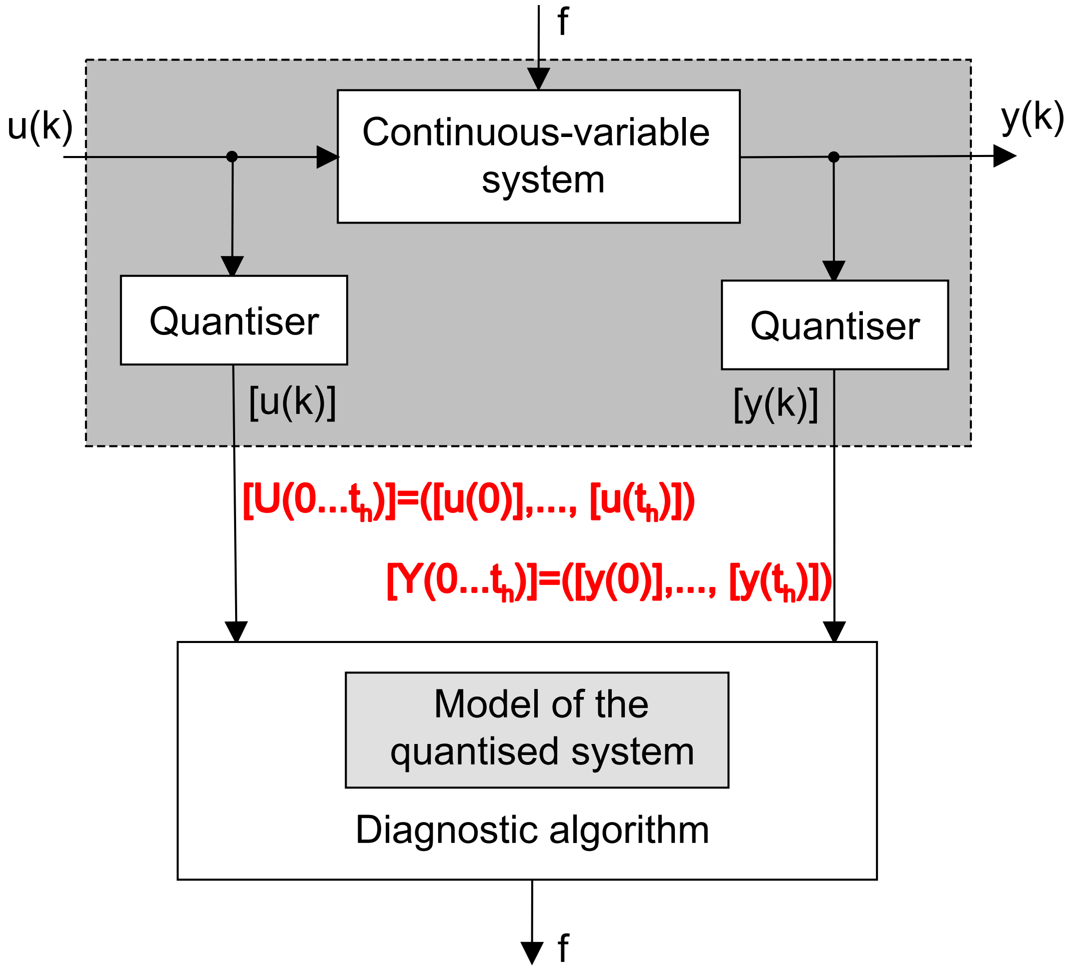




Solution:



Diagnosis of quantised systems



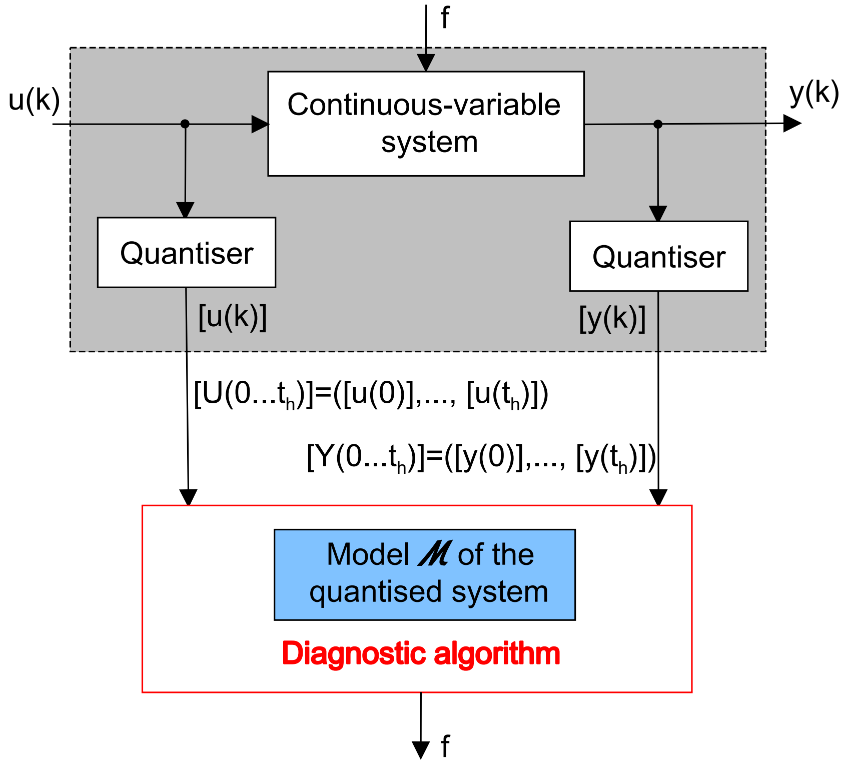
Diagnostic problem:

Given: Quantised system

I/O pair $([\boldsymbol{U}(0...t_h)], [\boldsymbol{Y}(0...t_h)])$

Find: Fault f

Diagnosis of quantised systems

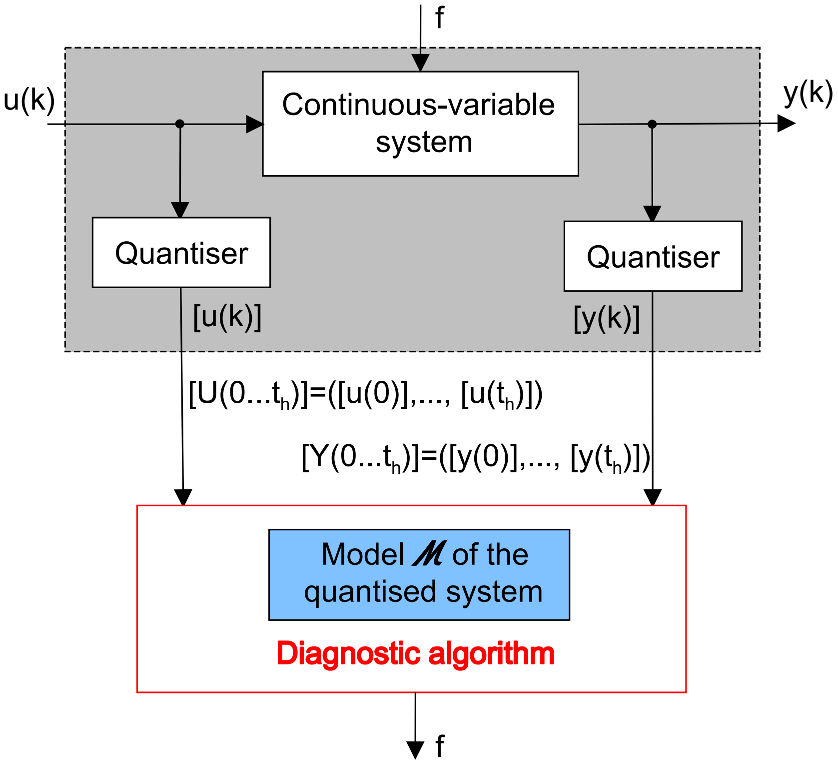


Solution steps

1. **Modelling**
   Determine a complete discrete-event model

2. **Diagnosis**
   Use the diagnostic method for stochastic automata to check whether $([\boldsymbol{U}], [\boldsymbol{Y}])$ is consistent with the discrete-event model

Diagnosis of quantised systems

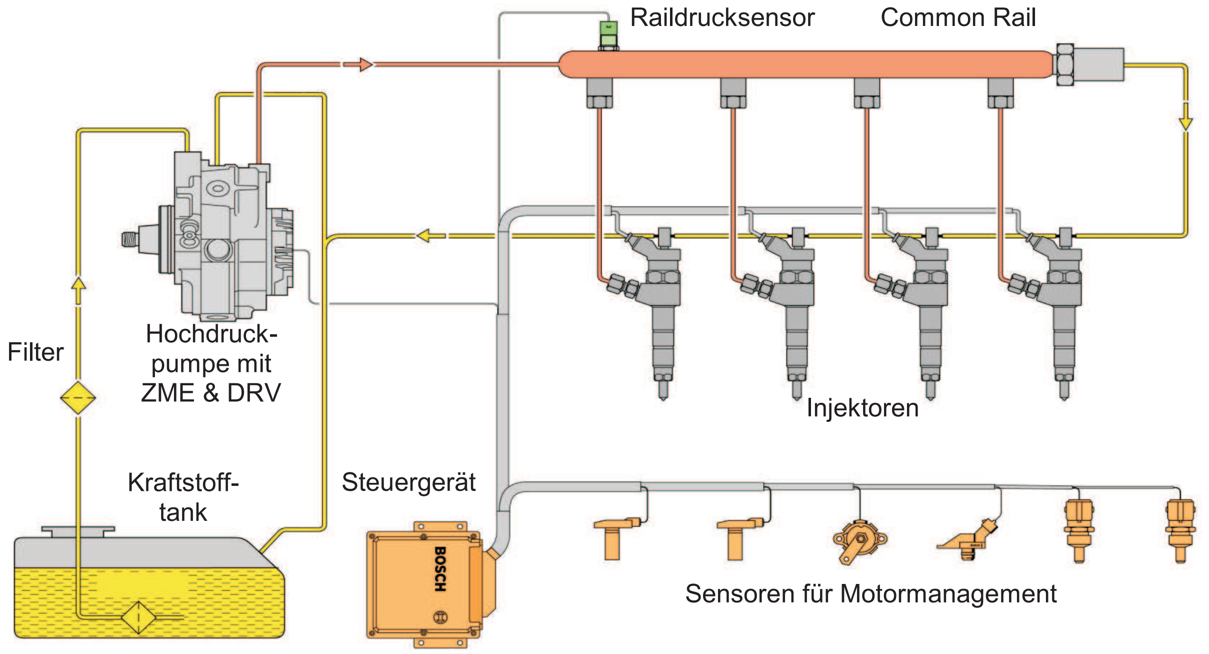


Diagnostic results:

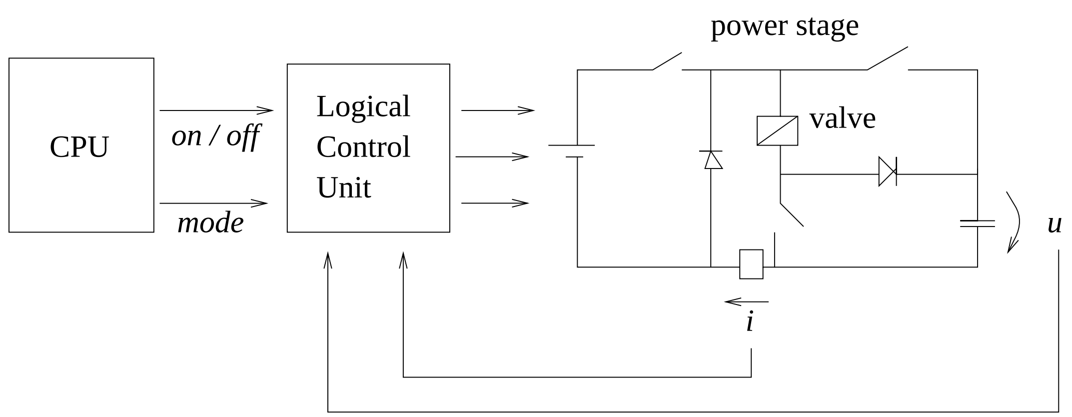
The results obtained for the discrete-event model hold for the quantised system because the model is complete.

- **Fault detection:**
  If $([\boldsymbol{U}], [\boldsymbol{Y}])$ is inconsistent with the model, a fault does exist.
- **Fault identification:**
  If $([\boldsymbol{U}], [\boldsymbol{Y}])$ is consistent with the model that holds for fault f, f is a fault candidate.

Example: Diagnosis of the common rail diesel injection system

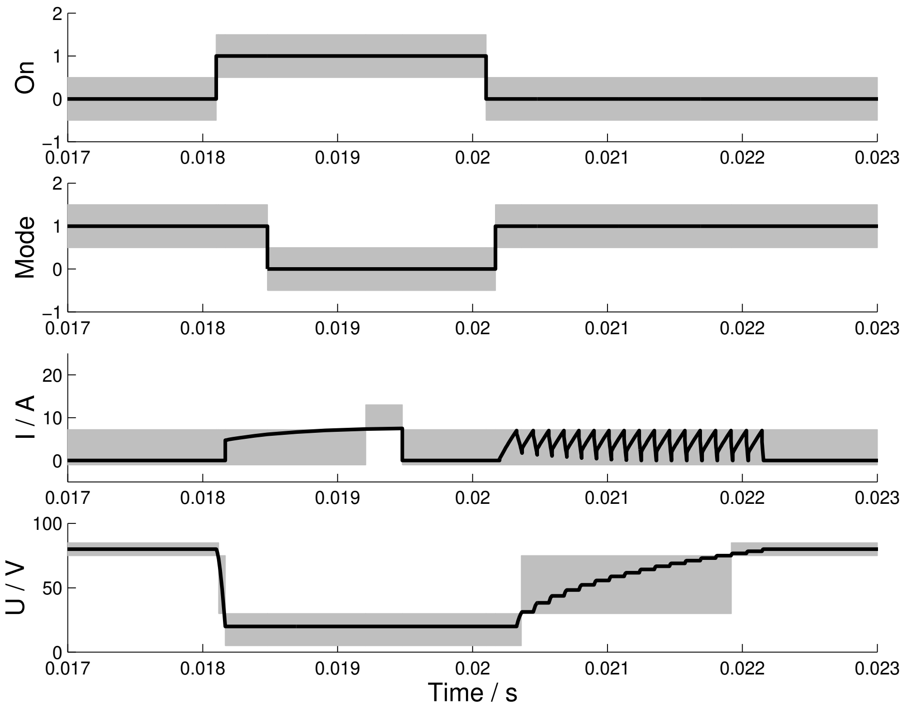


Injector block

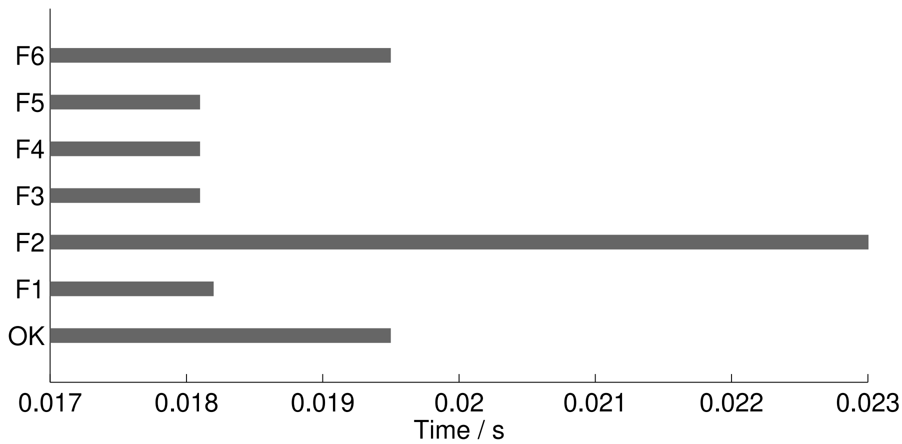


Example: Diagnosis of the common rail diesel injection system

Quantised measurements

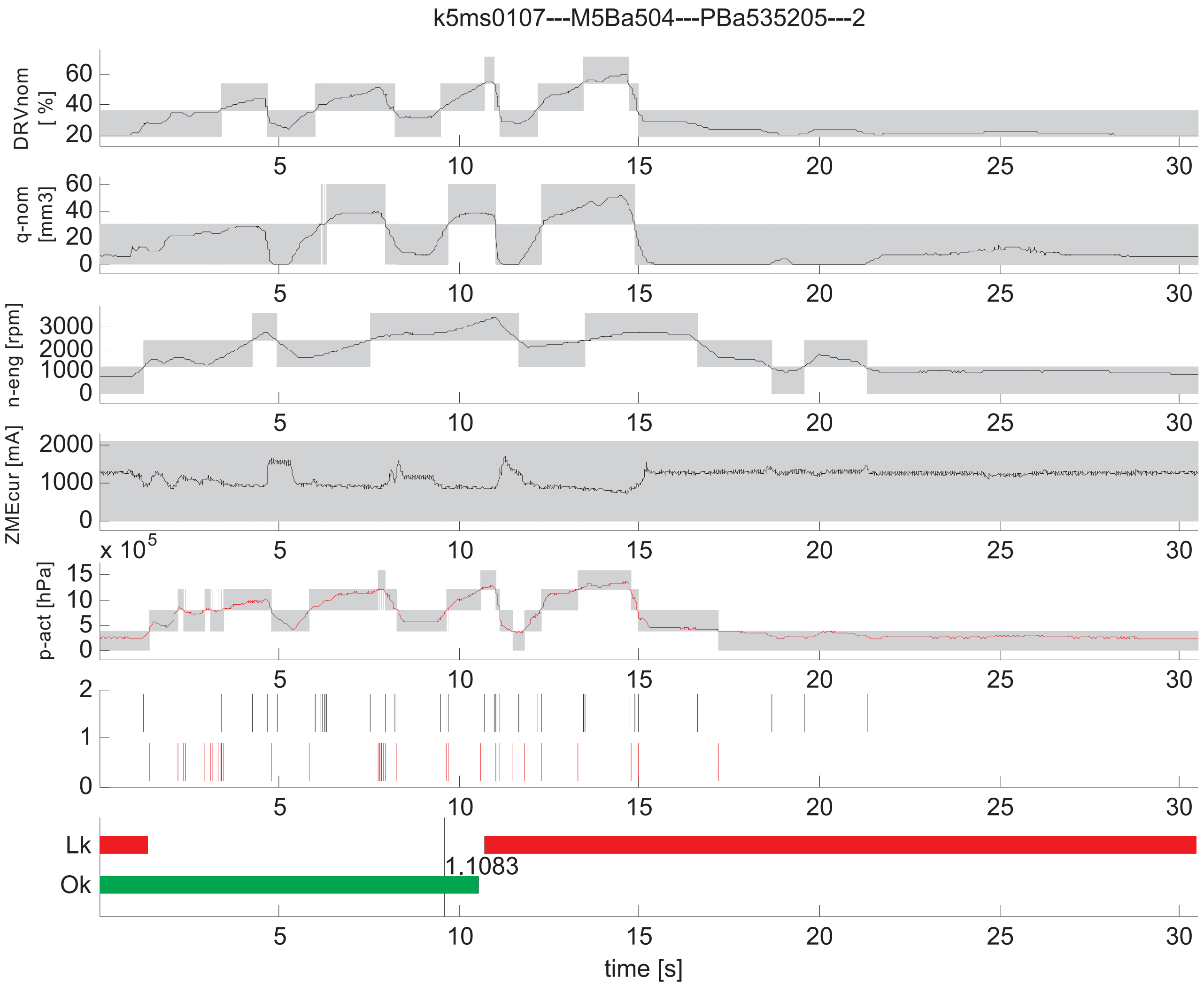


Diagnostic result for the injector

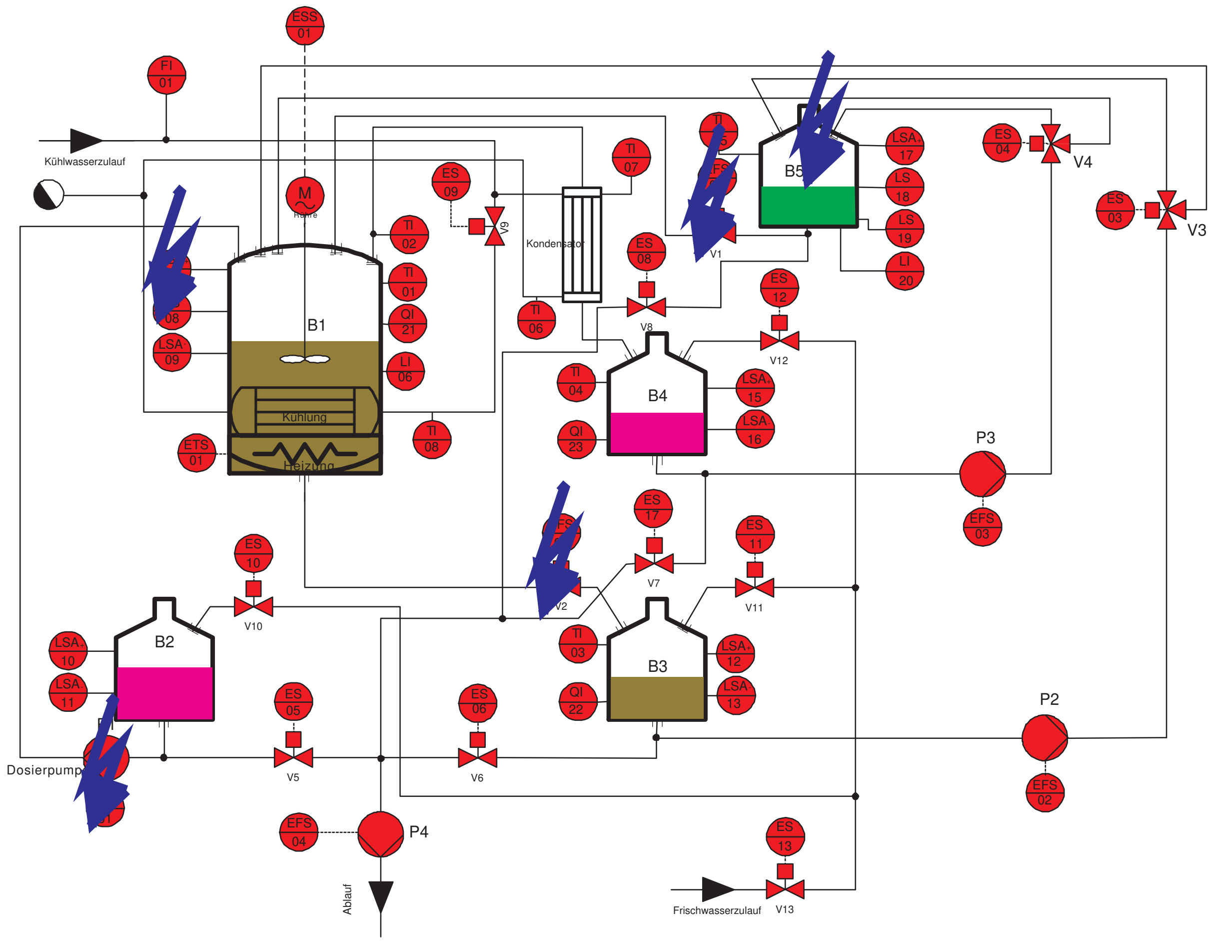


Example: Diagnosis of the common rail diesel injection system

Diagnostic result for the pressure control system:

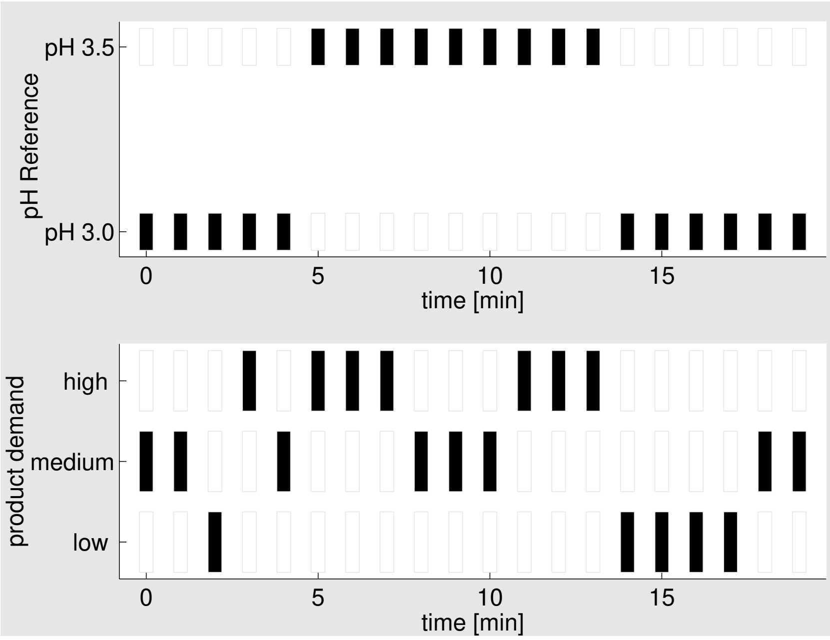


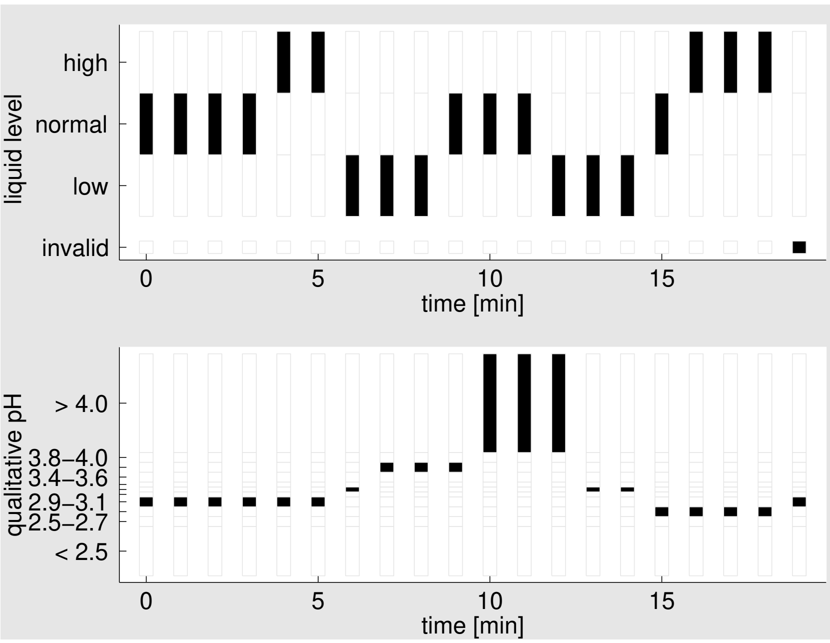
Example: Diagnosis of a neutralisation process



Example: Diagnosis of a neutralisation process

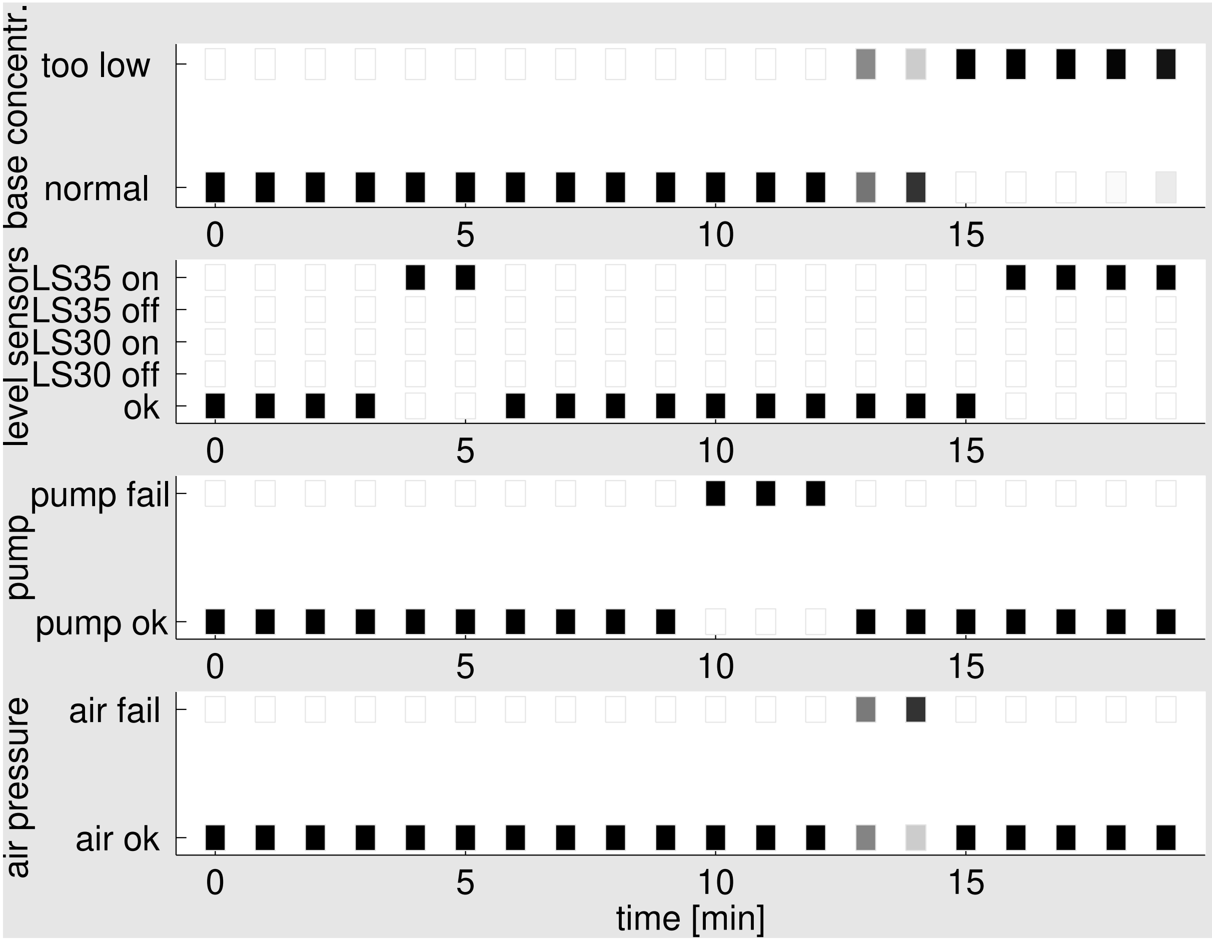
Quantised measurements



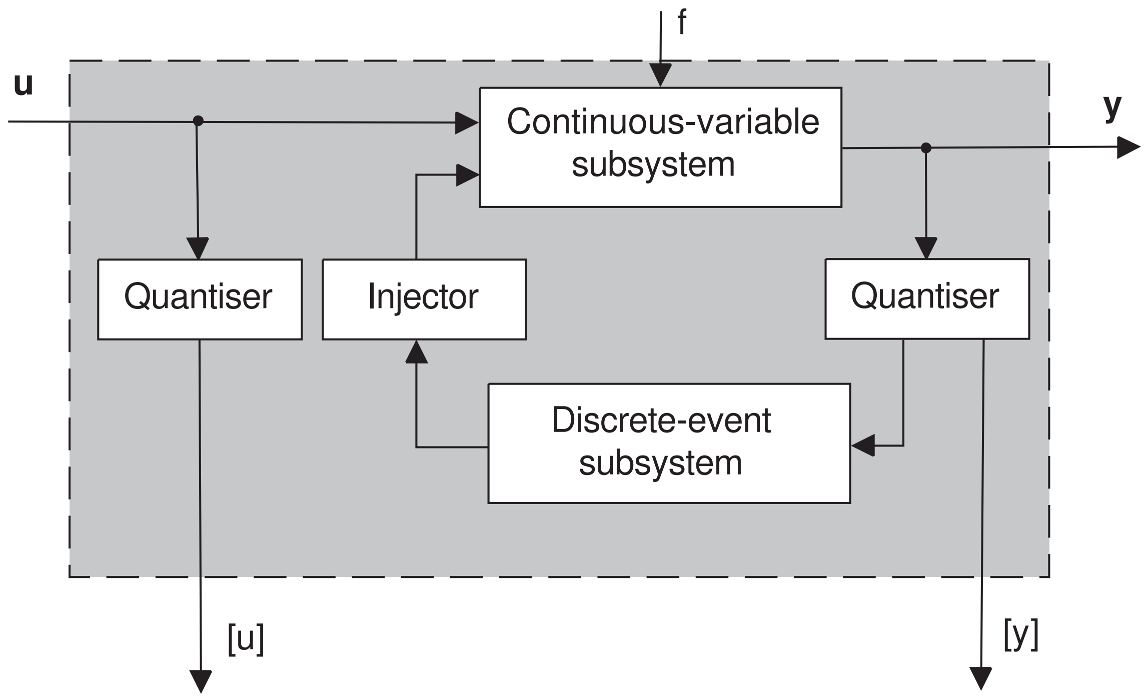


Example: Diagnosis of a neutralisation process

Diagnostic results



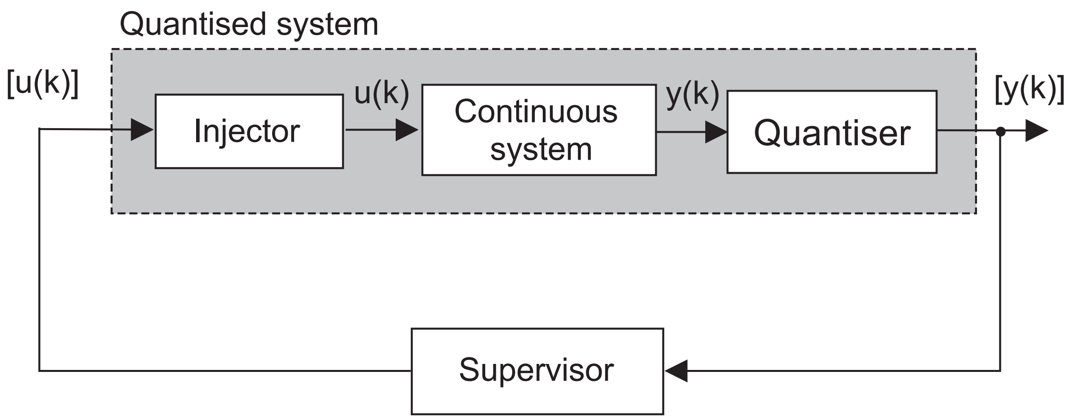
Conclusions



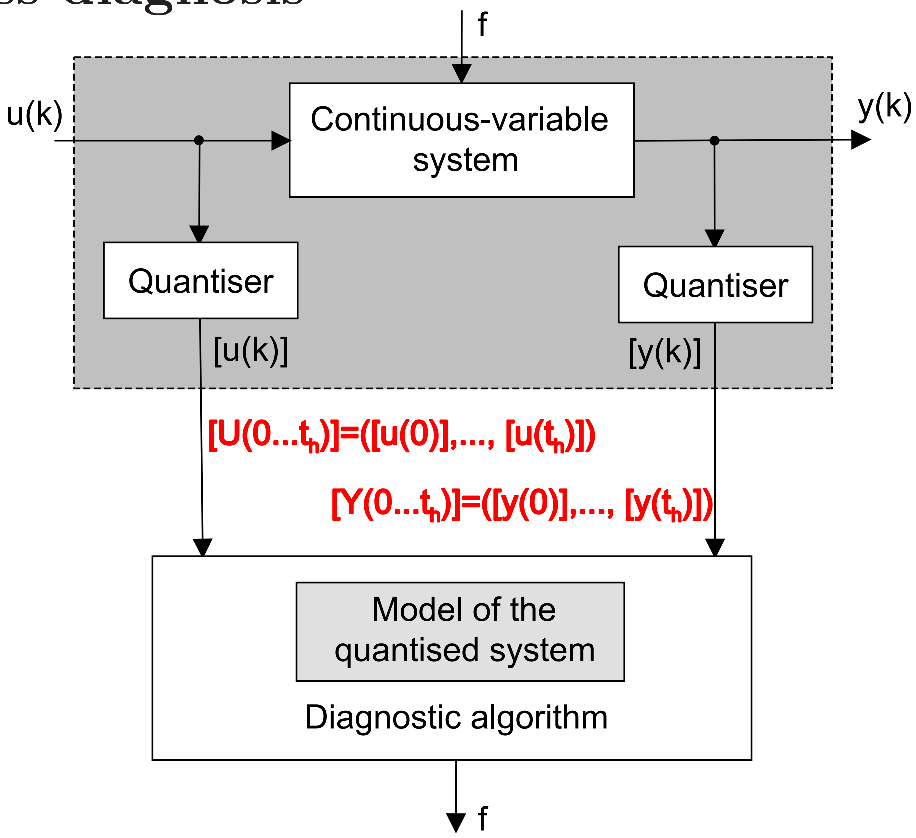
- Quantisers occur naturally within dynamical systems
- Quantisers reflect the uncertainties of the inputs and measurements
- Quantisers are introduced to reduce the information used during the diagnosis
  - Reduce the measurement information
  - Reduce the model complexity

Conclusions

Supervisory control



Process diagnosis



To solve process supervision tasks, ignore as many details as possible: Use discrete-event models of the hybrid system

Conclusions

- The dynamics of quantised systems can be described by event sequences
- Complete discrete-event representations of quantised systems can be obtained by abstraction.
- Diagnosis means to test whether the measured I/O pair is consistent with the model.
- Methods and algorithms are available for diagnosing quantised systems

The theory on quantised systems bridges the gap between continuous systems theory and discrete systems theory

References

- M. Blanke, M. Kinnaert, J. Lunze, M. Staroswiecki:
  Diagnosis and Fault-Tolerant Control
  Springer-Verlag 2003
- J. Schröder:
  Modelling, State Observation and Diagnosis of Quantised Systems
  Springer-Verlag 2002
- J. Lunze, J. Raisch:
  Discrete models for hybrid systems
  in *Modelling, Analysis, and Design of Hybrid Systems*
  Springer-Verlag 2002