CUPPAAL

optimal

Scheduling & Synthesis of Control Programs

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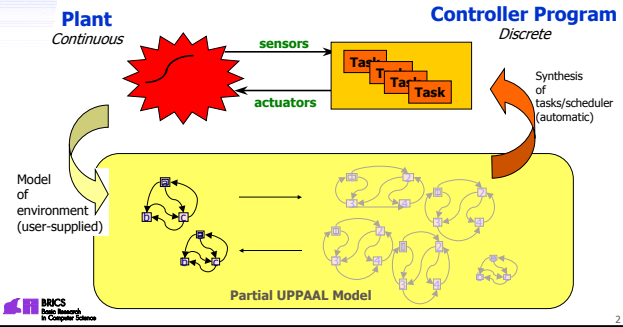
..., HSCC'01, TACAS'01, CAV'01, AIPS'02,.... AMETIST

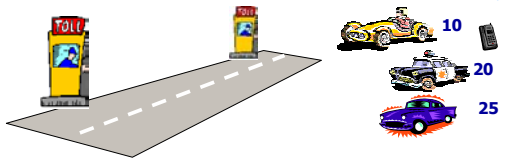
...and Beyond

Synthesis of Control Program



Real Time Scheduling

- Only 1 "BroBizz"
- Cheat is possible (drive close to car with "Bizz")



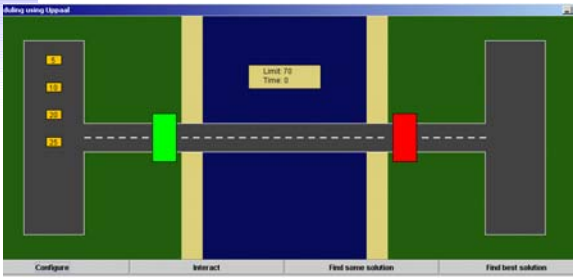
UNSAFE

Crossing Times

5
10
20
25

SAFE

CAN THEY MAKE IT TO SAFE WITHIN 70 MINUTES ???



OR use UPPAAL

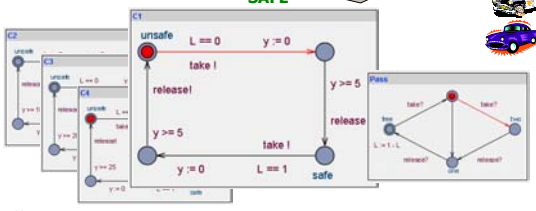
Real Time Scheduling

Solve Scheduling Problem using UPPAAL

UNSAFE

5
10
20
25

SAFE



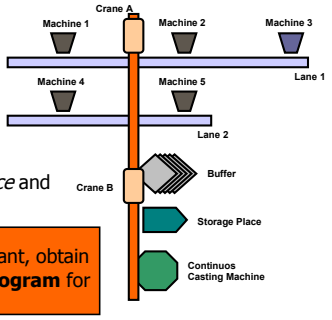
Rush Hour

ObJECTIVE: Get your CAR out

Steel Production Plant

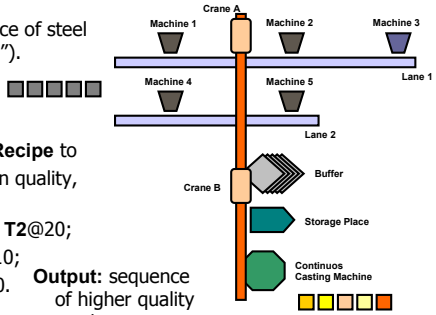
- A. Fehnker
- Hune, Larsen, Pettersson
- Case study of Esprit-LTR project 26270 VHS
- Physical plant of SIDMAR located in Gent, Belgium.
- Part between *blast furnace* and *hot rolling mill*.

Objective: model the plant, obtain **schedule** and control **program** for plant.



Steel Production Plant

Input: sequence of steel loads ("pigs").

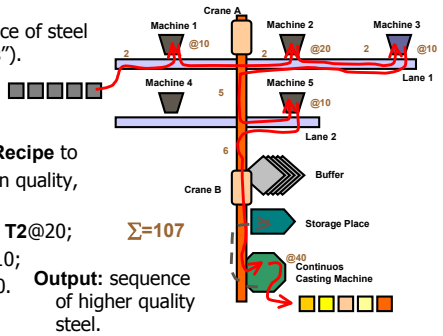


Load follows **Recipe** to become certain quality, e.g:
start; **T1**@10; **T2**@20; **T3**@10; **T2**@10; end within 120.

Output: sequence of higher quality steel.

Steel Production Plant

Input: sequence of steel loads ("pigs").



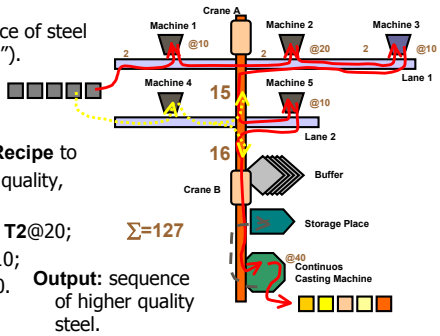
Load follows **Recipe** to become certain quality, e.g:
start; **T1**@10; **T2**@20; **T3**@10; **T2**@10; end within 120.

Σ=107

Output: sequence of higher quality steel.

Steel Production Plant

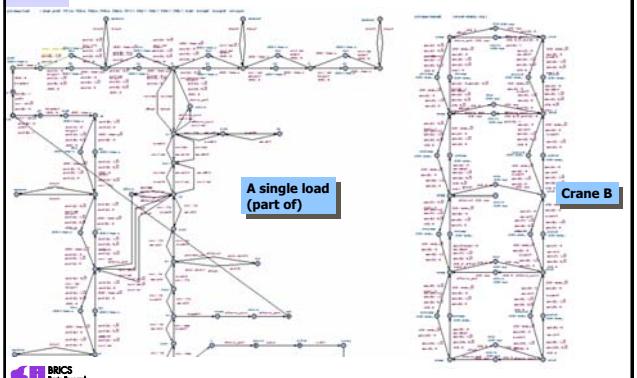
Input: sequence of steel loads ("pigs").



Load follows **Recipe** to obtain certain quality, e.g:
start; **T1**@10; **T2**@20; **T3**@10; **T2**@10; end within 120.

Σ=127

Output: sequence of higher quality steel.



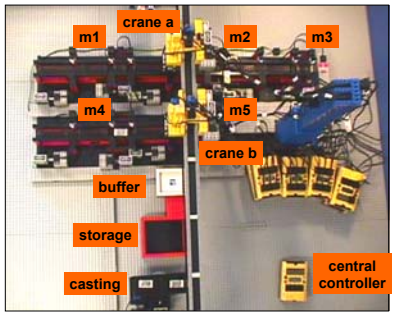
Experiment

n	All Guides						Some Guides						No Guides					
	BFS		DFS		BSH		BFS		DFS		BSH		BFS		DFS		BSH	
	s	MB	s	MB	s	MB	s	MB	s	MB	s	MB	s	MB	s	MB	s	MB
1	0,1	0,9	0,1	0,9	0,1	0,9	0,1	0,9	0,1	0,9	0,1	0,9	3,2	6,1	0,8	2,2	3.9	3.3
2	18,4	36,4	0,1	1	0,1	1,1	-	-	4,4	7,8	7,8	1,2	-	-	19,5	36,1	-	-
3	-	-	3,2	6,5	3,4	1,4	-	-	72,4	92,1	901	3,4	-	-	-	-	-	-
4	-	-	4	8,2	4,6	1,8	-	-	-	-	-	-	-	-	-	-	-	-
5	-	-	5	10,2	5,5	2,2	-	-	-	-	-	-	-	-	-	-	-	-
10	-	-	13,3	25,3	16,1	9,3	-	-	-	-	-	-	-	-	-	-	-	-
15	-	-	31,6	51,2	48,1	22,2	-	-	-	-	-	-	-	-	-	-	-	-
20	-	-	61,8	89,6	332	46,1	-	-	-	-	-	-	-	-	-	-	-	-
25	-	-	104	144	87,2	83,3	-	-	-	-	-	-	-	-	-	-	-	-
30	-	-	166	216	124,2	136	-	-	-	-	-	-	-	-	-	-	-	-
35			209	250	-	-	-	-	-	-	-	-	-	-	-	-	-	-

- **BFS** = breadth-first search, **DFS** = depth-first search, **BSH** = bit-state hashing,
- **"-"** = requires >2h (on 450MHz Pentium III), >256 MB, or suitable hash-table size was not found.
- **System size**: 2n+5 automata and 3n+3 clocks, if n=35: 75 automata and 108 clocks.
- **Schedule generated for n=60 on Sun Ultra with 2x300MHz with 1024MB in 2257s .**

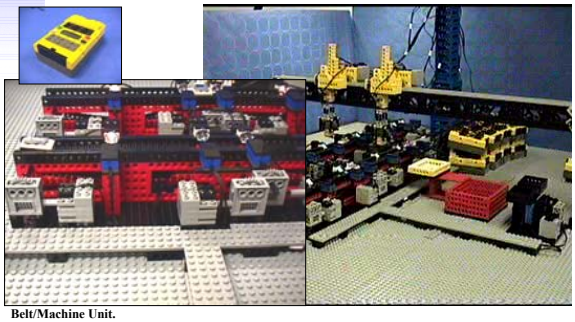
LEGO Plant Model

- LEGO RCX Mindstorms.
- Local controllers with control programs.
- IR protocol for remote invocation of programs.
- Central controller.



Synthesis

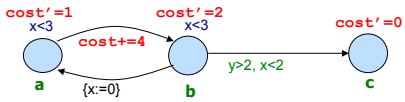
LEGO Plant Model



Belt/Machine Unit.

Linearly Priced Timed Automata:

Optimal Scheduling



- **Timed Automata + Costs on transitions and locations.**
  - Cost of performing transition: `Transition cost`.
  - Cost of performing delay **d**: (**d** x `Location cost`).

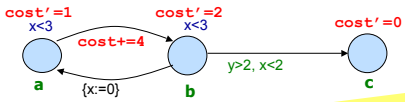
- Trace:

$$(a,x=y=0) \xrightarrow{4} (b,x=y=0) \xrightarrow[2.5 \times 2]{\varepsilon(2.5)} (b,x=y=2.5) \xrightarrow{0} (a,x=0,y=2.5)$$

- Cost of Execution Trace: Sum of costs: **4 + 5 + 0 = 9**

Linearly Priced Timed Automata:

Optimal Scheduling



Problem :

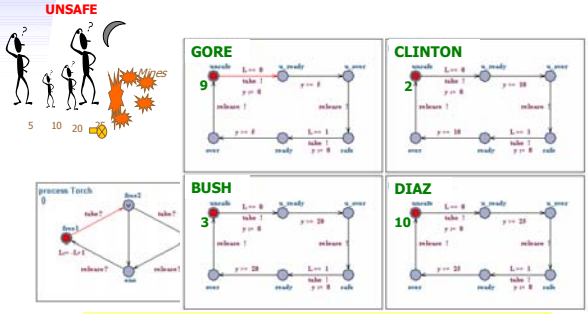
Find the minimum cost of reaching location **c**

- Tra

$$\xrightarrow[2.5 \times 2]{} (b,x=y=2.5) \xrightarrow{0} (a,x=0,y=2.5)$$

- Cost of Execution Trace: Sum of costs: **4 + 5 + 0 = 9**

EXAMPLE: Optimal rescue plan for important persons (Presidents and Actors)



OPTIMAL PLAN HAS ACCUMULATED COST=195 and TOTAL TIME=65!

Experiments *MC Order*

COST-rates G_5	C_{10}	B_{20}	D_{25}	SCHEDULE	COST	TIME	#Expl	#Pop'd
Min Time				CG> G< BD> C< CG>		60	1762 (1538)	2638
1	1	1	1	CG> G< BG> G< GD>	55	65	252	378
9	2	3	10	GD> G< CG> G< BG>	195	65	149	233
1	2	3	4	CG> G< BD> C< CG>	140	60	232	350
1	2	3	10	CD> C< CB> C< CG>	170	65	263	408
1	20	30	40	BD> B< CB> C< CG>	975 (1085)	85 (time<85)	-	-
0	0	0	0	-	0	-	406	447

Jobshop Scheduling

[TACAS'2001]

	Sport	Economy	Local News	Comic Stip
Kim	2. 5 min	4. 1 min	3. 3 min	1. 10 min
Jüri	1. 10 min	2. 20 min	3. 1 min	4. 1 min
Jan	4. 1 min	1. 13 min	3. 11 min	2. 11 min
Wang	1. 1 min	2. 1 min	3. 1 min	4. 1 min

Problem: compute the minimal **MAKESPAN**

NP-hard
- Simulated annealing
- Shiffted bottleneck
- Branch-and-Bound
- Gentic Algorithms

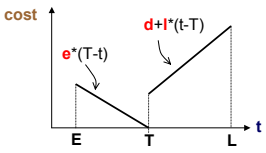
Experiments

B-&-B algorithm running for 60 sec.

		problem instance	BF cost	BF states	MC cost	MC states	MC+ cost	MC+ states	DF cost	DF states	RDF cost	RDF states	comb. heur. cost	comb. heur. states	minimal makespan
m=5	j=10	la01	-	-	-	-	-	-	2466	-	842	-	666	292	666
		la02	-	-	-	-	-	-	2360	-	806	-	672	-	655
		la03	-	-	-	-	-	-	2084	-	769	-	626	-	597
		la04	-	-	-	-	-	-	2212	-	783	-	639	-	590
	j=15	la05	-	-	-	-	593	9791	1955	-	696	-	593	284	593
		la06	-	-	-	-	-	-	3656	-	1076	-	926	480	926
		la07	-	-	-	-	-	-	3410	-	1113	-	890	-	890
		la08	-	-	-	-	-	-	3520	-	1009	-	863	400	863
		la09	-	-	-	-	-	-	3984	-	1154	-	951	425	951
	j=20	la10	-	-	-	-	-	-	3681	-	1063	-	958	454	958
		la11	-	-	-	-	-	-	4974	-	1303	-	1222	642	1222
		la12	-	-	-	-	-	-	4557	-	1271	-	1039	633	1039
		la13	-	-	-	-	-	-	4846	-	1227	-	1150	662	1150
		la14	-	-	-	-	1292	10653	5145	-	1377	-	1292	688	1292
	j=10	la15	-	-	-	-	-	-	5264	-	1459	-	1289	-	1207
		la16	-	-	-	-	-	-	4849	-	1298	-	1022	-	945
m=10		la17	-	-	-	-	-	-	4299	-	938	-	786	-	784
		la18	-	-	-	-	-	-	4763	-	1034	-	922	-	848
		la19	-	-	-	-	-	-	4566	-	1140	-	904	-	842
		la20	-	-	-	-	-	-	5056	-	1378	-	964	-	902
	j=15	la21	-	-	-	-	-	-	7008	-	1326	-	1149	-	(1040,1053)
		la22	-	-	-	-	-	-	6920	-	1413	-	1047	-	927
		la23	-	-	-	-	-	-	7676	-	1357	-	1075	-	1032
		la24	-	-	-	-	-	-	7237	-	1346	-	1061	-	935
		la25	-	-	-	-	-	-	7141	-	1290	-	1070	-	977

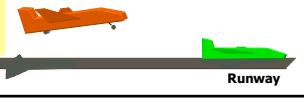
Lawrence Job Shop Problems

Example: Aircraft Landing

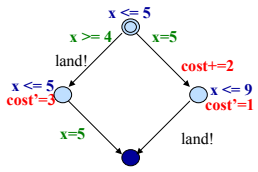


E earliest landing time
T target time
L latest time
e cost rate for being early
l cost rate for being late
d fixed cost for being late

Planes have to keep separation distance to avoid turbulences caused by preceding planes

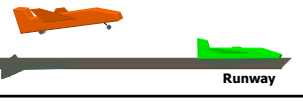


Example: Aircraft Landing



4 earliest landing time
5 target time
9 latest time
3 cost rate for being early
1 cost rate for being late
2 fixed cost for being late

Planes have to keep separation distance to avoid turbulences caused by preceding planes



Aircraft Landing

Source of examples: Baesley et al'2000

	problem instance	1	2	3	4	5	6	7
	number of planes	10	15	20	20	20	30	44
	number of types	2	2	2	2	2	4	2
1	optimal value	700	1480	820	2520	3100	24442	1550
	explored states	481	2149	920	5693	15069	122	662
	cputime (secs)	4.19	25.30	11.05	87.67	220.22	0.60	4.27
2	optimal value	90	210	60	640	650	554	0
	explored states	1218	1797	669	28821	47993	9035	92
	cputime (secs)	17.87	39.92	11.02	755.84	1085.08	123.72	1.06
3	optimal value	0	0	0	130	170	0	
	explored states	24	46	84	207715	189602	62	N/A
	cputime (secs)	0.36	0.70	1.71	14786.19	12461.47	0.68	
4	optimal value				0	0		
	explored states	N/A	N/A	N/A	65	64	N/A	N/A
	cputime (secs)				1.97	1.53		

Symbolic Semantics

Zones

Definition

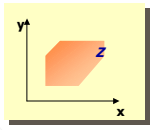
A *zone* Z over a set of clocks C is a finite conjunction of simple constraints of the forms:

$$x \geq l \qquad x \leq u \qquad x - y \geq l' \qquad x - y \leq u'$$

where $x, y \in C$, $l, u \in \mathbb{N}$ and $l', u' \in \mathbb{Z}$.

For $u \in \mathbb{R}^C$ and Z a zone we write $u \models Z$ if u satisfies all constraints of Z.

Operations

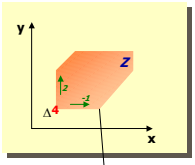


Reset: $\{x\}Z = \{u[0/x] \mid u \models Z\}$

Delay: $Z^{\uparrow} = \{u + d \mid u \models Z\}$

Offset: $\Delta_Z \models Z$ such that $\forall u \models Z. \forall x \in C. \Delta_Z(x) \leq u(x)$.

Priced Zone



Definition

A priced zone P is a tuple (Z, c, r), where:

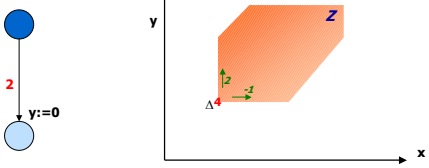
- Z is a zone
- $c \in \mathbb{N}$ describes the cost of Δ_Z
- $r : C \to \mathbb{Z}$ gives a *rate* for any clock $x \in C$.

We write $u \models P$ whenever $u \models Z$. For $u \models P$ we define $\mathrm{Cost}(u, P)$ as follows:

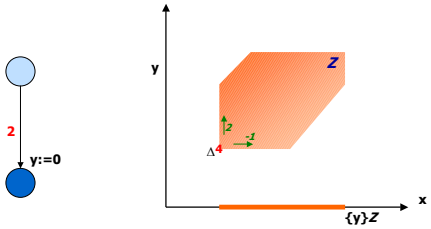
$$\mathrm{Cost}(u, P) = c + \sum_{x \in C} r(x) \cdot (u(x) - \Delta_Z(x))$$

$Cost(x, y) = 2y - x + 2$

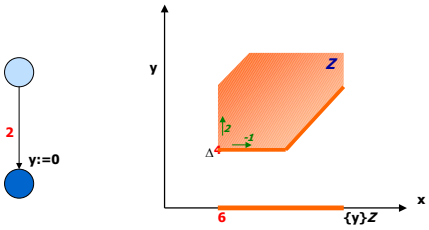
Reset



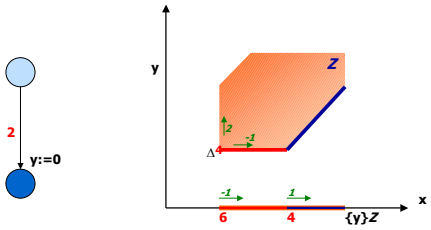
Reset



Reset



Reset



A split of $\{y\}Z$

Facets

The solution

Relative Facets

Definition

Whenever $x \geq y - l$ (resp. $x \geq l$) is a constraint of Z, then $Z \wedge (x = y - l)$ (resp. $Z \wedge x = l$) is a *lower facet* of Z *relative* to x. We denote by $LF_x(Z)$ the collection of lower facets of Z relative to x.

Similarly the collection of upper facets $UF_x(Z)$ relative to x may be defined.

Theorem

Any "point" in a projection of a zone will be in the projection of one of the lower (upper) facets of the zone:

$$\{y\}Z = \bigcup_{F \in LF_y(Z)} \{y\}F \qquad \{y\}Z = \bigcup_{F \in UF_y(Z)} \{y\}F$$

Reset

Definition

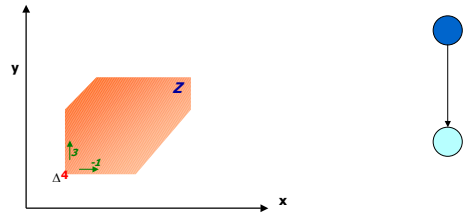
Let $P = (F, c, r)$ where F is a facet relative to y in the sense that $y = x - l$ or $y = l$ is a constraint of F. Now $\{y\}P = (F', c', r')$ where:

$$\begin{aligned} F' &= \{y\}F \\ c' &= c \\ r'(z) &= \begin{cases} r(z) & z \neq x \\ r(y) + r(x) & z = x \end{cases} \end{aligned}$$

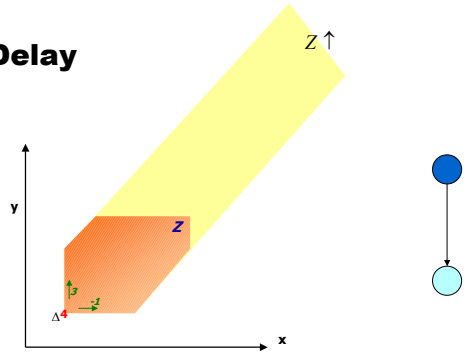
Let $P = (Z, c, r)$ be a priced zone with $r(y) \geq 0$. Then:

$$\{y\}P = \left\{ \{y\}Q \mid Q \in LF_y(P) \right\}$$

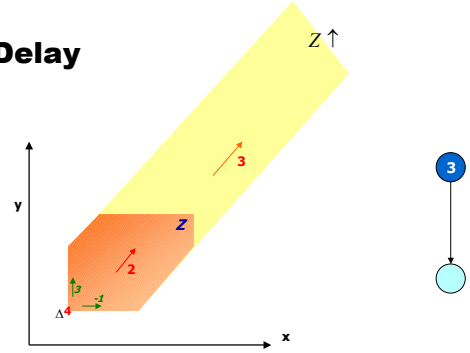
Delay



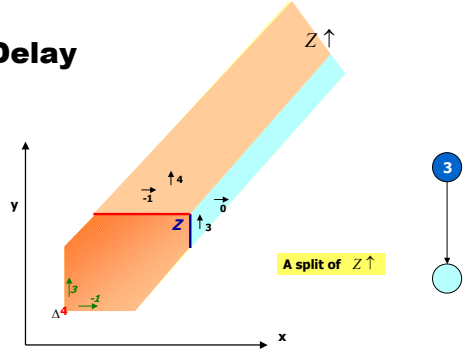
Delay



Delay



Delay



Facets

The solution

Facets

Definition

Whenever $x \geq l$ is a constraint in Z then $Z \wedge (x = l)$ is a *lower facet* of Z.

We denote by $LF(Z)$ the collection of all lower facets of Z.

Theorem

Any state reachable by delay from "points" in Z are reachable from one of the lower facets of Z, i.e.:

$$Z^{\uparrow} = \bigcup_{F \in LF(Z)} F^{\uparrow}$$

Theorem

Any state reachable by delay from "points" in Z are either within Z or reachable from one of the upper facets of Z, i.e.:

$$Z^{\uparrow} = Z \cup \bigcup_{F \in UF(Z)} F^{\uparrow}$$

Delay

Definition

Let $P = (F, c, r)$ where F is a lower (upper) facet and let $p \in \mathbb{R}_{\geq 0}$ (the price rate of a location). Then $Delay(p, P) = (F', c', r')$ where:

$$F' = F^{\uparrow}$$
$$c' = c$$
$$r'(z) = \begin{cases} r(z) & F \text{ not a lower facet wrt } z \\ p - \sum_{x \neq z} r(x) & F \text{ is a lower facet wrt } z \end{cases}$$

NOTE: By adding a clock which is always reset when taking a transition we would only be required to apply $Delay$ on facets! There is a general construction however.

Branch & Bound Algorithm

```
Cost := ∞
Passed := ∅
Waiting := {(l0, R0)}
while Waiting ≠ ∅ do
  select (l, R) from Waiting
  if l = lg and mincost(R) < Cost then
    Cost := mincost(R)
  if for all (l, R') in Passed: R' ⋠ R then
    add (l, R) to Passed
    for all (l', R') such that (l, R) → (l', R'): add (l', R') to Waiting
return Cost
```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```

$R' \leq R$

R'* is bigger & cheaper than *R

≤ is a well-quasi ordering which guarantees termination!

Branch & Bound Algorithm

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```


Related Work

- Alur, Courcourbetis, Henzinger (1993)
  Accumulated delays in Realtime Systems
- Alur, Torre, Pappas (HSCC'01)
  Optimal Paths in Weighted Timed Automata
- Behrmann, Fehnker, et all (HSCC'01)
  Minimum-Cost Reachability for Priced Timed Automata

Related Work (cont)

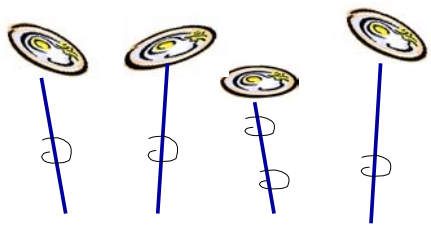
- Asarin & Maler (1999)
  Time optimal control using backwards fixed point computation
- Niebert, Tripakis & Yovine (2000)
  Minimum-time reachability using forward reachability
- Behrmann, Fehnker et all (TACAS'2001)
  Minimum-time reachability using Branch-and-Bound
- Abdeddaim, Maler (CAV'01)
  Job-Shop Scheduling using Timed Automata
- General Trend (AAAI'01):
  Integrating Scheduling/Planning and Model Checking

Optimal Infinite Scheduling

work in progress

Ed Brinksma **Kim G Larsen**
Patricia Bouyer
Emmanuel Fleury

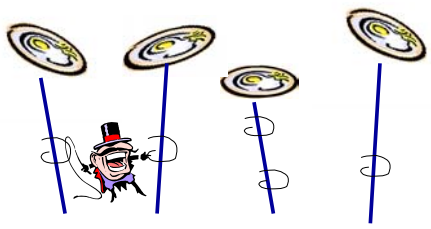
The Plate Juggling Problem



Problem: avoid having the plates falling down

The Plate Juggling Problem

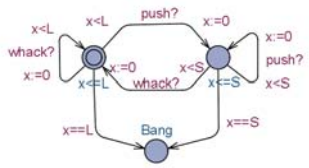
thanks to Oded



Problem: avoid having the plates falling down

The Plate Juggling Problem

using Timed Automata



A Plate

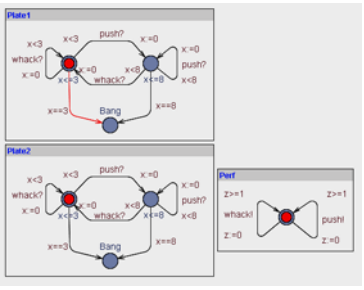
The Joggler

Infinite Scheduling

E□ ¬(Plate1.Bang or Plate2.Bang or ...)

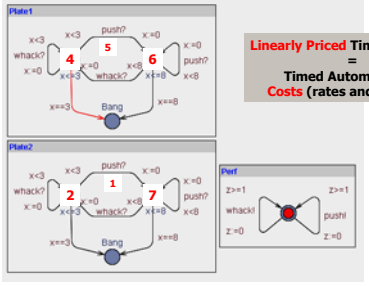
Optimal Infinite Scheduling

with respect to what ??



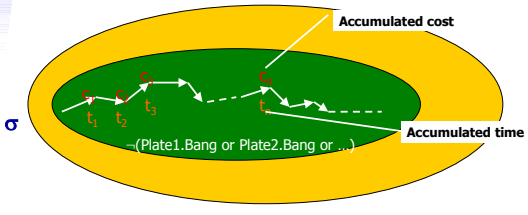
Optimal Infinite Scheduling

with respect to what ??



Linearly Priced Timed Automata
=
Timed Automata with
Costs (rates and impulses)

Optimal Infinite Scheduling

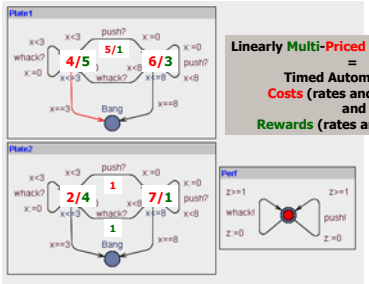


Value of path σ: $val(\sigma) = \lim_{n\to\infty} c_n/t_n$

Optimal Schedule σ^*: $val(\sigma^*) = \inf_\sigma val(\sigma)$

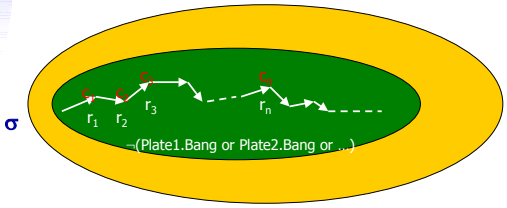
Optimal Infinite Scheduling

with respect to what ??



Linearly Multi-Priced Timed Automata
=
Timed Automata with
Costs (rates and impulses)
and
Rewards (rates and impulses)

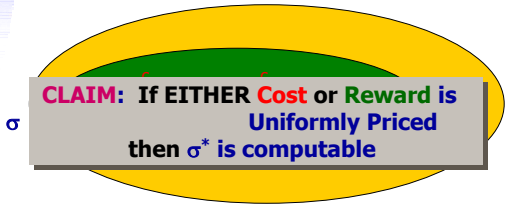
Optimal Infinite Scheduling



Value of path σ: $val(\sigma) = \lim_{n\to\infty} c_n/r_n$

Optimal Schedule σ^*: $val(\sigma^*) = \inf_\sigma val(\sigma)$

Optimal Infinite Scheduling



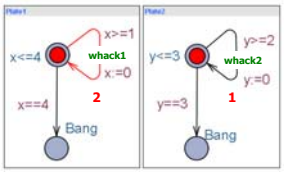
Value of path σ: $val(\sigma) = \lim_{n\to\infty} c_n/r_n$

Optimal Schedule σ^*: $val(\sigma^*) = \inf_\sigma val(\sigma)$

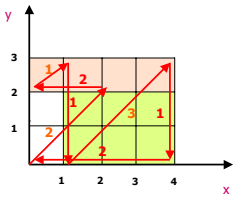
Appr Optimal Schedules

using UPPAAL

cost: impulse
reward: time



Simplified Juggling Problem

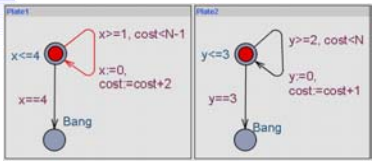


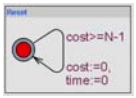
Infinite Schedule:
σ = (ε(2);whack1;epsilon(1);
whack2;ε(3);whack2;whack1)*

val(σ)= (2+1+1+2)/(2+1+3) = 1

σ optimal ??

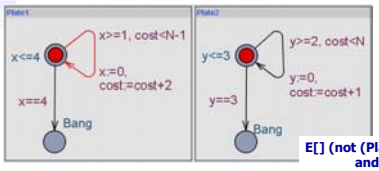
Appr Optimal Schedules

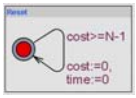




int[0,N] cost;
clock time;

Appr Optimal Schedules

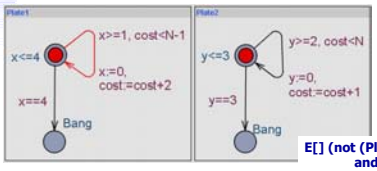


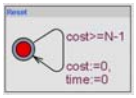


int[0,N] cost;
clock time;

E[] (not (Plate1.Bang or Plate2.Bang)
and
(cost>=N-1 imply time >= M))
=
E[] ϕ(N,M)

Appr Optimal Schedules





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clock time;

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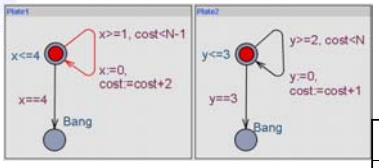
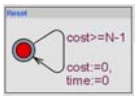
E[] ϕ(N,M)

σ ⊨ [] ϕ(N,M) imply val(σ)≤ N/M

Appr Optimal Schedules





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E[] ϕ(N,M)

(N,M)	∃σ. σ⊨[] ϕ(N,M)
(3,3)	YES
(3,4)	NO
(7,8)	YES
(7,9)	NO
(10,12)	YES
(10,13)	NO

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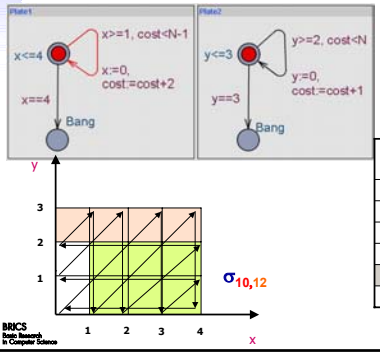
Appr Optimal Schedules



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clock time;

(N,M)	∃σ. σ⊨[] ϕ(N,M)
(3,3)	YES
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(7,8)	YES
(7,9)	NO
(10,12)	YES
(10,13)	NO

BRICS Basic Research in Computer Science

To be Continued ....