An introduction to hybrid systems theory and applications

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DISC Summer School on
Modeling and Control of Hybrid Systems
Veldhoven, The Netherlands
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http://lcewww.et.tudelft.nl/~disc_hs/

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Goals for this mini-course

Why hybrid systems ?
- Emphasis on engineering and biological examples

Modeling of hybrid systems
- Emphasis on abstraction and refinement

Analysis of hybrid systems
- Emphasis on algorithmic verification

Synthesis of hybrid controllers
- Emphasis on temporal logic synthesis

Warning : All questions and answers are biased and incomplete!

Some references

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George J. Pappas
Automatica. To appear in 2003.

Model checking LTL over controllable linear systems is decidable
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Hybrid Systems : Computation and Control, Lecture Notes in Computer Science, Prague, Czech Republic, April 2003

Modeling and analyzing biomolecular networks
Rajeev Alur, Calin Belta, Vijay Kumar, Max Mintz, George J. Pappas, Harvey Rubin, and Jonathan Schug
Computing in Science and Engineering, 4(1):20-31, January 2002.

Symbolic reachability computations for families of linear vector fields
G. Lafferriere, G. J. Pappas, and S. Yovine
Journal of Symbolic Computation, 32(3):231-253, September 2001.

Discrete abstractions of hybrid systems
R.Alur, T. Henzinger, G. Lafferriere, G. Pappas
Proceedings of the IEEE, 88(2):971-984, July 2000.

Hierarchically consistent control systems
George J. Pappas, Gerardo Lafferriere, and Shankar Sastry
IEEE Transactions on Automatic Control, 45(6):1144-1160, June 2000.

O-minimal hybrid systems
G. Lafferriere, G. J. Pappas, and S. Sastry
Mathematics of Control, Signals, and Systems, 13(1):1-21, March 2000.

Decidable controller synthesis for classes of linear systems
Omid Shakernia, George J. Pappas, and Shankar Sastry
Hybrid Systems : Computation and Control, Lecture Notes in Computer Science, volume 1790, Springer, 2000

Outline of this mini-course

Lecture 1 : Monday, June 23
- Examples of hybrid systems, modeling formalisms

Lecture 2 : Monday, June 23
- Transitions systems, temporal logic, refinement notions

Lecture 3 : Tuesday, June 24
- Discrete abstractions of hybrid systems for verification

Lecture 4 : Tuesday, June 24
- Discrete abstractions of continuous systems for control

Lecture 5 : Thursday, June 26
- Bisimilar control systems

Why hybrid ?

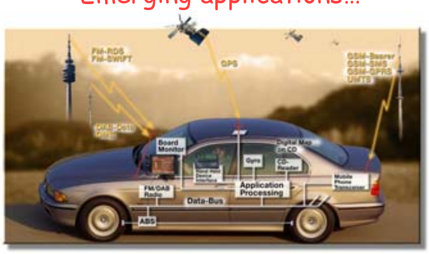
Enabling technologies

Advances in sensor and actuator technology
GPS, control of quantum systems

Invasion of powerful microprocessors in physical devices
Sophisticated software/hardware on board

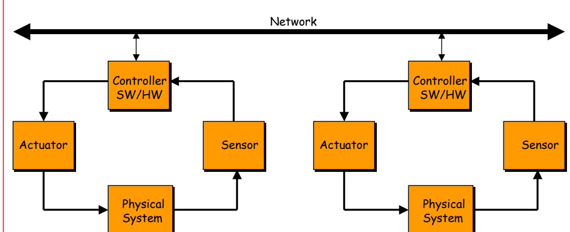
Networking everywhere
Interconnects subsystems

Emerging applications…

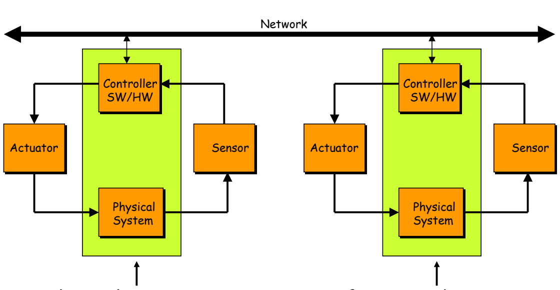


Latest BMW : 72 networked microprocessors
Boeing 777 : 1280 networked microprocessors

Networked embedded systems…

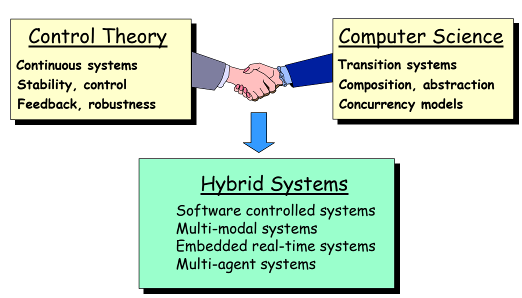


Networked embedded systems…

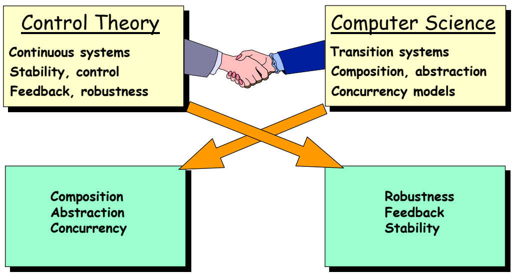


Physical system is continuous, software is discrete

Discrete and Continuous



Exporting Science



Different views…

Computer science perspective

View the physics from the eyes of the software

Modeling result : Hybrid automaton

Control theory perspective

View the software from the eyes of the physics

Modeling result : Switched control systems

Hybrid behavior arises in

Hybrid dynamics

Hybrid model is a simplification of a larger nonlinear model

Quantized control of continuous systems

Input and observation sets are finite

Logic based switching

Software is designed to supervise various dynamics/controllers

Partial synchronization of many continuous systems

Resource allocation for competing multi-agent systems

Hybrid specifications of continuous systems

Plant is continuous, but specification is discrete or hybrid…

Logic based switching

Nuclear reactor example

Without rods

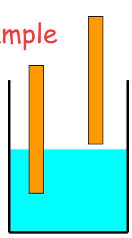
$$\dot{T} = 0.1T - 50$$

With rod 1

$$\dot{T} = 0.1T - 56$$

With rod 2

$$\dot{T} = 0.1T - 60$$

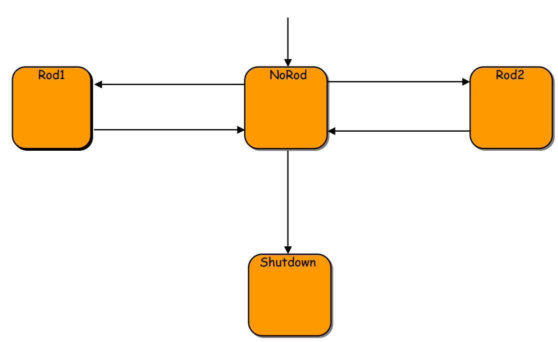


Rod 1 and 2 cannot be used simultaneously

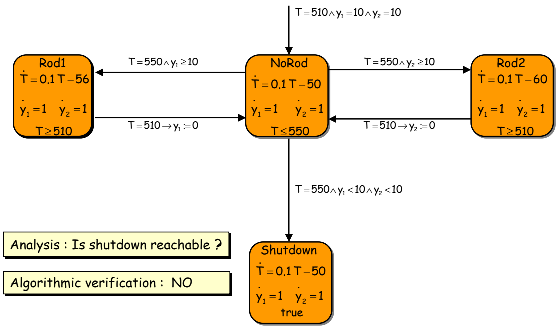
Once a rod is removed, you cannot use it for 10 minutes

Specification : Keep temperature between 510 and 550 degrees.
If T=550 then either a rod is available or we shutdown the plant.

Software model of nuclear reactor



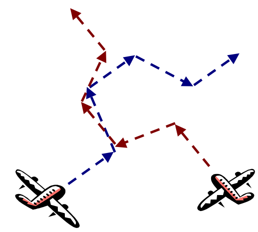
Hybrid model of nuclear reactor



Analysis : Is shutdown reachable ?

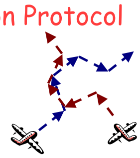
Algorithmic verification : NO

Conflict Resolution in ATM*



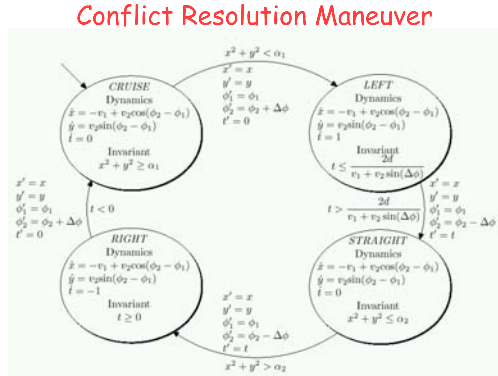
Conflict Resolution Protocol

1. Cruise until a_1 miles away
2. Change heading by $\Delta\Phi$
3. Maintain heading until lateral distance d
4. Change to original heading
5. Change heading by $-\Delta\Phi$
6. Maintain heading until lateral distance -d
7. Change to original heading



Is this protocol safe ?

Conflict Resolution Maneuver



Computing Unsafe Sets

$$v_1=4;\ v_2=5;\ \lambda=0$$

$$\begin{aligned}
\texttt{unsafeCruise} &= \texttt{Resolve}\left[\exists t>0 \wedge (x-v_1t+\lambda v_2 t)^2+(y+\sqrt{1-\lambda^2}v_2t)^2\leq 25\right]\\
&= \left(y<-\tfrac{20}{\sqrt{41}} \wedge -\sqrt{41}-\tfrac{4y}{5}\leq x\leq\sqrt{41}-\tfrac{4y}{5}\right)\vee\left(y=-\tfrac{20}{\sqrt{41}}\wedge-\sqrt{41}-\tfrac{4y}{5}<x\leq\sqrt{41}-\tfrac{4y}{5}\right)\vee\\
&\quad\left(y=\tfrac{20}{\sqrt{41}}\wedge-\sqrt{25-y^2}<x<\sqrt{41}-\tfrac{4y}{5}\right)\vee\left(\tfrac{20}{\sqrt{41}}\leq y<5\wedge-\sqrt{25-y^2}<x<\sqrt{25-y^2}\right)\vee\\
&\quad\left(-\tfrac{20}{\sqrt{41}}<y<\tfrac{20}{\sqrt{41}}\wedge-\sqrt{25-y^2}<x\leq\sqrt{41}-\tfrac{4y}{5}\right)
\end{aligned}$$

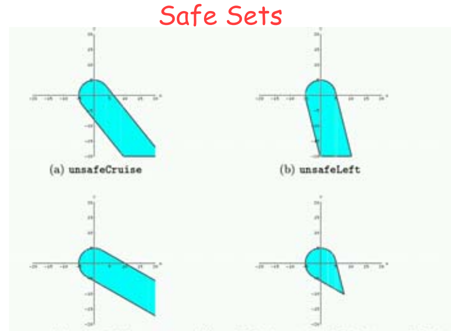
$$v_1=4;\ v_2=5;\ \lambda=\tfrac{3}{5}$$

$$\begin{aligned}
\texttt{unsafeLeft} &= \texttt{Resolve}\left[\exists t>0 \wedge (x-v_1t+\lambda v_2 t)^2+(y+\sqrt{1-\lambda^2}v_2t)^2\leq 25\right]\\
&= \left(y<-\tfrac{5}{\sqrt{17}} \wedge -\tfrac{5\sqrt{17}}{4}-\tfrac{x}{4}\leq x\leq\tfrac{5\sqrt{17}}{4}-\tfrac{x}{4}\right)\vee\left(y=-\tfrac{5}{\sqrt{17}}\wedge-\tfrac{5\sqrt{17}}{4}-\tfrac{x}{4}<x\leq\tfrac{5\sqrt{17}}{4}-\tfrac{x}{4}\right)\vee\\
&\quad\left(y=\tfrac{5}{\sqrt{17}}\wedge-\sqrt{25-y^2}<x<\tfrac{5\sqrt{17}}{4}-\tfrac{x}{4}\right)\vee\left(\tfrac{5}{\sqrt{17}}<y<5\wedge-\sqrt{25-y^2}<x<\sqrt{25-y^2}\right)\vee\\
&\quad\left(-\tfrac{5}{\sqrt{17}}<y<\tfrac{5}{\sqrt{17}}\wedge-\sqrt{25-y^2}<x\leq\tfrac{5\sqrt{17}}{4}-\tfrac{x}{4}\right)
\end{aligned}$$

$$v_1=4;\ v_2=5;\ \lambda=-\tfrac{3}{5}$$

$$\begin{aligned}
\texttt{unsafeRight} &= \texttt{Resolve}\left[\exists t>0 \wedge (x-v_1t+\lambda v_2 t)^2+(y+\sqrt{1-\lambda^2}v_2t)^2\leq 25\right]\\
&= \left(y<-7\sqrt{\tfrac{5}{13}} \wedge -\tfrac{5\sqrt{65}}{4}-\tfrac{7y}{4}\leq x\leq\tfrac{5\sqrt{65}}{4}-\tfrac{7y}{4}\right)\vee\left(y=-7\sqrt{\tfrac{5}{13}}\wedge-\tfrac{5\sqrt{65}}{4}-\tfrac{7y}{4}<x\leq\tfrac{5\sqrt{65}}{4}-\tfrac{7y}{4}\right)\vee\\
&\quad\left(y=7\sqrt{\tfrac{5}{13}}\wedge-\sqrt{25-y^2}<x<\tfrac{5\sqrt{65}}{4}-\tfrac{7y}{4}\right)\vee\left(7\sqrt{\tfrac{5}{13}}<y<5\wedge-\sqrt{25-y^2}<x<\sqrt{25-y^2}\right)\vee\\
&\quad\left(-7\sqrt{\tfrac{5}{13}}<y<7\sqrt{\tfrac{5}{13}}\wedge-\sqrt{25-y^2}<x\leq\tfrac{5\sqrt{65}}{4}-\tfrac{7y}{4}\right)
\end{aligned}$$

Safe Sets



(a) unsafeCruise

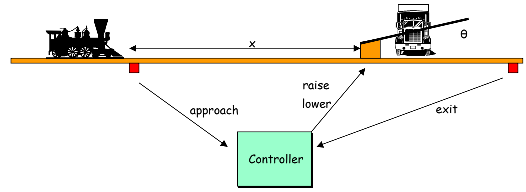
(b) unsafeLeft

(c) unsafeRight

(d) unsafeCruise ∧ unsafeLeft ∧ unsafeRight

Partial synchronization (Concurrency)

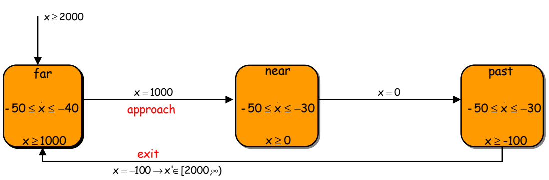
The train gate



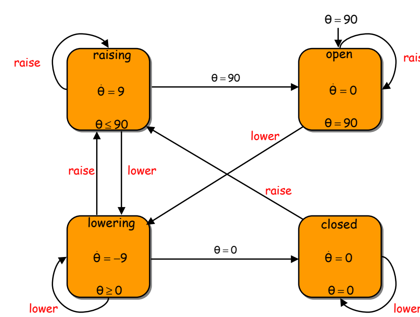
System = Train || Gate || Controller

Safety specification : If train is within 10 meters of the crossing, then gate should completely closed.
Liveness specification : Keep gate open as much as possible.

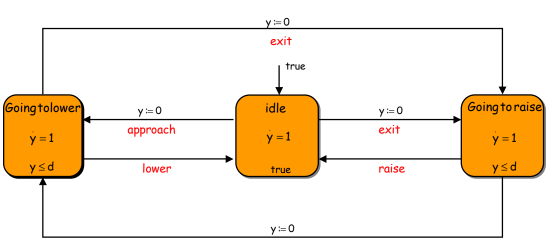
Train model



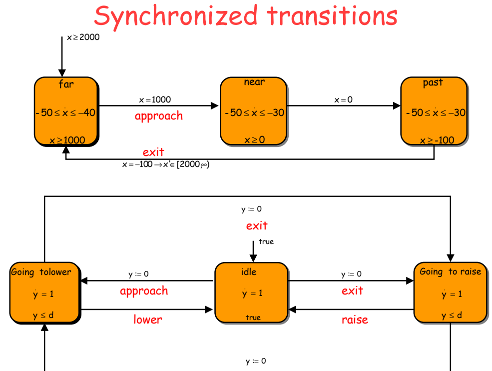
Gate model



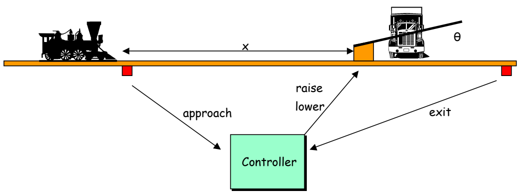
Controller model



Synchronized transitions



Verifying the controller



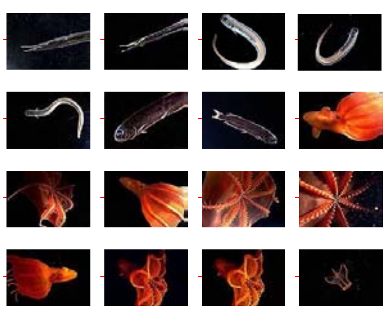
System = Train || Gate || Controller

Safety specification : Can we avoid the set $\theta > 0 \wedge (-10 \leq x \leq 10)$?

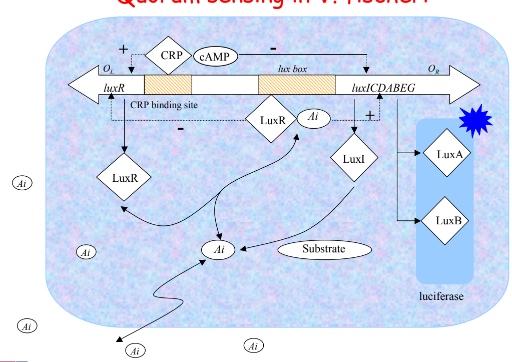
Parametric HyTech verification : YES if $d \leq \frac{49}{5}$

Hybrid dynamics

Quorum sensing in V. fischeri



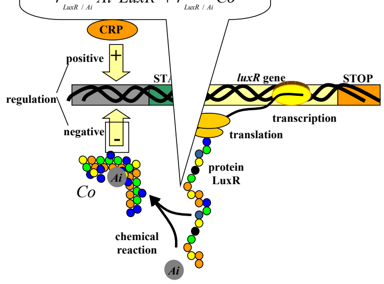
Quorum sensing in V. fischeri



Modeling of biological systems

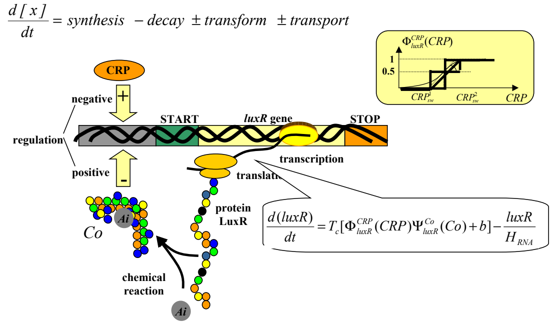
$$\frac{d(LuxR)}{dt} = T_l\, luxR - \frac{LuxR}{H_{sp}} - r^b_{LuxR/Ai}\, Ai\, LuxR + r^d_{LuxR/Ai}\, Co$$

$d[\ldots$ $\pm$ *transport*



Modeling of biological systems

$$\frac{d[x]}{dt} = synthesis - decay \pm transform \pm transport$$



$$\frac{d(luxR)}{dt} = T_c[\Phi^{CRP}_{luxR}(CRP)\Psi^{Co}_{luxR}(Co) + b] - \frac{luxR}{H_{RNA}}$$

V. fischeri mathematical model

$$\frac{d(luxR)}{dt} = T_c(pl_{ij} + b) - \frac{luxR}{H_{RNA}}$$

$$\frac{d(luxICDABEG)}{dt} = T_c(pr_{kl} + b) - \frac{luxICDABEG}{H_{RNA}}$$

$$\frac{d(LuxR)}{dt} = T_l luxR - \frac{LuxR}{H_{sp}} - k_1 LuxRAi\_i + k_{-1}Co$$

$$\frac{d(LuxI)}{dt} = T_l luxICDABEG - \frac{LuxI}{H_{sp}}$$

$$\frac{d(Ai\_i)}{dt} = (k_2 LuxI)S - \frac{Ai\_i}{H_{Ai}} - k_1 LuxRAi\_i + k_{-1}Co + n(Ai\_e - Ai\_i)$$

$$\frac{d(Co)}{dt} = -\frac{Co}{H_{sp}} + k_1 LuxRAi\_i - k_{-1}Co$$

$$\frac{d(Ai\_e)}{dt} = -\frac{Ai\_e}{H_{Ai}} - n(Ai\_e - Ai\_i)$$

$$\frac{d(CRP)}{dt} = S_2$$

Co	CRP	% act
Co<Co_sw_l	CRP<CRP_sw_l	pl_{00}
Co<Co_sw_l	CRP>CRP_sw_l	pl_{01}
Co>Co_sw_l	CRP<CRP_sw_l	pl_{10}
Co>Co_sw_l	CRP>CRP_sw_l	pl_{11}

luxR

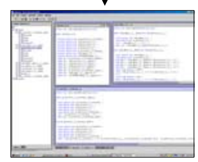
Co	CRP	% act
Co<Co_sw_r	CRP<CRP_sw_r	pr_{00}
Co<Co_sw_r	CRP>CRP_sw_r	pr_{01}
Co>Co_sw_r	CRP<CRP_sw_r	pr_{10}
Co>Co_sw_r	CRP>CRP_sw_r	pr_{11}

luxICDABEG

BioCharon = BioSketchPad + Charon

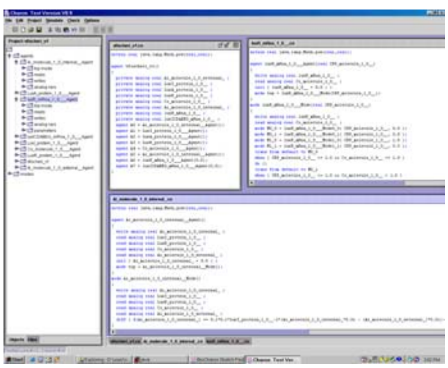
BioSketchPad

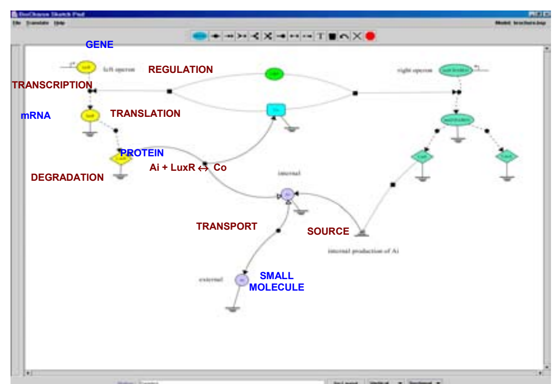
Biologist-friendly environment for representing, storing, simulating, and analyzing biomolecular networks.

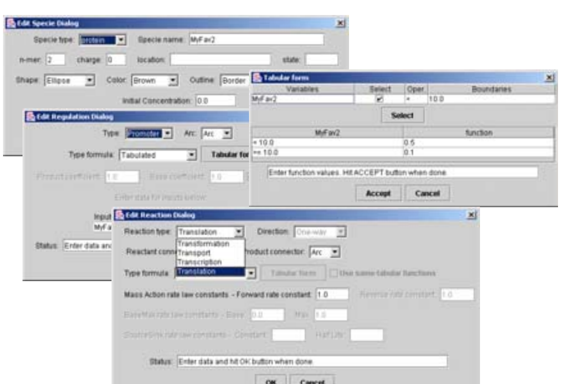


Charon

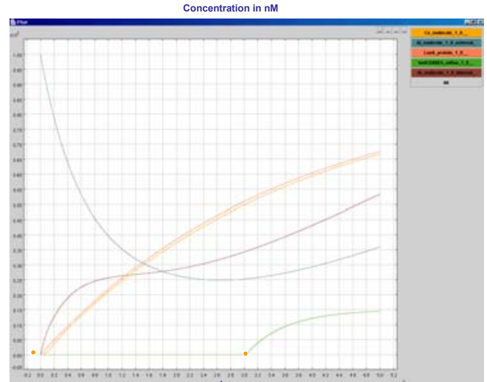
A programming language for modeling, simulating, analyzing, and designing hybrid systems







Concentration in nM



Research Issues

Modeling Issues
- Well posedness, robustness, zenoness

Analysis
- Stability issues, qualitative theory, parametric analysis

Verification
- Algorithmic methods that verify system performance

Controller Synthesis
- Algorithmic methods that design hybrid controllers

Simulation
- Mixed signal simulation, event detection, modularity

Code generation
- From hybrid models to embedded code

Complexity
- Compositionality and hierarchies

Tools : HyTech, Checkmate, d/dt, HYSDEL, Stateflow, Charon