Exam — Modeling and Control of Hybrid Systems (SC42075)

June 26, 2025

This exam has **three** questions; the maximal score for each question is marked in red next to the given question.

Questions:

10 1. For each of the following concepts,

   - **explain** the given concept in your own words, and
   - also give a **new** example (i.e., one that has ***not*** been discussed in the lecture notes, the slides, the lectures, or the assignment of this and previous years) that illustrates the given concept — also briefly explain and motivate how and why the given example relates to the given concept.
     Note: The examples do not have to correspond to real-life systems or situations; they can be simplified, purely mathematical, given as a picture, ..., as long as they are correct and well-explained.

   (a) invariant set

   (b) generalized gradient

   (c) mixed logical dynamical system

   (d) region graph

   (e) Zeno behavior

8 2. For each of the following systems or functions with a specific property, give a **new** example (i.e., one that has ***not*** been discussed in the lecture notes, the slides, the lectures, or the assignment of this and previous years) of a system or function that satisfies the given property — *if possible at all*.
   Motivate your answer.
   Note: The examples do not have to correspond to real-life systems or situations; they can be simplified, purely mathematical, given as a picture, ..., as long as they are correct and well-explained.

   (a) a deterministic timed automaton with Zeno behavior but no live-lock behavior

   (b) a deterministic single-input single-output LC system with $\mathbb{R}^3$ as state space

   (c) a nondeterministic unconstrained MMPS system with $\mathbb{R}^2$ as state space

   (d) a function $f : \mathbb{R} \to \mathbb{R} : x \mapsto f(x)$ without any affine parts and with a generalized gradient in $x = 1$ that is equal to $[-1, 4]$

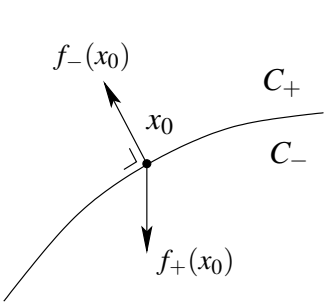
5.5 3. Consider a switched system with the following dynamics:

$$\dot{x}(t) = \begin{cases} f_-(x(t)) & \text{if } x(t) \in C_- \\ f_+(x(t)) & \text{if } x(t) \in C_+ \end{cases}$$

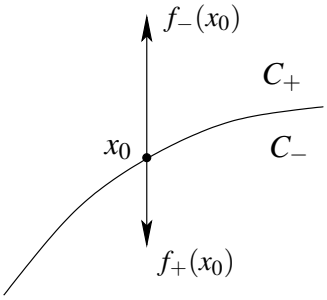
   where C_- and C_+ are two parts of the state space with a common boundary defined by a hyper-surface $\phi(x) = 0$.

../..

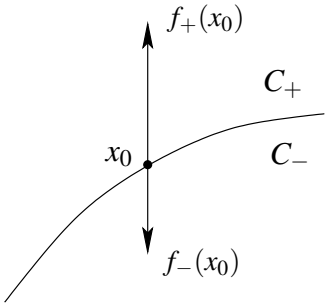
- Define and explain in your own words the meaning and the purpose of the concept Filippov solution.
- Now let x_0 be a point on the boundary of the regions C_- and C_+ as indicated in the figures below.



Case a)



Case b)



Case c)

Indicate how the system will evolve if it starts from the point x_0 in each of the cases a), b), and c),

1) if Filippov solutions are not allowed
2) if Filippov solutions are allowed

If there are multiple solutions possible, give *all* of them. Explain and motivate your answer.

End of the exam