Exam — June 2023 – Grading template
Modeling and Control of Hybrid Systems (SC42075)

As many answers are correct, below you can find the general guidelines that have been used to assign scores to the different questions and subquestions.

The maximal score for each subquestion is marked in red next to the given subquestion. This score can only be obtained if the answer is complete and fully correct, and if it does not contain any irrelevant or wrong statements.

Question 1 (10 points)

2 is the maximum score per subquestion for a fully correct answer (i.e., new/innovative, correct ,and explained well).
[in principle 1 for example and 1 for explanation, except in the cases listed below]

The examples do not have to correspond to real-life systems or situations, they can be simplified, purely mathematical, given as a picture, . . . , as long as they are correct and well-explained.

Subquestions:

2 (a) guard: make sure to mention that transition can occur, but is not forced

2 (b) bisimulation: make sure to mention and explain(!) that there should be full equivalence (if and only if) for a given property, else: -0.5
Self-loop forgotten in example: -0.25

2 (c) Zeno behavior: make sure you really have infinite number of switches in finite time interval
Note that damping/friction will in general not lead to Zeno behavior!: -1
For example it should also be argued why the time span is finite, else -0.75
Newton's cradle: not new (essentially same as 3 balls of lecture notes)

2 (d) global uniform asymptotic stability (GUAS): both global and uniform should be mentioned and explained, if none: -1; if one: -0.5
If only Lyapunov (i.e., a property, not a definition) is used to explain: -1
If no switching signal in example: -1 (not illustrative)

2 (e) Filippov solution: make sure to mention switched systems, vector field pointing towards other region, convex combination, sliding along switching surface: +0.25 for each of these items
If the given example does not exhibit sliding mode behavior (but e.g. just stays stuck in 1 point): -1

Question 2 (4.5 points)

1.5 is the maximum score per subquestion for a fully correct answer (i.e., new/innovative, correct and explained well)

If example wrong: -1.

If the example is both new and completely correct, but there is no explanation, the score is 0.75.

Examples do not have to correspond to real-life systems or situations, they can be simplified, purely mathematical, given as a picture, . . . , as long as they are correct and well-explained.

Subquestions:

1.5 (a) timed automaton with 3 modes and live-lock:
If initial state is not specified: -0.5
If 2 modes are in fact equivalent $\rightarrow$ only 2 modes $\rightarrow$ wrong example: -0.75

1.5 (b) non-deterministic PWA with 2 modes: can have non-deterministic initial state or multiple values on boundary
If no k: -0.25
If no $x(k+1)$ or no $\dot{x}$: -0.5
If overlapping interiors: -0.75

1.5 (c) fully deterministic hybrid automaton: if no or no unique initial condition: -0.75
If one of the modes is linear: -0.5
If invariant sets and guards not fully complementary: -0.25

Question 3 (8 points)

Subquestions:

1.5 (a) constrained MMPS deterministic?: not always, except if constraints yield unique solution for auxiliary variables [+ initial state deterministic]: +1
unconstrained deterministic?: yes, except possibly for initial condition: +0.5

1.5 (b) LC deterministic?: not always, except if LCP has unique solution [+ initial state deterministic]

5 (c) conditions: bounded variables +0.5
chain: MMPS $\rightarrow$ ELC $\rightarrow$ MLD $\rightarrow$ LC +1
steps:

* max,min = ELC: +1
* complementarity condition: mention this can be done via δ_i with $\sum_i \delta_i \leqslant \#\phi_i - 1$: +1
* $\delta \in \{0,1\}$ is equivalent to $0 \leqslant \delta \perp 1-\delta \geqslant 0$, introduce auxiliary variables q and r with $0 \leqslant q \perp r \geqslant 0$, split real-valued variables z in positive z^+ and negative parts z^- with $0 \leqslant z^+ \perp z^- \geqslant 0$: +1.5

Question 4 (4 points)

Subquestions:

2.5 (a) generalized gradient:
definition: mention nonsmooth+Lipschitz, sequence+limit, convex hull of gradients: +1.5
why useful: optimality conditions for non-smooth problems, in particular for optimal control/MPC for hybrid systems: +1

1.5 (b) timed automaton:
advantage: formal verification / reachability analysis possible: +0.75
disadvantage: limited modeling power: 0.75