Exam — June 2025 – Grading template
Modeling and Control of Hybrid Systems (SC42075)

As many answers are correct, below you can find the general guidelines that have been used to assign scores to the different questions and subquestions.

The maximal score for each subquestion is marked in red next to the given subquestion. This score can only be obtained if the answer is complete and fully correct, and if it does not contain any irrelevant or wrong statements.
Irrelevant or wrong answers result in a penalty of 50% or 100% of the overall (sub)score respectively.
If more than one answer is given the worst one will be graded.

Question 1 (10 points)

2 is the maximum score per subquestion for a fully correct answer (i.e., new/innovative, correct ,and explained well).
[in principle 1 for example and 1 for explanation, except in the cases listed below]

Subquestions:

2 (a) invariant set:
if only 1 mode, then it is the same set of feasible states; hence, not good example; so score is 0 then

2 (b) generalized gradient:
mention non-smooth+Lipschitz, sequence+limit, convex hull of gradients: -0.5 per missed item
explanation of subgradient instead of generalized: 0

2 (c) MLD:
give dynamics, else: -1
if no k: -0.25

2 (d) region graph:
mention purpose, i.e., the word "bisimulation" should be mentioned and explained, i.e., related to reachability, else: -0.5
also mention timed automaton, else: -0.5
example: if not region graph does not extend beyond upper bounds of x_i: -0.25
for proper explanation the example should also give the timed automaton that is considered, else: 0

2 (e) Zeno behavior:
make sure you really have infinite number of switches in finite time interval
Note that damping/friction will in general not lead to Zeno behavior!: -1
For example it should also be argued why the time span is finite, else -0.5
Newton's cradle: not new (essentially same as 3 balls of lecture notes)

Question 2 (8 points)

1.5 is the maximum score per subquestion for a fully correct answer (i.e., new/innovative, correct and explained well)

If example wrong: -1.

If the example is both new and completely correct, but there is no explanation, the score is 0.75.

Subquestions:

2 (a) does not exist if deterministic; this is due to clock dynamics $\dot{x} = 1$, the fact that guards, invariant sets, etc. involve constants (which can be scaled to be integers, and then the distance between events is either 0 (resulting in live-lock) or 1 (resulting in the absence of Zeno behavior)

2 (b) deterministic LC: create e.g., case where the condition $0 \leq v(k) = Mw(k) + q(k) \perp w(k) \geq 0$ (where $q(k)$ depends on $x(k)$ and $u(k)$), always yields a unique solution.
An example could then be to set $M = 1$ (to create a so-called P-matrix, as this yields a unique solution of the LCP). Indeed, if $q(k) > 0$ we have $v(k) > 0$ (as $w(k) \geq 0$) and thus $w(k) = 0$ because of orthogonality, while if $q(k) < 0$ we should have $w(k) > 0$ to make $v(k) \geq 0$, so then $v(k) = 0$ and $w(k) = -q(k)$. Finally, if $q(k) = 0$ then $w(k) = 0$.
If initial state is not specified: -0.5
If no k: -0.25
If no $x(k+1)$ or no $\dot{x}$: -0.5
If given system is actually linear and not fully LC: -1

2 (c) nondeterministic unconstrained MMPS:
only possible if there is nondeterminism in initial state as evolution itself is deterministic
If no k: -0.25
If no $x(k+1)$ or no $\dot{x}$: -0.5

2 (d) function without affine parts and generalized gradient equal to $[-1,4]$ in $x = 1$:
consider e.g. $f : x \mapsto \max\left(-(x-1)+(x-1)^2, 4(x-1)+(x-1)^2\right)$ i.e. example based on PWA parts but with extra nonlinear term with 0 derivative in $x = 1$
if affine parts: 0
if not Lipschitz: -1

Question 3 (5.5 points)

Subquestions:

2.5 (a) vector field points towards other region +1
convex combination +0.5
slides along switching surface +0.5
define solution in cases where otherwise no solution would exist +0.5

3×1 (b) a1) no solution
a2) 1 Filippov solution, slides to left

b1) no solution
b2) 1 Filippov solution, sticks to x_0

c1) 2 solutions, 1 to C_+ and 1 to C_-
c2) 2 solutions, 1 to C_+ and 1 to C_-