Assignment

Modeling and Control of Hybrid Systems (SC4160)
2017-2018 (Version 2)

Delft Center for Systems and Control, Delft University of Technology

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General remarks

- This assignment consists of two parts. In each part, you will see several steps that are outlined in a road map. You should follow this road map and present the results in a clear and concise report. In this report you should clearly explain and motivate all the choices you have made while solving the practical exercise.
  In your report you should also add an evaluation and conclusions section of 1–1.5 pages, in which you briefly outline the main insights you have obtained while making this assignment. You should also add the Matlab files you have written in an appendix to the report.

- The deliverable of this assignment is a written report of max. 50 pages (including cover pages and appendices) about the assignment that also contains the *appendix* with your m files. Make sure to mention your group number and student number on the cover page. The report is to be emailed as a single **pdf** file to
  `f.alavi@tudelft.nl`
  The *hard* deadline for emailing the report is Monday, June 26, 2017 at 17:00 p.m.

- You will be graded on the contents and the presentation of the report, on the originality[1] of your answers, and on the correctness, *efficiency*, and readability of the Matlab files (i.e., do not forget to include explanatory comments in your Matlab files).

- We recommend you to keep the computations symbolic or analytic as long as possible and not to hardcode any of the parameters in your Matlab programs (instead, write one separate Matlab function or script that defines the parameters) so that you can easily take other parameter values, longer control horizons, other reference signals, etc. into account.
  Furthermore, since each step of this assignment depends on the preceding ones, we recommend that after Steps 2.2, 2.3, and 2.6 of the roadmap, you check your intermediate results with the teaching assistant (during the office hours).

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[1]I.e., a correct answer that differs from the answers given by the other groups will be graded higher than a correct answer that is an almost literal copy of the answer of another group.

1   Part 1: Hybrid system example

Select a system in your own field of research or interests that can be considered as a hybrid system (and that is not yet discussed in the lecture notes).

Step 1.1: Give a description of the system.

Step 1.2: Describe the system as a hybrid automaton (if needed, extend the definition of the (autonomous) hybrid automaton given in the lecture notes with inputs).

2   Part 2: Energy management of microgrids

Set-up

We consider a microgrid that is connected to the main power grid. Figure 1 presents a schematic diagram of the considered microgrid, where diesel generators (DG) units and their fuel tanks, batteries, and an energy management system (EMS) are the devices considered in this assignment. Assume that the EMS has an accurate prediction of the load in the microgrid and is able to communicate with diesel generators and batteries without any delay. We assume that the microgrid always satisfies the power balance condition, i.e., the amount of power used by the load and the charging batteries is equal to the power provided by the power grid, diesel generators, and the discharging batteries.
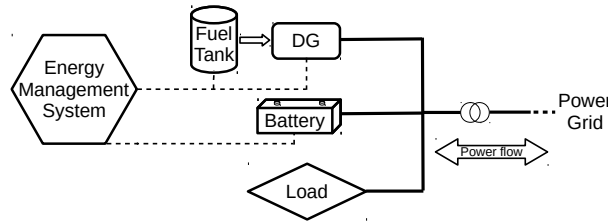

Figure 1: Schematic structure of the considered microgrid.

A price of electrical energy per kWh is defined for the exchanged energy between the microgrid and the power grid. As a result, whenever the electrical power is imported from the power grid, there is an additional operational cost of the microgrid due to the price of importing energy. Conversely, exporting energy to the power grid will result in a benefit for the microgrid. The energy management system is able to control the operation of the diesel generators and batteries and the aim of this road map is to design a hybrid model predictive control algorithm for the energy management system such that the operational cost of the system is minimized.

Tasks & Road map

Step 2.1: *Modeling a battery*

We consider a simple model for batteries, where a battery is considered as a storage of electrical energy with a different charging and discharging efficiency.

By considering the stored energy in the battery as the system state and using the variables of Table 1, construct a discrete-time piecewise affine (PWA) model of a battery. Note: The exchanged power of the battery denotes the charging/discharging power of the battery from

Table 1: Parameters related to a battery

Variables related to a battery	Name	Range	Unit
Stored energy	x_b	$[0, \bar{x}_b]$	kWh
Exchanged power	u_b	$[\underline{u}_b, \bar{u}_b]$	kW
Operational mode (charge/discharge)	s_b	$\{0, 1\}$	–
Charging efficiency	η_c	*CONSTANT*	–
Discharging efficiency	η_d	*CONSTANT*	–

the grid side and it can have a negative or positive sign. We consider a positive sign for the exchanged power whenever the battery is discharged, i.e., $s_b(k) = 0$. The sampling time of our system is expressed by T_s.

Step 2.2: Using the result of the previous step and the following constraints:

$$0 \leq x_b \leq \bar{x}_b$$
$$\underline{u}_b \leq u_b \leq \bar{u}_b$$
$$\text{if } s_b = 1 \Rightarrow u_b \leq 0$$
$$\text{if } s_b = 0 \Rightarrow u_b > 0,$$

construct a mixed logical dynamical (MLD) model of a battery. Note: Determine the matrices $A, B_1, \ldots, B_4, E_1, \ldots, E4, g_5$ in the following general MLD formulation:

$$x(k+1) = Ax(k) + B_1 u(x) + B_2 \delta(k) + B_3 z(k) + B_4$$
$$E_1 x(k) + E_2 u(k) + E_3 \delta(k) + E_4 z(k) \leq g_5.$$

Also indicate the definition of $x(k)$, $u(k)$, $\delta(k)$, and $z(k)$.

Step 2.3: *Modeling a diesel generator*

Suppose that the fuel consumption of our specific diesel generators can be determined as follows:

$$f(k) = f(u_d) = C_{base} + mu_d^2(k),$$

where $f(k)$ is the consumed fuel of the diesel generator at time step k in [kg/h], C_{base} is a constant number that represents a constant rate of fuel consumption whenever the diesel generator is on, and the value of $u_d(k)$ represents the output power of the diesel generator at time step k. The coefficient m is a constant number that is determined based on the specifications of the diesel generator.

Construct a PWA approximation $\hat{f} : [0, \bar{u}_d] \to \mathbb{R}$ with 2 regions of the fuel consumption curve f as follows. We want a perfect match for $u_d = 0$ and $u_d = \bar{u}_d$. This implies that we still have two degrees of freedom, i.e., the coordinates (a, b) of the middle edge point of the PWA curve (see Figure 2). Now determine a and b such that the *squared* area between f and $\hat{f}$ (i.e., the squared area corresponding the hashed region in Figure 2) is minimized, or equivalently, such that:

$$\int_0^{\bar{u}_d} \left(f(u_d) - \hat{f}(u_d) \right)^2 du_d$$

Table 2: Parameters related to a diesel generator

Variables related to a diesel generator	Name	Range	Unit
Remaining fuel	x_d	$[\underline{x}_d, \bar{x}_d]$	kg
generated power	u_d	$[0, \bar{u}_d]$	kW
Operational mode (on/off)	s_d	$\{0,1\}$	–
Filling rate of fuel tank	R_f	*CONSTANT*	kg/h

is minimized.

Note: Use analytic computations (by hand or by using Mathematica or Maple, etc.) to determine the optimal a and b (and *not* numerical computations/optimization).

From now on, we consider the PWA function $\hat{f}$ as the fuel consumption function of the diesel generators.
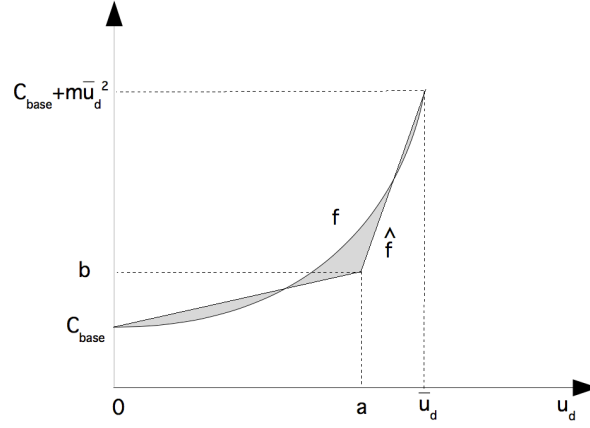

Figure 2: Fuel consumption of the considered diesel generator, f, and its PWA approximation, $\hat{f}$.

Step 2.4: Consider a switching signal, s_d, that indicates the on/off operational mode of the generator. In addition, suppose that the fuel tank is constantly refilled with R_f [kg] of fuel per time step. By considering the fuel level in the tank, x_d, as the system state and using the result of Step 2.3, construct a discrete-time PWA model for a diesel generator.

Step 2.5: Using the results of the previous step and the following constraints:

$$\underline{x}_d \leq x_d \leq \bar{x}_d$$
$$0 \leq u_d \leq \bar{u}_d$$
$$\text{if } s_d(k) = 0 \Rightarrow \text{there is no power generation at time step } k,$$

construct an MLD model of a diesel generator. Report your final result in the general MLD formulation as explained in Step 2.2.

Step 2.6: *System model*

Now consider 2 diesel generators and 2 batteries with the specifications described in Table 3 and Table 4, respectively. Write a Matlab code to create the MLD model of a system that contains all the devices. From now on, assume that $T_s = 0.25$ [h].

Table 3: Specifications of the diesel generators

DG number	$\underline{x}_d$ [kg]	$\bar{x}_d$ [kg]	$\bar{u}_d$ [kW]	R_f [kg/h]	$m\left[\frac{kg}{h.kW^2}\right]$	C_{base} [kg/h]
1	10	100	15	0.4	0.44	2
2	20	120	15	0.4	0.52	2.4

Table 4: Specification of the batteries

Battery number	η_c	η_d	$\bar{x}_b$ [kg]	$\underline{u}_b$ [kW]	$\bar{u}_b$ [kW]
1	0.9	0.8	48	-3	2
2	0.95	0.77	64	-4	3

Step 2.7: *Defining the cost function*

The operational cost of the microgrid at time step k is defined as:

$$J(k) = \sum_{j=1}^{N_p} \left(\sum_{i=1}^{N_b} W_{b,i} |\Delta s_{b,i}(k+j)| + \sum_{i=1}^{N_g} W_{d,i} |\Delta s_{d,i}(k+j)| \right) \tag{1}$$

$$- W_{fuel} \sum_{i=1}^{N_g} \left(x_{d,i}(k+N_p) - x_{d,i}(k) \right) - W_e \sum_{i=1}^{N_b} \left(x_{b,i}(k+N_p) - x_{b,i}(k) \right)$$

$$+ \sum_{j=1}^{N_p} P_{imp}(k+j) C_e(k+j), \tag{2}$$

where $P_{imp}(k)$ is the imported power to the microgrid at time step k and $C_e(k)$ is the price (benefit) of importing electricity to (exporting electricity from) the microgrid. In the power balance condition, we have:

$$P_{imp}(k+j) = P_{load}(k+j) - \sum_{i=1}^{N_g} u_{d,i}(k+j) - \sum_{i=1}^{N_b} u_{b,i}(k+j).$$

The parameters N_g and N_b determine the number of diesel generators and the number of batteries in the microgrid, respectively. The weights in the cost function are given in Table 5. Note that for a signal $s(.)$, we have $\Delta s(k) = s(k) - s(k-1)$.

Write a Matlab code that computes the optimal MPC input sequence for a given sample step k for values of N_p and N_c (i.e. the control horizon). Assume that the values of $\tilde{C}_e$ and $\tilde{P}_{load}$ are known, where $\tilde{C}_e = \begin{bmatrix} C_e(k) & \dots & C_e(k+N_p-1) \end{bmatrix}^T$ and $\tilde{P}_{load}(k) = \begin{bmatrix} P_{load}(k) & \dots & P_{load}(k+N_p-1) \end{bmatrix}^T$.

Hints & notes:

Table 5: Value of the weight factors in the cost function

Weight factor	Value
$W_{b,i}$	$2+i$
$W_{d,i}$	$10+i$
W_{fuel}	4
W_e	0.4

- The MPC optimization problem at time step *k* can be transformed into a mixed-integer linear programming problem (MILP).

- In order to solve an MILP problem, you can use one of the following options:

  – Use the `glpk` function of the MPT toolbox version 3 (for MATLAB R2011a and newer) (see `http://people.ee.ethz.ch/~mpt/3/`)

  – Use the Matlab interface of the Gurobi optimizer. See: `http://www.gurobi.com/products/gurobi-optimizer`

  – Use the `mpt_solveMILP` command of the previous version of the MPT toolbox – version 2.6.3 (see
  `http://people.ee.ethz.ch/~mpt/2/downloads/`).
  To install and activate this toolbox, see the instructions at
  `http://control.ee.ethz.ch/~mpt/docs/install.php`
  For step 2 of the installation procedure we recommend you to use the
  `addpath(genpath(...))`
  approach. Note that this command should be typed every time you (re)start MATLAB and want to use the Multi-Parametric Toolbox. The command to solve MILP problems is `mpt_solveMILP`
  Note that there is an on-line reference guide for the Multi-Parametric Toolbox at
  `http://control.ee.ethz.ch/~mpt/docs/`
  The main page for `mpt_solveMILP` is
  `http://people.ee.ethz.ch/~mpt/2/docs/refguide/mpt/solvers/mpt_solveMILP.html`

  – Use the `cplex` command of the TOMLAB cplex toolbox (note that you need a license, see `http://tomopt.com/tomlab/`). With this command you can solve MILP problems (see also the `milp_solve_tomlab_cplex` script on the course website).

- As we want you to get some insight into the hybrid MPC method and the relation with integer programming, you are *not* allowed to solve the entire exercise using the `mpt_control` or related commands of the MPT toolbox or the Hybrid Toolbox (see `http://cse.lab.imtlucca.it/~bemporad/hybrid/toolbox/`). However, feel free to compare the results obtained with your own programs to those obtained with the MPT toolbox or the Hybrid Toolbox, and to discuss the differences, if any.

Step 2.8: Write a Matlab code to simulate the closed-loop behavior of the system (i.e. apply the receding horizon approach in which at each step the optimal MPC control input is recomputed

and applied to the system). Assume that:

$$N_{\mathrm{p}} = N_{\mathrm{c}} = 24$$
$$x_{\mathrm{d},i}(0) = 50 \ \forall i$$
$$x_{\mathrm{b},i}(0) = 10 \ \forall i$$
$$s_{\mathrm{d},i}(-1) = s_{\mathrm{b},i}(-1) = 0 \ \forall i$$
$$C_{\mathrm{e}}(k) = 0.005 + 0.005 \sin\left(\frac{\pi T_{\mathrm{s}} k}{12}\right)$$
$$P_{\mathrm{load}}(k) = \begin{cases} 0 & \text{if } k \le 20 \\ 30 + 2k & \text{if } 21 \le k \le 50 \\ 45 & \text{if } k \ge 51 \end{cases}$$

Report your results.