Short solutions for Exam of November 2015
"Optimization in Systems and Control" (SC4091)

This document concisely lists the solutions for the exam of November 2015. Note that other solutions might also be correct to some degree, and that

you should extensively motivate your answers in the actual exam!

(cf. the worked solutions for Sample Exams 1 and 2 and for the exams of October 2013 and October 2014).

Short answers for Question 1

P1. M10: penalty function + steepest descent (convex problem, (sub)gradient easily computable (so actually use subgradient instead of gradient))

P2. multi-start M3: gradient projection method, or multi-start M10: penalty function + steepest descent (nonconvex due to x_1x_3 and x_4x_5, gradient easily computable)

P3. M1 (linear programming)

P4. M9: penalty function + Powell, or multi-start M12: genetic algorithm (convex, (sub)gradient is very time-consuming to compute)

P5. multi-start M7: Lagrange method + Davidon-Fletcher-Powell quasi-Newton algorithm (nonconvex, constraint cannot be used to eliminate variable, gradient easily computable)

P6. multi-start M10: penalty function + steepest descent (nonconvex due to second constraint, gradient easily computable)

P7. M2 (convex quadratic optimization problem)

P8. multi-start M9: penalty function + Powell, or multi-start M12: Genetic algorithm, or multi-start M10: penalty function + steepest descent (using subgradient and using convex penalty function) (nonconvex due to second constraint, in principle $|\cdot|$ is non-smooth so gradient cannot be used)

P9. multi-start M6: line search method with Broyden-Fletcher-Goldfarb-Shanno direction (constraint can be used to eliminate x_1, nonconvex, gradient easily computable)

P10. M10: penalty function + steepest descent (convex problem, gradient easily computable)

The answers below are short answers only;

you should extensively motivate your answers in the actual exam!

(cf. the worked solutions for Sample Exams 1 and 2 and for the exams of October 2013 and October 2014).

Short answers for Question 2

1a. See lecture notes (give definition, mention multi-objective, non-unique & non-interior solutions, add pictures + explain each of these elements in your own words)

1b. A, B, D, E, G can *possibly* be obtained via heuristic
A, E, G can be obtained via weighted sum

1c. For $\varepsilon = 1$: no solution
for $\varepsilon = 3$: (3,5)
for $\varepsilon = 7$: (6,1), e.g. G

2. Define $\tilde{S} = S \cap \{x | g(x) \geqslant 0, h(x) = 0\}$
f should be convex on $\tilde{S}$
$\tilde{S}$ should be convex (i.e., $-g$ convex on $\tilde{S}$ and h affine on $\tilde{S}$)

Short answers for Question 3

a. Fill out the equation for $x(k+1)$ and use the fact that x and d are independent, so $E[A_{\text{cl}} x(k) d^T(k) B_{\text{cl}}^T] = 0$

b. Similar but use the equation for $y(k)$ and use the fact that x and d are independent. This yields: $E[y(k)y^T(k)] = C_{\text{cl}} X C_{\text{cl}}^T + D_{\text{cl}} D_{\text{cl}}^T \sigma_d^2$

c. Replace C_{cl} and D_{cl} by affine expressions in θ. Note that X does not depend on θ. Finally, we get a quadratic-affine expression with a positive definite quadratic term.

d. In this case X would depend on θ too.