Short solutions for Exam of November 2017 "Optimization in Systems and Control" (SC42055)

This document concisely lists the solutions for the exam of November 2017. Note that other solutions might also be correct to some degree, and that **you should extensively motivate your answers in the actual exam!** (cf. the worked solutions for Sample Exams 1 and 2 and for the exams of October 2013 and October 2014).

Short answers for Question 1

P1. NCC: Nonconvex constrained (nonconvex due to term x_1x_3 in the cost function)
multi-start M8 (penalty + steepest descent) or – if the original problem is split into 2 problems with convex linear constraints (due to the nonconvexity of the 2nd constraint) – multi-start M3 (gradient projection)

P2. NCC: Nonconvex constrained (nonconvex due to integer variables)
multi-run M11 (simulated annealing)

P3. $2 \times$ CP: Convex (the 3rd constraint is actually nonconvex, but it can be split in two sets of convex constraints[1], so *two* convex problems result)
M10 (interior point)

P4. CP: Convex
M10 (interior point)

P5. CP: Convex
M10 (interior point)

P6. $10 \times$ LP: Linear (the 3rd constraint is actually nonconvex, but it can be written as a union of 10 affine constraints[2]; so *10* linear programming problems result)
M1 (simplex)

P7. NCC: Nonconvex constrained (nonconvex due to the equality constraint, which is not affine)
multi-start M6 (Lagrange + DFP)

P8. LP: Linear
M1 (simplex)

P9. NCU: Nonconvex unconstrained (nonconvex due to nonconvex dependence of α_i on θ)
multi-start M11: Simulated annealing (gradient and Hessian not analytically computable, and objective function requires time-consuming numerical evaluation)

[1]Note that $\max(L_1, L_2) \geq 3$ with L_1 and L_2 linear expressions, is equivalent to $L_1 \geq 3$ **or** $L_2 \geq 3$.

[2]In particular, $\|x\|_\infty \geqslant 2$ is equivalent to $\max_{i\in\{1,2,4,5,\}}(|x_i|) \geq 2$ or $\max_{i\in\{1,2,4,5,\}}(x_i, -x_i) \geq 2$ or $x_1 \geq 2$ **or** $-x_1 \geq 2$ **or** $x_2 \geq 2$ **or** $-x_2 \geq 2$ **or** $\ldots$ $x_5 \geq 2$ **or** $-x_5 \geq 2$, i.e., the union of $2 \cdot 5 = 10$ affine constraints.

P10. QP: Quadratic
M2 (modified simplex)

The answers below are short answers only; **you should extensively motivate your answers in the actual exam!** (cf. the worked solutions for Sample Exams 1 and 2 and for the exams of October 2013 and October 2014).

Short answers for Question 2

1. See lecture notes (explain/stress linear inequality constraints; orthogonal projection of negative gradient on active set (if any); line minimization; and add an illustrative drawing)

2a. The objective function is convex since it can be written as a sum of squares and an affine term. In addition, the constraints are affine and thus convex. So the overall problem is convex.

2b. The negative gradient in $(1,0)$ is $[-2\ 5]^{\mathrm{T}}$, which points towards the feasible set (so no projection is needed). The line minimization yields $(70/128, 145/128)$. This point is strictly feasible so for the next search no projection is needed either. The negative gradient is $[5/128\ 2/128]^{\mathrm{T}}$, which yields the point $(3365/5888, 6728/5888)$.
Note: in this particular case we are very mild w.r.t. computations errors in computing the 2nd point.

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Short answers for Question 3

a. We have

$$M = \frac{1}{1+PK_1+PQK_2}\begin{bmatrix} K_2 & -(K_1+K_2Q) \\ PQK_2 & Q \end{bmatrix}$$

b. After substitution we obtain

$$M = \begin{bmatrix} S_2 & -(S_1+S_2Q) \\ PQS_2 & Q(1-PS_1-PQS_2) \end{bmatrix}$$

Since the entries of M are sums and products of stable transfer functions and since the set of stable transfer functions is closed under addition and multiplication, we can conclude that all entries of M are stable transfer functions. So the overall closed-loop system with inputs r and d and outputs u and y is stable.