Exam — November 2023 – Grading template
Optimization for Systems and Control (SC42056)

Important: Please recall the following instructions from the exam procedure:

- Note that – just as in previous years – **correct results without proper and correct motivation will not receive any marks.**

For an example on how proper and correct motivations look like, please consult the worked solutions for Sample Exams 1 and 2 and for the exams of October 2013 and October 2014.

Additional scoring guidelines

S0: correct result without proper and correct motivation: 0
likewise: if in Question 1 the answer for (c) and/or (d) is formally correct, but an error is made in (a) or (b) that affects the result for (c) and/or (d): 0 for (c) and/or (d)

S1: partially incomplete motivation for (non)convexity or simplification: -50%

S2a: small computation error that *does* affect result: -0.5

S2b: small computation error that does *not* affect result: -0.25

S3: missing, wrong, or not properly motivated $N\times$ in Question 1(c): -2 for (c) and (d) together

S4: multi-start listed when it is not needed: -0.5

S5: ∇f as row vector: -0.5

S6: redundant function in stopping criterion: 0

S7: introduction of redundant variables or constraints that are not needed at all: -1

S8: introduction of additional wrong constraints and/or wrong classification of extra/unsimplified constraint: -1

S10: as indicated in instructions: if 2 or more solutions are given, the worst one is assumed to have been selected

CE: Even if the answers to (a)-(d) are wrong, you can sometimes still score marks for (e) if and only if (a)-(d) are internally consistent and if they all result in the answer given in (e) and if (e) is **100 %** correct and complete.

On the next pages **concise** answers are given with scores marked in red. To earn the indicated score the corresponding answer has to be given completely, including the information inside the brackets; else the score is 0.

QUESTION 1 $(8 \times 9 = 72$ points)

- **P1**

1 (a) First we turn the maximization problem into a minimization problem:

$$\min_{x\in\mathbb{Z}^3} f_s(x) := -\exp(x_1 + 5x_2 + x_3 - 8) - \sinh(x_1 + 5x_2 + x_3 - 8)$$

Since $-\exp(\cdot)$ is an decreasing function of its argument, and the same holds for $-\sinh(\cdot)$ and since they both have the same argument $x_1 + 5x_2 + x_3 - 8$ we can maximize the argument instead. Turning this again into a minimization problem and dropping the constant -8 we obtain: 1

$$\min_{x\in\mathbb{Z}^3} -x_1 - 5x_2 - x_3$$

For simplifications of the constraint, see (b)

4 (b) The simplified objective function f_s is linear: 1.5
Constraint (1) is equivalent to $|x_1| + |x_2| + |x_3| \leqslant 12$ can be written as the intersection of $2^3 = 8$ affine constraints: $\pm x_1 \pm x_2 \pm x_3 \leqslant$: 1
Since $(\cdot)^3$ is an increasing function, constraint (2) can be rewritten as $5 + 3x_1 + x_2 - x_3 \leqslant 3\sqrt[3]{3}$, which is an affine constraint: 0.5
Constraint (3) can be rewritten as the **union** of 3 affine constraints: $3 + 2x_1 + 3x_2 \geqslant 5$ **or** $4x_2 + 3x_3 \geqslant 5$ **or** $8 - x_1 + x_2 - x_3 \geqslant 5$: 1

1 (c) $3 \times$ MILP: mixed-integer linear programming problem

2.5 (d) M12: Branch-and-bound method for mixed-integer linear programming

0.5 (e) Optimum is found once entire tree is explored

- **P2**

1 (a) As the function $\sqrt[5]{\cdot}$ is a nondecreasing function, we can minimize the argument instead. Note that the $(\cdot)^2$ function has to stay. Furthermore, since $(\cdot)^2 = |\cdot|^2$ and since $(\cdot)^2$ is increasing for nonnegative arguments, we can minimize $|\cdot|$: 0.5
The constraint can be used to express x_4 as a function of the other variables: 0.5

$$x_4 = \frac{1}{2}\left(123 - 3^{(\dots)} - 3x_1 - x_2^2 - \log(\dots)\right)$$

Next, x_4 can be eliminated from the objection function, resulting in an unconstrained problem of the form $\min_{x_1,x_2,x_3} f_{\text{elim}}$ with

$$f_{\text{elim}} = \left|x_1 + 6x_2 + 8x_3 - 553.5 + 4.5 \cdot 3^{(\dots)} + 13.5x_1 + 4.5x_2^2 + 4.5\log(\dots)\right|$$

4 (b) The function f_{elim} is nonlinear and nonconvex: indeed, as the argument of the absolute-value function is nonlinear and can be negative, we cannot conclude that f_{elim} is convex.

1 (c) NCU: nonconvex unconstrained optimization problem

2.5 (d) The gradient and the Hessian of the objective function can be computed analytically. As there is no pure second-order algorithm available, the best choice is
multi-start : 0.5 + M5: BFGS quasi-Newton algorithm: 1

0.5 (e) $\|\nabla f_{\text{elim}}(x_k)\| \leqslant \varepsilon$

- **P3**

1 (a) As $\arctan(\cdot)$ is an increasing function, we can maximize its argument; next, the constant 1 can be dropped: 0.25
Since $(\cdot)^4$ is an increasing function for **nonnegative** arguments (note that the term $-8x_3x_4$ can be absorbed, see below, so the argument is positive definite and thus nonnegative), we can minimize the argument. So we finally get $\min_x f_\mathrm{s}(x) := \min_{x\in\mathbb{R}^4} 2x_1^2 + 4x_2^2 + x_3^2 + (x_3 - 4x_4)^2$: 0.75
For simplifications of the constraints, see (b)

4 (b) The objective function f_s can be written as a nonnegative sum of squares (see (a)), so it is a convex quadratic function: 2
Constraint (1) can be rewritten as the **union** of 2 affine constraints:
$-12 \leqslant x_1 + 2x_2 + 3x_3 + 4x_4 \leqslant -\sqrt{2}$ **<u>or</u>** $\sqrt{2} \leqslant x_1 + 2x_2 + 3x_3 + 4x_4 \leqslant 12$: 1
Constraint (2) can be rewritten as $3^{x_1} + |3x_2 - 2x_3 + 5x_4|^3 \leqslant 12$. Since $3^{(\cdot)}$ is a convex function, the first term of this constraint is convex. Since $(\cdot)^3$ is convex and increasing for nonnegative arguments and since its argument is convex and nonnegative, $|3x_2 - 2x_3 + 5x_4|^3$ is convex. Since the sum of convex functions is convex, constraint (2) is convex: 1

1 (c) $2 \times$ CP: convex optimization problem

2.5 (d) M3: Ellipsoid algorithm
if multi-start is checked: -0.5

0.5 (e) $|f_\mathrm{s}(x_k) - f_\mathrm{s}(x^*)| \leqslant \varepsilon_f$ **and/or** $\|x_k - x^*\|_2 \leqslant \varepsilon_x$
and $g_\mathrm{s}(x_k) \leqslant \varepsilon_g$
if the condition $g_\mathrm{s}(x_k) \leqslant \varepsilon_g$ is missing: 0

- **P4**

1 (a) Since $\cosh(\cdot)$ is increasing for nonnegative arguments, we can minimize the argument instead. The constant 2 can be dropped: 0.5
So the objective function to be minimized is $7|x_1|+4|x_2|+|x_3|+2|x_4$, which is a nonnegative sum of absolute values. By introducing dummy variables $\alpha_i \geqslant |x_i|$ (which can be rewritten as affine constraints $\alpha_i \geqslant x_i$, $\alpha_i \geqslant -x_i$), we can instead consider $\min_{\alpha,x} 7\alpha_1 + 4\alpha_2 + \alpha_3 + 2\alpha_4$: 0.5
For simplifications of the constraints, see (b)

4 (b) The simplified objective function is linear: 1
Since an even power is U-shaped, constraint (1) can be rewritten is $-15 \leqslant 6+3x_1+x_2-x_3-x_4 \leqslant 15$, which is an affine constraint: 0.5
Constraint (2) can be rewritten as the **intersection** of 3 affine constraints: $3+2x_1+3x_2-x_4 \geqslant 9$ and $4x_2+3x_4 \geqslant 9$ and $2x_1-8 \geqslant 9$: 0.5
Constraint (3) can be rewritten as[1] $\max(|x_1|,|x_2|,|x_3|,|x_4|) \geqslant \pi$ or as a **union** of $2 \cdot 4 = 8$ affine constraints $x_1 \geqslant \pi$ **or** $x_1 \leqslant -\pi$ **or** $\ldots x_4 \geqslant \pi$ **or** $x_4 \leqslant -\pi$: 2

1 (c) $8\times$ LP: mixed-integer linear programming problem

2.5 (d) M1: Simplex algorithm for linear programming

0.5 (e) The simplex algorithm will always find a global optimum in a finite number of iterations

[1] Note that we cannot substitute this for $\max(\alpha_1,\alpha_2,\alpha_3,\alpha_4) \geqslant \pi$, as this could cause α_i^* to be larger than $|x_i^*|$, e.g., if we replace $\min_{x\in Rset} |x|$ s.t. $|x| \geqslant 1$ by $\min_{x,\alpha\in\mathbb{R}} \alpha$ s.t. $\alpha \geqslant x$, $\alpha \geqslant -x$, $\alpha \geqslant 1$ then $\alpha^* = 1$ and $x^* \in [-1,1]$ so we do not always have $\alpha^* = |x^*|$ anymore.

- **P5**

  1 (a) The objective function cannot be simplified.
  For simplifications of the constraints, see (b)

  4 (b) Due to presence of cos or sin (or a similar correct argument) in the argument of the integral, the objective function is a nonconvex function: 2.5

  The constraints can be rewritten as $-2 \leqslant v_i \leqslant 2$ for $i = 1, 2, \ldots, 10$, which are affine constraints: 1.5

  1 (c) NCC: nonconvex constrained optimization problem

  2.5 (d) The integral that appears in the objective function cannot be computed analytically, so numerical computation is required, which will be time-consuming. Therefore, and also due to the high number of variables, a gradient-free method is recommended. So the best choice is[2]:
  multi-run : 0.5 + M11: Simulated annealing: 2

  0.5 (e) Temperature becomes less than some threshold ($T \leqslant T_{\text{final}}$)

[2]Note that in view of the number of variables (10), M10: barrier + Nelder-Mead is not an acceptable choice here.

- **P6**

1 (a) The objective function cannot be simplified
For simplifications of the constraints, see (b)

4 (b) The terms $2x_1x_3$ and $2x_4x_5$ cannot be absorbed, so f is a nonconvex function: 1
Since $(\cdot)^4$ is a U-shaped function, constraint (1) can be rewritten as $-2 \leqslant x_1 + 2x_2 + 8x_3 - 9x_4 + 8x_5 \leqslant 2$, which is an affine constraint: 0.5
Since $2^{(\cdot)}$ is an increasing function, constraint (2) can be rewritten as $-x_1 + 3x_2 - x_3 + 6x_4 + x_5 \geqslant 2$, which is an affine constraint: 0.5
Constraint (3) can be rewritten as $|x_1| + |x_2| + |x_3| + |x_4| + |x_5| + \max(|x_1|, |x_2|, |x_3|, |x_4|, |x_5|) \leqslant 9$, which is equivalent to the set (intersection!) of the 5 following constraints:

$$\begin{aligned}
2|x_1| + |x_2| + |x_3| + |x_4| + |x_5| &\leqslant 9 \\
|x_1| + 2|x_2| + |x_3| + |x_4| + |x_5| &\leqslant 9 \\
|x_1| + |x_2| + 2|x_3| + |x_4| + |x_5| &\leqslant 9 \\
|x_1| + |x_2| + |x_3| + 2|x_4| + |x_5| &\leqslant 9 \\
|x_1| + |x_2| + |x_3| + |x_4| + 2|x_5| &\leqslant 9
\end{aligned}$$

which results in $5 \cdot 2^5 = 5 \cdot 32 = 160$ affine constraints:

$$\begin{aligned}
\pm 2x_1 \pm x_2 \pm x_3 \pm x_4 \pm x_5 &\leqslant 9 \\
\pm x_1 \pm 2x_2 \pm x_3 \pm x_4 \pm x_5 &\leqslant 9 \\
\pm x_1 \pm x_2 \pm 2x_3 \pm x_4 \pm x_5 &\leqslant 9 \\
\pm x_1 \pm x_2 \pm x_3 \pm 2x_4 \pm x_5 &\leqslant 9 \\
\pm x_1 \pm x_2 \pm x_3 \pm x_4 \pm 2x_5 &\leqslant 9 \quad : 2
\end{aligned}$$

1 (c) NCC: nonconvex constrained optimization problem

2.5 (d) multi-start : 0.5 + M2: Gradient projection: 2
(as this uses 1st-order information and as gradient is easy to compute, while no methods that use 2nd-order information like SQP are available)
multi-start : 0.5 + M9: penalty + steepest descent would also be accepted: 2

0.5 (e) KKT conditions with ε (list them!) **or**
$\|\nabla f_{\text{penalty+}}(x_k)\| \leqslant \varepsilon$ where $f_{\text{penalty+}}$ is the sum of the simplified objective function after elimination and the penalty function

- **P7**

1 (a) We turn the maximization problem into a minimization problem:

$$\min_x f_s(x) := -3\log(x_1+2x_2+6x_3+x_4+1) + (|x_1+x_2-4x_3-x_4+3|-1)^4$$

For simplifications of the constraints, see (b)

4 (b) Although the first term f_s is convex, the second term is not as the argument of $(\cdot)^4$ can become negative due to the term -1 (this occurs, e.g., for $x_1 = x_2 = x_4 = 0$ and $x_3 = 3/4$). In fact, the function $(|\cdot|-1)^4$ looks like a smooth W-shaped function (e.g., the function equals 1 if the argument is 0, 0 if the argument equals ± 1, and it is positive again if the argument is less than -1 or larger than 1). So the simplified objective function is not convex: 2

Constraint (1) con be written as the intersection of two affine constraints: $x_1+2x_2 \geqslant 1$ and $6x_3+x_4 \geqslant 1$: 0.5
Constraint (2) contains a positive sum of even powers (which are convex) and is thus convex: 0.5
Constraint (3) contains 2 terms. As $(\cdot)^2$ is convex and increasing for positive arguments (note that $2^{(\cdot)}$ and $3^{(\cdot)}$ both yield positive values), and as both $2^{(\cdot)}$ and $3^{(\cdot)}$ are convex, the first term of (3) is convex. Since the absolute value function is convex in its argument and as that argument is affine here, the second term of (3) is also convex. Hence, constraint (3) is convex: 1

1 (c) NCC: nonconvex constrained optimization problem

2.5 (d) multi-start : 0.5 + M9: Penalty function approach + steepest descent method: 2
(as this uses 1st-order information and as gradient is easy to compute[3], while no methods that use 2nd-order information like SQP are available)

0.5 (e) KKT conditions with ε (list them!) **or**
$\|\nabla f_{\text{penalty+}}(x_k)\| \leqslant \varepsilon$ where $f_{\text{penalty+}}$ is the sum of the simplified objective function after elimination and the penalty function

Alternative solution in view of the full analysis of the constraints:

1 (a) First we turn the maximization problem into a minimization problem:

$$\min_x -3\log(x_1+2x_2+6x_3+x_4+1) + (|x_1+x_2-4x_3-x_4+3|-1)^4$$

Next we consider two cases, depending on whether or not $x_1+x_2-4x_3-x_4+3 \geqslant 0$. Then we get 2 problems:

$$\begin{aligned}\text{P7a:} \min_x \; & -3\log(x_1+2x_2+6x_3+x_4+1) + (x_1+x_2-4x_3-x_4+3-1)^4 \\ \text{s.t. } & \text{(1)–(3) and } x_1+x_2-4x_3-x_4+3 \geqslant 0\end{aligned}$$

and

$$\begin{aligned}\text{P7b:} \min_x \; & -3\log(x_1+2x_2+6x_3+x_4+1) + (-x_1-x_2+4x_3+x_4-3-1)^4 \\ \text{s.t. } & \text{(1)–(3) and } x_1+x_2-4x_3-x_4+3 \leqslant 0\end{aligned}$$

For simplifications of the constraints, see (b)

[3]Note that although $|\cdot|$ is a non-smooth V-shaped function, the 4th power makes the function smooth; as a result, the gradient is defined for the entire domain of the function.

4 (b) As $-\log(\cdot)$ and $(\cdot)^4$ are convex functions, as they have affine arguments in P7a and P7b, and as the (positive) sum of two convex functions is convex, the objective functions of P7a and P7b are convex: 2

Constraint (1) can be written as the intersection of two affine constraints: $x_1 + 2x_2 \geqslant 1$ and $6x_3 + x_4 \geqslant 1$: 0.5
Constraint (2) contains a positive sum of even powers (which are convex) and is thus convex: 0.5
Constraint (3) contains 2 terms. As $(\cdot)^2$ is convex and increasing for positive arguments (note that $2^{(\cdot)}$ and $3^{(\cdot)}$ both yield positive values), and as both $2^{(\cdot)}$ and $3^{(\cdot)}$ are convex, the first term of (3) is convex. Since the absolute value function is convex in its argument and as that argument is affine here, the second term of (3) is also convex. Hence, constraint (3) is convex: 1

1 (c) $2\times$ CP: convex optimization problem

2.5 (d) M3: Ellipsoid algorithm
if multi-start is checked: -0.5

0.5 (e) $|f_s(x_k) - f_s(x^*)| \leqslant \varepsilon_f$ **and/or** $\|x_k - x^*\|_2 \leqslant \varepsilon_x$
and $g_s(x_k) \leqslant \varepsilon_g$
where f_s represents the objective function of P7a or P7b, and g_s represents the constraint function of P7a or P7b after all the constraints have been simplified as explained in (b)
if the condition $g_s(x_k) \leqslant \varepsilon_g$ is missing: 0

- **P8**

1 (a) The maximization problem is first transformed into a minimization problem. The constant -1 can be dropped. Moreover, since $\exp(\cdot)$ is increasing, we can minimize the argument instead. In addition, the constant -8 can be dropped, which leads to: 1

$$\min_x f_s(x) = 5x_1^2 + 2x_2^2 - 8x_2x_3 + 2x_3^2 - 5x_1 - 6x_2 + 2x_3$$

For simplifications of the constraints, see (b)

4 (b) The term $-8x_2x_3$ cannot be absorbed, so f_s is a nonconvex (quadratic) function: 1.25

As $(\cdot)^2$ is a U-shaped function, constraint (1) can be rewritten as $-10^3 \leqslant \cosh(x_1 + x_2 - x_3) \leqslant 10^3$. Since $\cosh(\cdot)$ is always positive, we only have to consider the part $\cosh(x_1 + x_2 - x_3) \leqslant 10^3$. As cosh is a U-shaped function, this results in the affine constraint $-\operatorname{acosh}(10^3) \leqslant x_1 + x_2 - x_3 \leq \operatorname{acosh}(10^3)$: 1.5
Constraint (2) results the **union** of (the intersection of) 2 affine constraints and 1 affine constraint: $-625 \leqslant 1 + x_1 + 20x_2 + 30x_3 - 6 \leqslant 625$ **or** $x_1 + 8x_2 - x_3 - 5 \leqslant 5\sqrt[3]{5}$: 1.25

1 (c) 2×[4] NCC: nonconvex constrained optimization problem

2.5 (d) multi-start: 0.5 + M2: Gradient project method with variable step size line minimization: 2
(as this uses 1st-order information and as gradient is easy to compute, while no methods that use 2nd-order information like SQP are available)
multi-start : 0.5 + M9: penalty + steepest descent would also be accepted: 2

0.5 (e) KKT conditions with ε (list them!)
for M9 we could also use: $\|\nabla f_{\text{penalty}+}(x_k)\| \leqslant \varepsilon$ where $f_{\text{penalty}+}$ is the sum of the simplified objective function after elimination and the penalty function

[4] We need to put the factor 2 here, due to the **or** in the simplified constraint (2)

QUESTION 2 $(9+15+4=28$ points)

- **Question 2.1**

7 (a) Mention/provide at least the following:

[1] used for convex optimization problems: 0.5

[2] use of ***convex*** barrier function ϕ defined by $\phi(x) = -\sum_{i=1}^{m} \log(-g_i(x))$ if x is feasible and $+\infty$ otherwise: 1

[3] minimization of $tf + \phi$ for $\boldsymbol{t} \geqslant \mathbf{0}$, which is a convex unconstrained optimization problem: 1

[4] resulting optimal point $x^*(t)$ is always inside feasible set: 1

[5] start with low t and gradually increase t: 0.5

[6] increasing t makes optimal point $x^*(t)$ converge to the optimum x^* of the original convex problem: 1

[7] stopping criterion on t of the form $f(x^*(t)) - f(x^*) \leqslant \frac{m}{t} \leqslant \varepsilon$ with m the number of inequality constraints: 1

(1) illustrative drawing: 1

1 (b) A required condition to be able to apply the interior-point method is that the feasible set should have a non-empty interior (as else the barrier function would always be $+\infty$): 0.5
At first sight, the latter condition would exclude the presence of constraints of the form $h(x) = 0$, but as these should be affine, we can use them to eliminate variables (as an affine function of the other variables, thus preserving convexity after substitution). The resulting problem is thus still convex and has only inequality constraints: 0.5

1 (c) We would need multi-start if the selected barrier or penalty function would not be convex in the optimization variable x, as then resulting unconstrained optimization problem would be nonconvex in general.
If the selected barrier or penalty function is convex in x, then the overall objective function is convex, and as the resulting unconstrained optimization problem is convex then, a local optimum would also be a global optimum. So in that case multi-start would not be needed.

- **Question 2.2**

2 (a) If the problem is characterized as convex, the score for the entire subquestion (a) will be 0
If the problem is first simplified[5], the score for the entire subquestion (a) will be 0

First of all we transform the maximization problem into a minimization problem, and we rewrite the constraints: 0.5

$$\begin{aligned} &\min_{x\in\mathbb{R}^2} -|x_1+x_2-2|^4 \\ &\text{s.t.}\ \ -4 \leqslant x_i \leqslant 4 \quad \text{for } i=1,2 \\ &\qquad x_1+x_2 \leqslant 6 \\ &\qquad -x_1-x_2 \leqslant 7 \end{aligned}$$

As $-|\cdot|^4 = -(\cdot)^4$ is concave, the objective function is not convex: 1.5
Hence, the given problem is nonconvex.

12 (b) As $-(\cdot)^4$ is a decreasing function for nonnegative arguments (as is the case here due to the absolute value), we can maximize the argument instead; we also merge constraint (2) and (3), which yields: 1 [1]

$$\begin{aligned} &\max_{x\in\mathbb{R}^2} |x_1+x_2-2| \\ &\text{s.t.}\ \ -4 \leqslant x_i \leqslant 4 \quad \text{for } i=1,2 \\ &\qquad -7 \leqslant x_1+x_2 \leqslant 6 \end{aligned}$$

This leads to 2 linear programming problems: 2 [2]

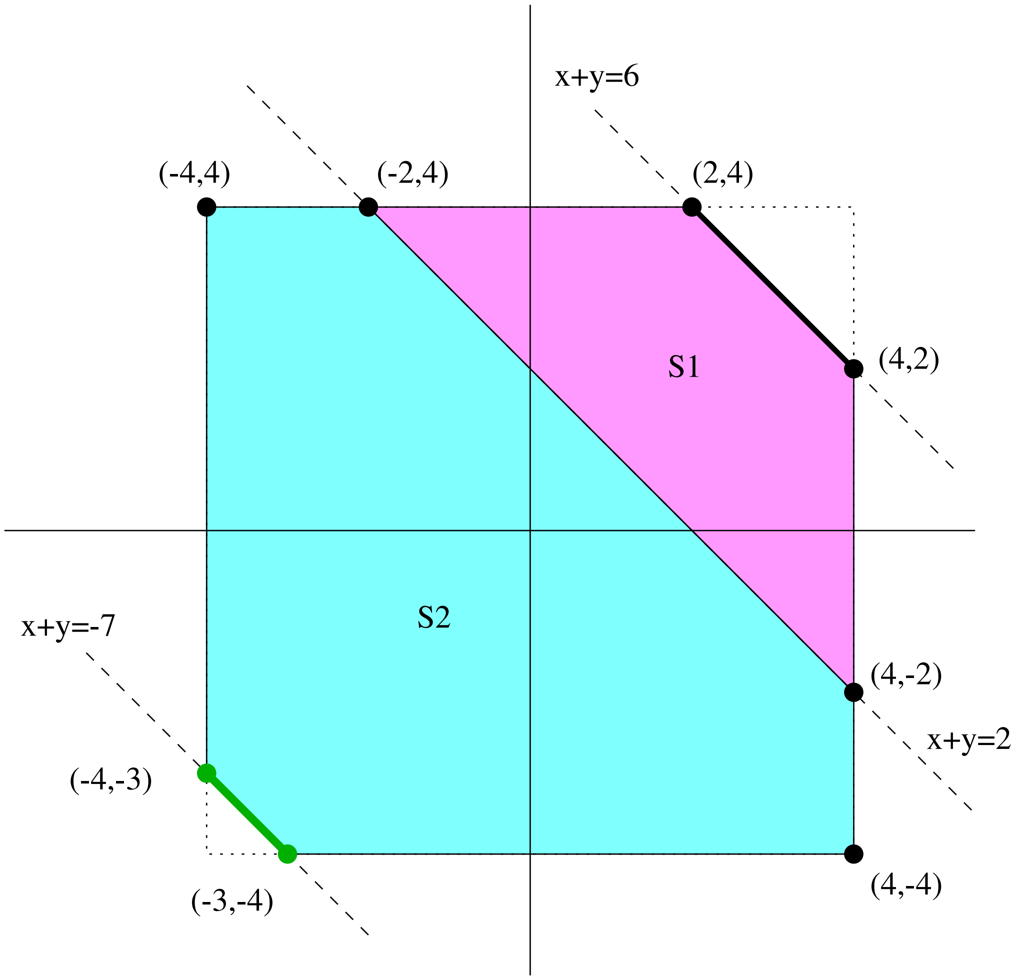
$$\begin{aligned} &\text{P1} : \max_{x\in\mathbb{R}^2} x_1+x_2-2 \\ &\text{s.t.}\ \ -4 \leqslant x_i \leqslant 4 \quad \text{for } i=1,2 \\ &\qquad -7 \leqslant x_1+x_2 \leqslant 6 \\ &\qquad x_1+x_2-2 \geqslant 0 \end{aligned}$$

and

$$\begin{aligned} &\text{P2} : \max_{x\in\mathbb{R}^2} -x_1-x_2+2 = -\min_{x\in\mathbb{R}^2} x_1+x_2-2 \\ &\text{s.t.}\ \ -4 \leqslant x_i \leqslant 4 \quad \text{for } i=1,2 \\ &\qquad -7 \leqslant x_1+x_2 \leqslant 6 \\ &\qquad x_1+x_2-2 \leqslant 0 \end{aligned}$$

The feasible sets S_1 (magenta) and S_2 (cyan) of P1 and P2 are given in the following figure: 2 [3]

[5] Note that simplification can affect convexity: e.g., the problem $\min_{x\in[-1,1]} x^3$ is not convex, while the simplified problem $\min_{x\in[-1,1]} x$ is convex.



The boundary lines of the form $x+y=c$ of the feasible sets are also indicated in the figure.
In order to solve P1 we have to move this line as far up as possible while still have an intersection with the feasible set. This yields the line between the points $(2,4)$ and $(4,2)$ as optimal solutions with function value 4: 2 [4]
In order to solve P2 we have to move the line $x+y=c$ as far down as possible while still have an intersection with the feasible set. This yields the line between the points $(-3,-4)$ and $(-4,-3)$ as optimal solutions with function value 9: 2 [5]
So the set of globally optimal solutions of the given optimization problem is given by the line between the points $(-3,-4)$ and $(-4,-3)$: 3 [6]

If due to error, only problem P2 is solved, but P1 is not considered: -6
If only endpoints are given and not the entire line segment in [6]: -2.5

1 (c) As we have split the original problem in the union of two linear programming problems and as for each of the two linear programming problems we have determined the set of globally optimal solutions, the optima we found are indeed globally optimal solutions of the given problem.

- **Question 2.3**

4 Mention/provide at least the following:

  [1] multi-objective optimization problem: 1

  [2] no other *feasible* points x such that $F(x) \leqslant F(x^*)$ and $F_i(x) < F_i(x^*)$: 2

  [3] illustrative drawing with some ***correct*** explanation: 1