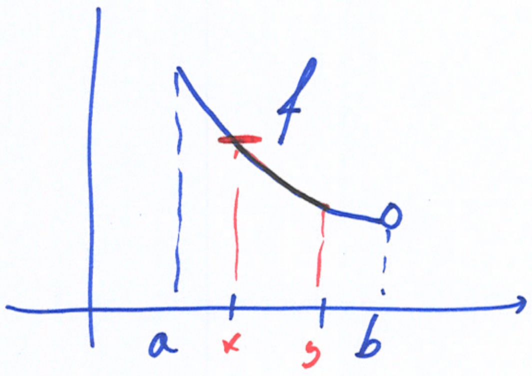
2024 - 9 - 26

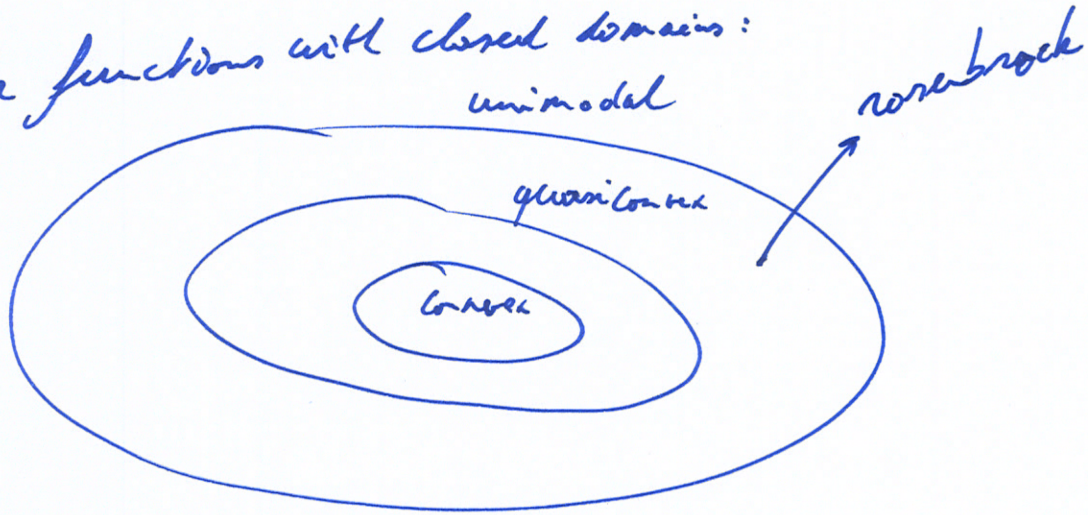


$\text{dom} f = [a, b)$ $\quad$ $\cancel{? \, x}$ $\quad$ $b \notin \text{dom} f$

unimodal? **no** $\quad$ dom f convex ✓

quasiconvex? **yes** $\quad$ dom f convex ✓ $\quad$ $f((1-\lambda)x + \lambda y) \leq \max(f(x), f(y))$ ✓

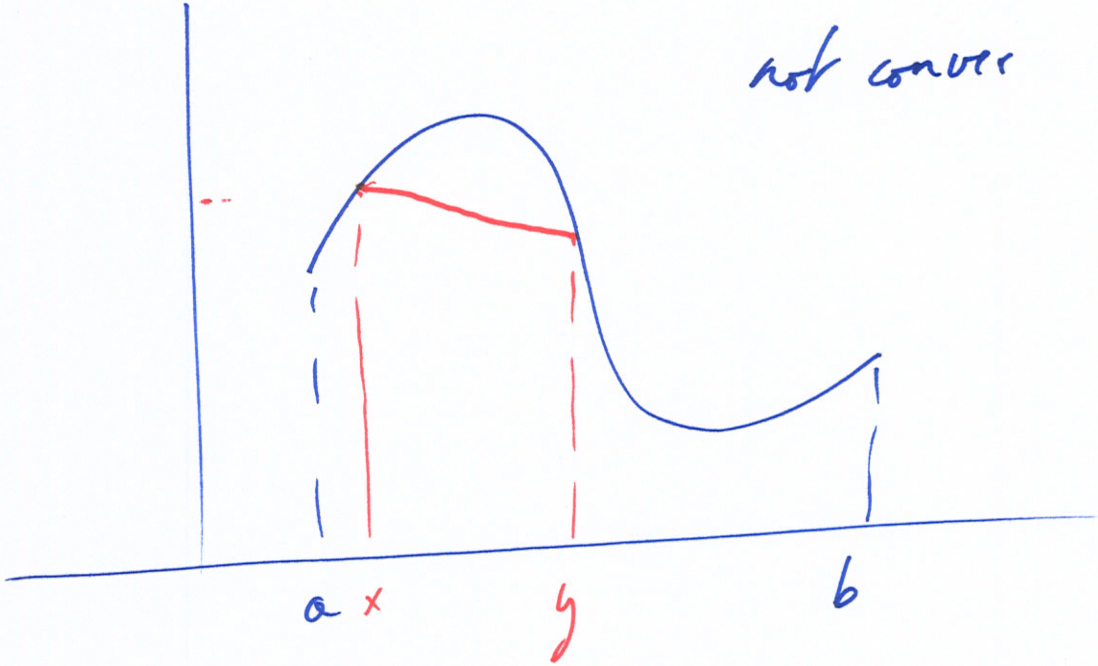
for functions with closed domains:

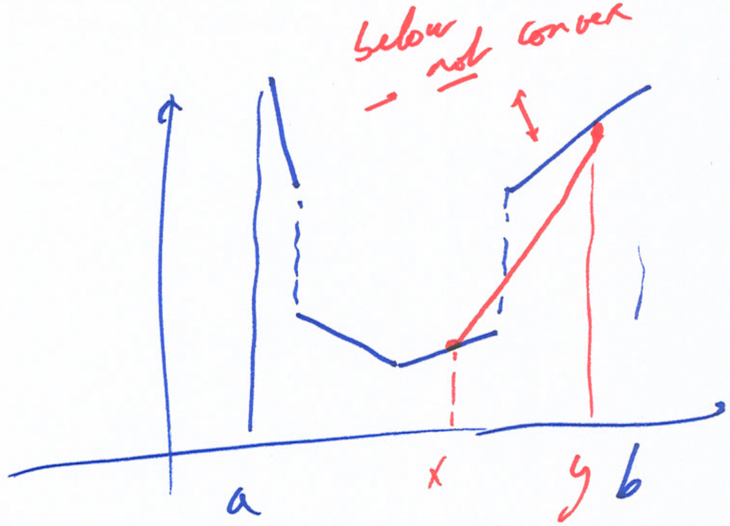


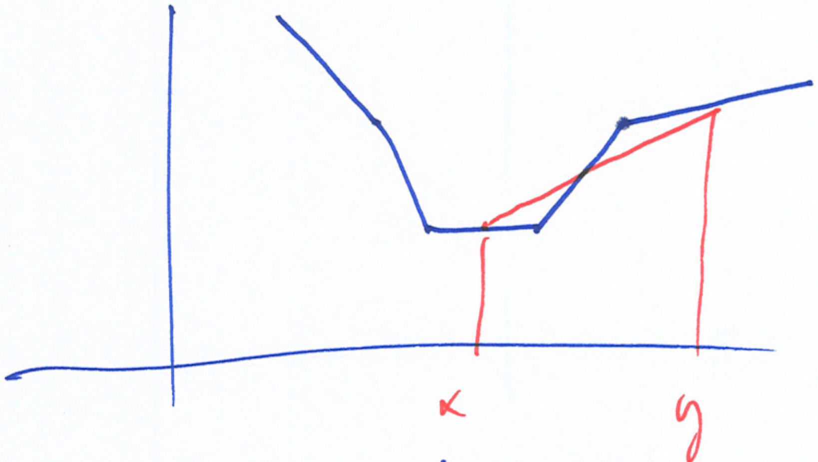
+ for 1D: unimodal = quasiconvex

if domain is closed

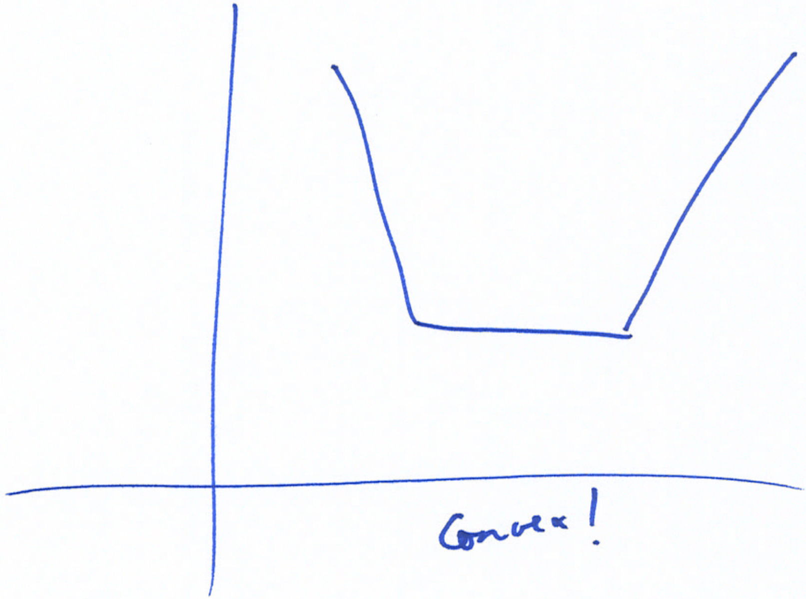
2

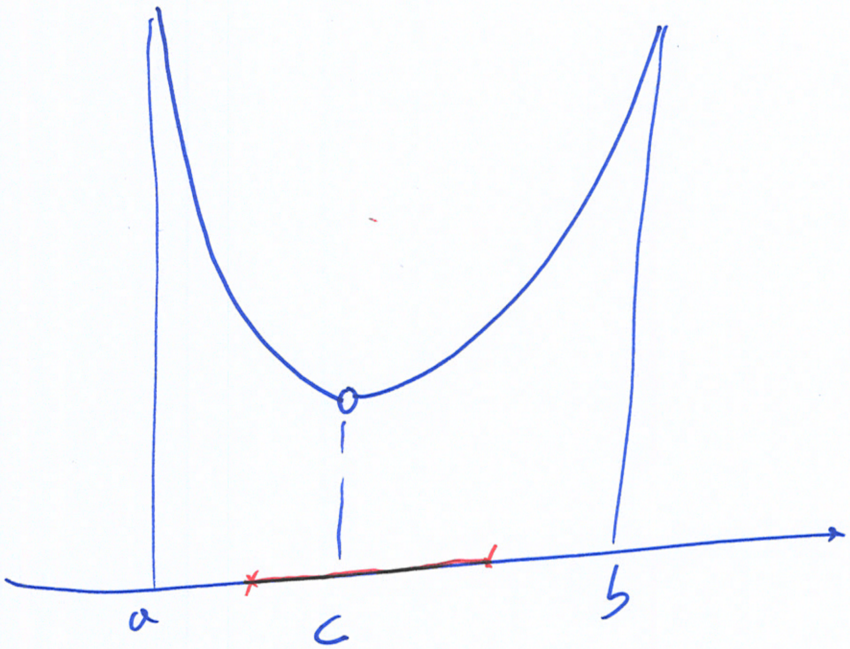






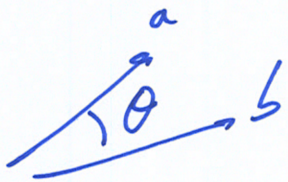
not convex



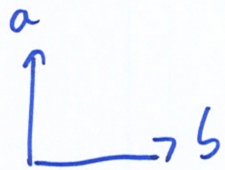


$\mathrm{dom}\, f = [a\, b] \setminus \{c\}$

→ non convex domain

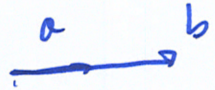


$$a^T \cdot b = \sum_{i=1}^{n} a_i b_i = \|a\| \cdot \|b\| \cdot \cos \underbrace{(a,b)}_{\theta}$$

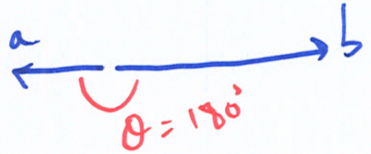


$a \perp b \Rightarrow a^T \cdot b = 0$

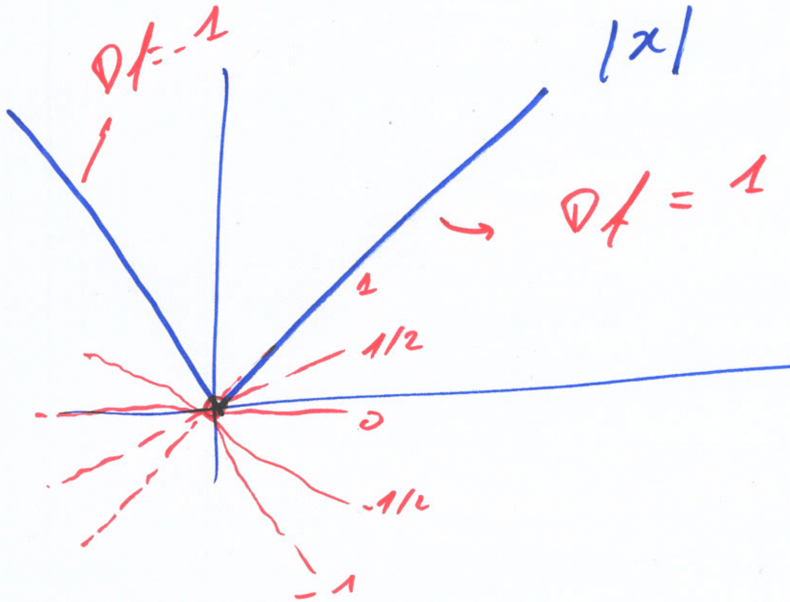
a // b, same direction: $a^T b = \|a\| \cdot \|b\| \cdot 1$



a // b, opposite direction: $a^T b = \|a\| \cdot \|b\| \cdot (-1)$



subgradient



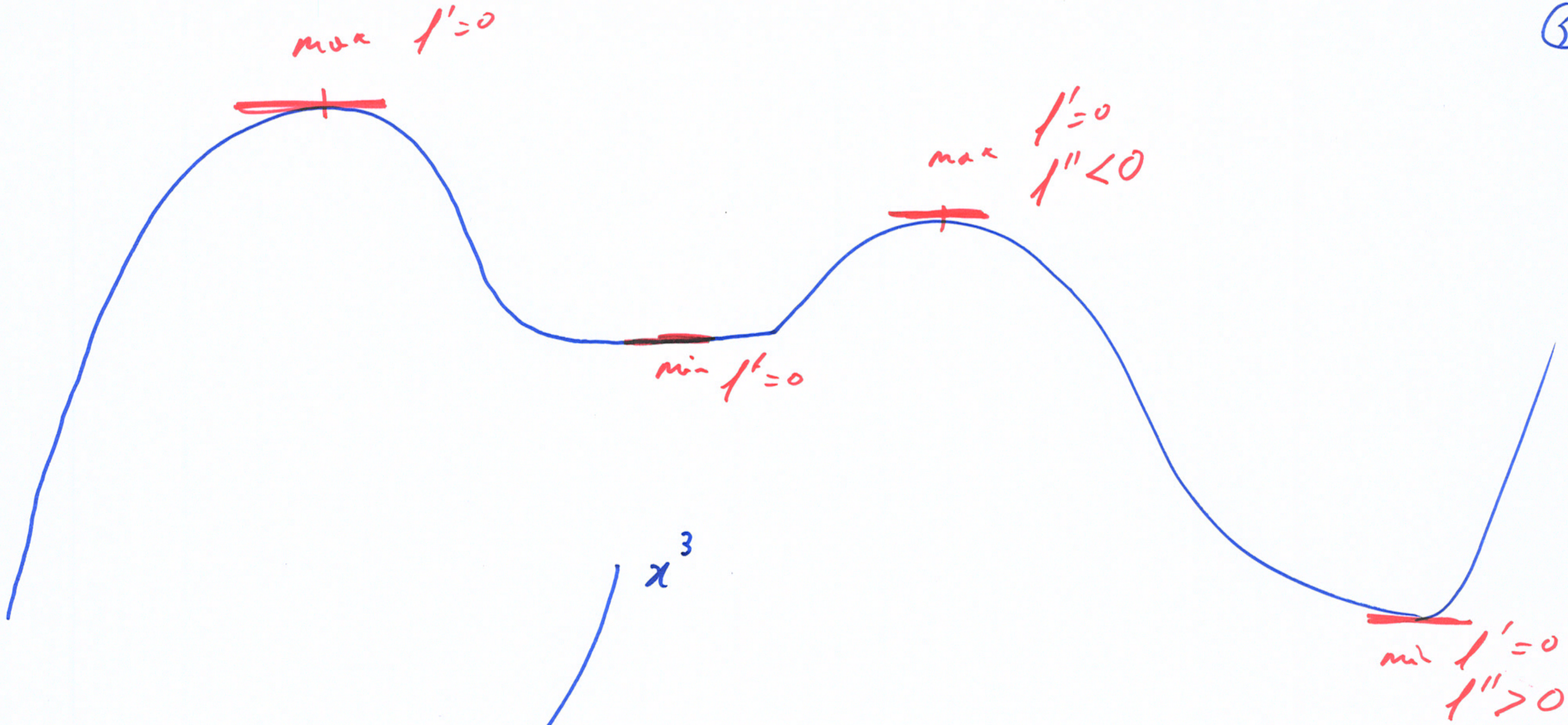
$$f \geq f(x_0) + \nabla^T f(x_0)(x - x_0)$$

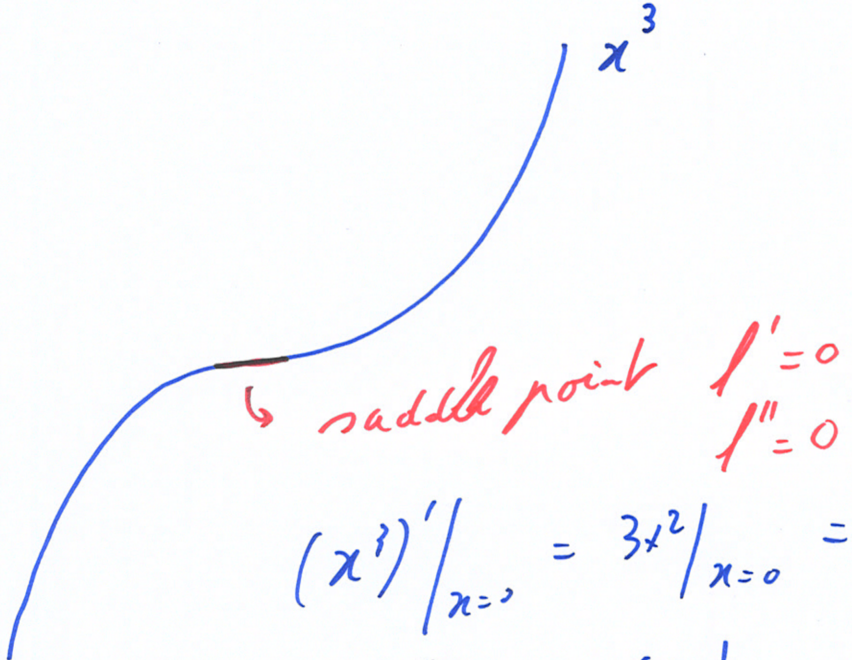
$f(x_0) = 0$

$\geq$ slope $\cdot\, x$

$b \in [-1, 1]$

$\nabla f(0) \in [-1, 1]$





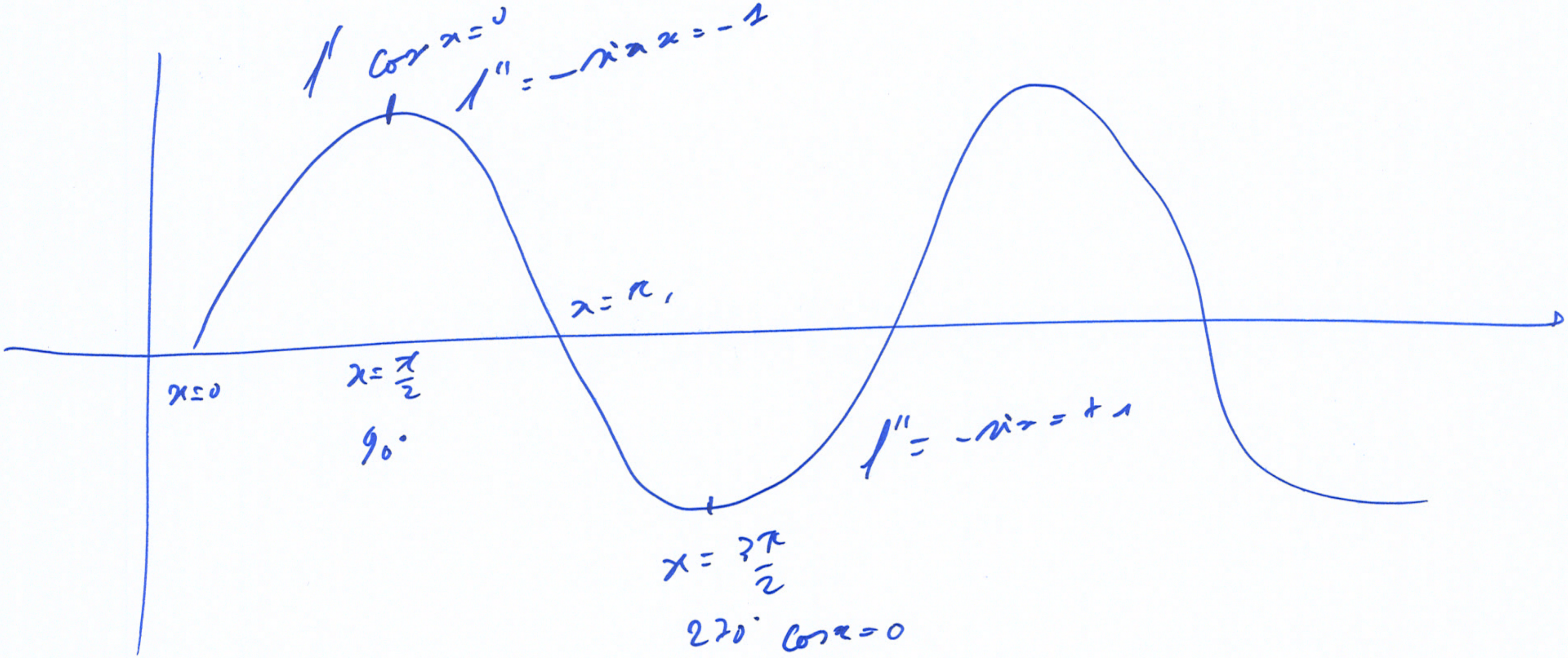
$$(x^3)'\Big|_{x=0} = 3x^2\Big|_{x=0} = 0$$

$$(x^3)''\Big|_{x=0} = 6\cdot x\Big|_{x=0} = 0$$

$f'=0$

example

$f(x) = \sin x$ $\xrightarrow{'} \cos x \xrightarrow{''} -\sin x$



$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} > 0 \quad \succ 0$$

↓

positive definite matrix

$$\underbrace{[x\ y]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}} = \underbrace{[x\ y]\begin{bmatrix} x \\ y \end{bmatrix}}$$

$$= x^2 + y^2 > 0$$

except $(x,y)=(0,0)$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \geq 0 \quad \succcurlyeq 0$$

↓

positive semi-definite

$$[x\ y]\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = x^2 \geq 0 \quad \forall (x,y)$$

$x=0,\ y=5$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\left.\begin{aligned} A_{\{1\}\{1\}} &= 1 > 0 \\ A_{\{1,2\}\{1,2\}} &= \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1 > 0 \end{aligned}\right\} \text{positive definite}$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 4 & 0 \\ 1 & 0 & 6 \end{bmatrix}$$

$|2| = 2 > 0$

$\begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix} = 2 \cdot 4 - 1 \cdot 1 = 7 > 0$

$\begin{vmatrix} 2 & 1 & 1 \\ 1 & 4 & 0 \\ 1 & 0 & 6 \end{vmatrix} = 2 \cdot 4 \cdot 6 - 4 - 6 = 38 > 0$

→ positive definite

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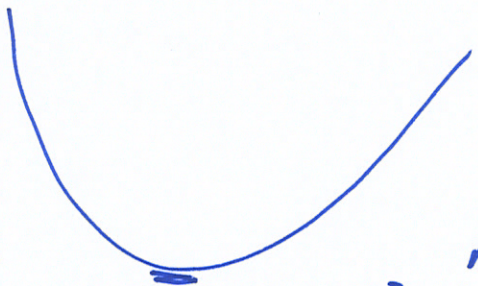
f is convex



$f' = 0$ → minimum

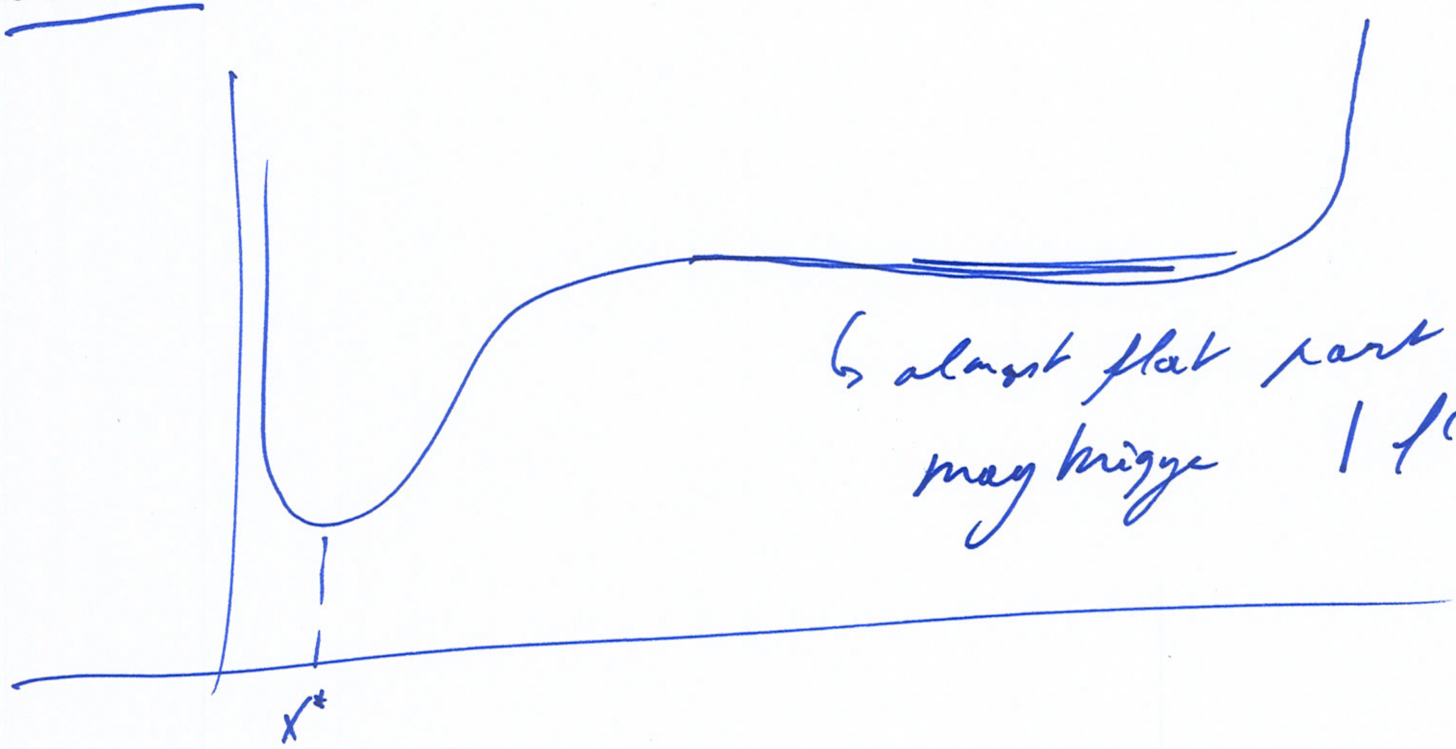
$v \in \mathbb{R}^n$

$$\|v\|_2 = \sqrt{v^T \cdot v} = \sqrt{\sum_{i=1}^{n} v_i^2}$$

$$\|v\|_1 = \sum |v_i|$$

$$\|v\|_\infty = \max |v_i|$$

$$\|v\|_p = \sqrt[p]{\sum_{i=1}^{n} |v_i|^p} \qquad p \geq 1$$



↳ almost flat part may trigger $|f(x_{k+1}) - f(x)| \leq \epsilon$