① Convex

$$\min_x f(x) \qquad g(x) \le 0$$

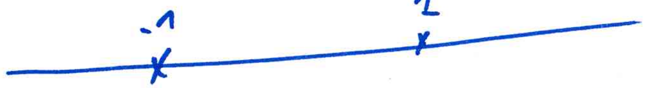
f, g convex.

$$\min_x f(x)$$
$$g(x) \le 0$$
$$\underline{h(x) = 0}$$

conditions on h to get convex problem

$h = x^2 - 1$ convex

$x^2 - 1 = 0 \rightarrow x = 1$ or $x = -1$



set $\{-1, 1\}$ is not convex

$x^2 - 1 = 0$ is not convex constraint.

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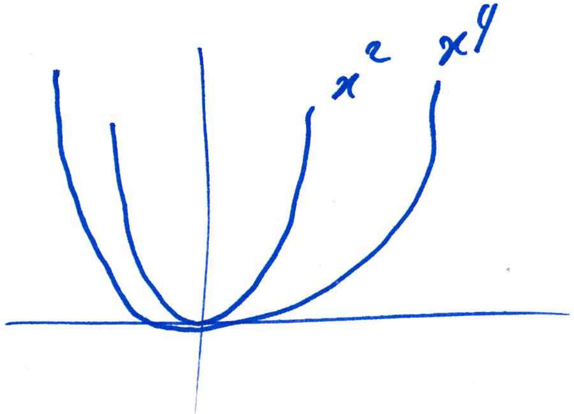
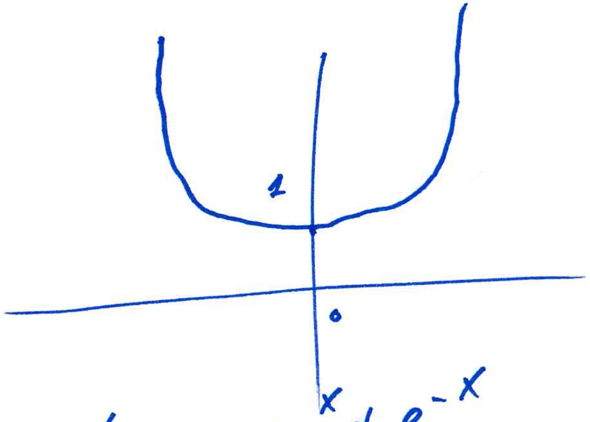
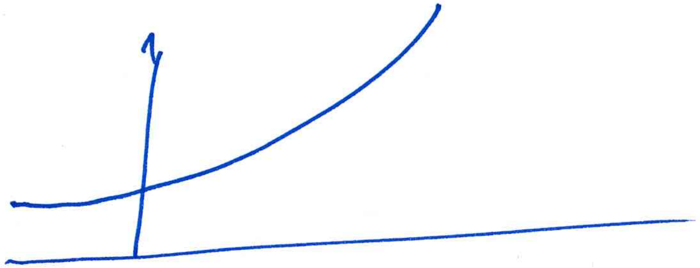


x^{2n} $\quad n \geq 1 \quad \rightarrow$ Convex

$\cosh x$



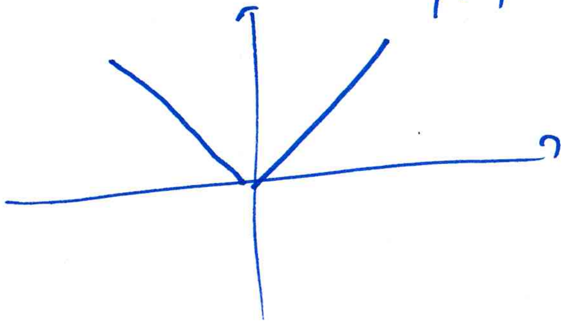
e^x $\rightarrow$ Convex



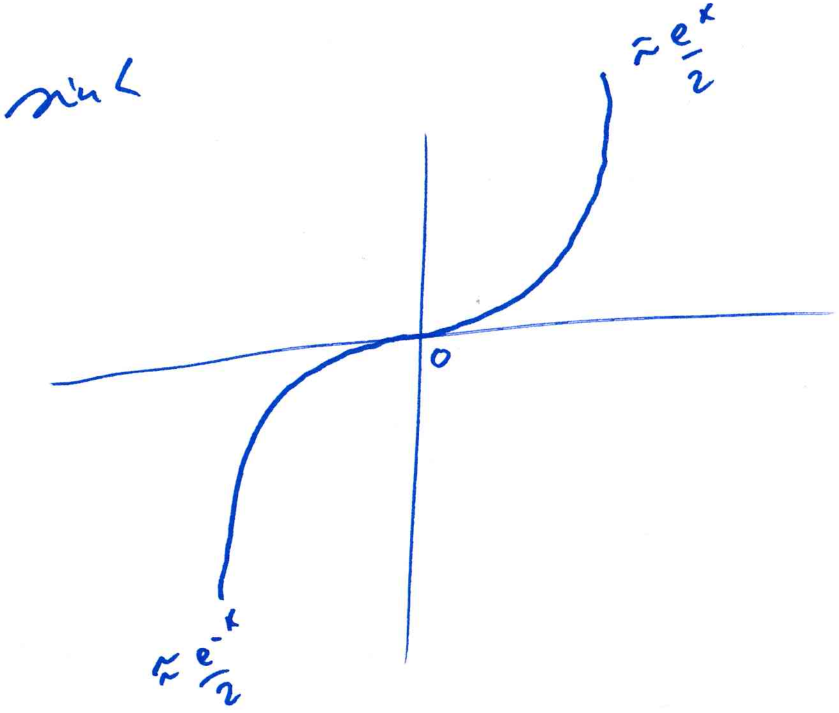
$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$\rightarrow$ Convex

$|x|$ $\rightarrow$ Convex

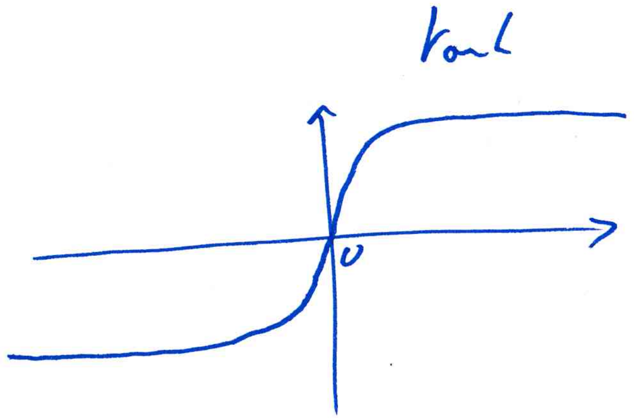


(3)



$\sinh x = \dfrac{e^x - e^{-x}}{2}$

→ <u>not</u> convex



$\tanh x = \dfrac{\sinh x}{\cosh x}$

→ <u>not</u> convex.

(4) $\|\underbrace{\lambda x}_{u} + \underbrace{(1-\lambda)y}_{v}\| \overset{?}{\leq} \lambda\|x\| + (1-\lambda)\|y\|$

$\overset{(4)}{\leq} \|\lambda x\| + \|(1-\lambda)y\|$

$\lambda \in [0,1]$

$\overset{(3)}{\leq} \underbrace{|\lambda|}_{\geq 0}\|x\| + |\underbrace{1-\lambda}_{\geq 0}|\,\|y\|$

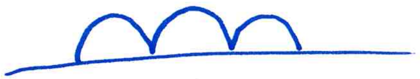
$\leq \lambda\|x\| + (1-\lambda)\|y\|$!

$\rightarrow \|x\|$ is convex (in its argument)

$|x|$ is convex

$|x + 2y + 3z + 33v + 5q - 7|$ is convex

$|\sin x|$ <u>not</u> convex.



(5)

$$\|x\|_2 = \sqrt{\Sigma x_i^2}$$

$$\|x\|_1 = \Sigma |x_i|$$

$$\|x\|_\infty = \max_i |x_i|$$

$$\|x\|_p = \sqrt[p]{\sum_{i=1}^{n} |x_i|^p} \qquad p \geq 1$$

$$2^p + 3^p \approx \left[\max(2,3)\right]^p$$

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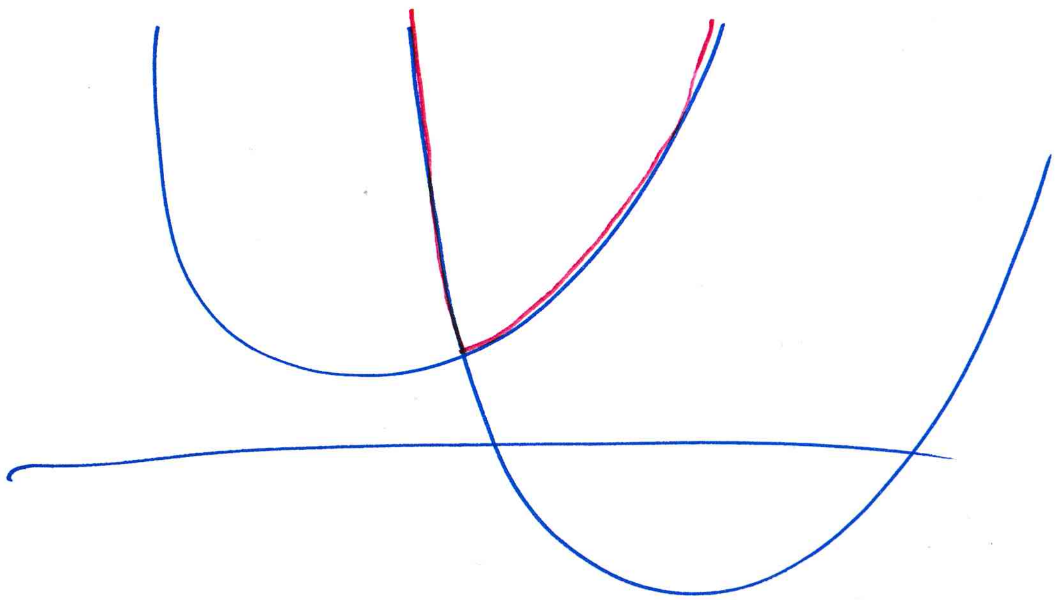
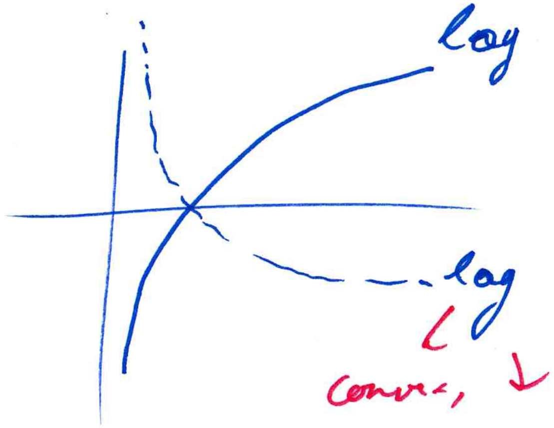


f_i convex $\searrow$ $\max f_i$ is convex



- $e^{x^2+y^2}$

  x^2+y^2 convex
  e^x convex ↗

- $\underbrace{-\log}_{h}\,(\underbrace{1-x^2}_{g})$ if $x \in (-1,1)$

  $-g = x^2-1$ convex
  $h = -\log$ convex, ↓