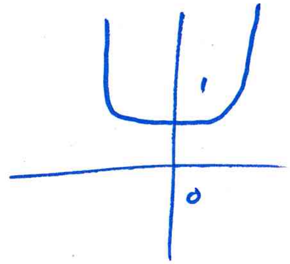
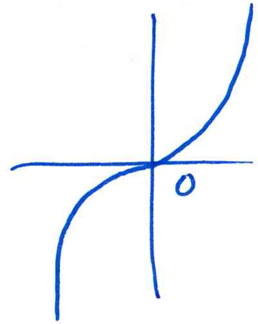
① 2024-10-22

$$\min_x \; e^x(3\cosh x - 4\sinh x + 8) \quad \text{s.t. } x \le \log 2$$

$\cosh x = \dfrac{e^x+e^{-x}}{2}$



$\sinh x = \dfrac{e^x-e^{-x}}{2}$



$$\min_x \; e^x\left(\frac{3}{2}e^x + \frac{3}{2}e^{-x} - \frac{4}{2}e^x + \frac{4}{2}e^{-x} + 8\right) \quad \text{s.t. } x \le \log 2$$

$$\min_x \; \frac{3}{2}e^x \cdot e^x + \frac{3}{2} - 2e^x \cdot e^x + 2 + 8e^x \quad \text{s.t. } x \le \log 2$$

$$\min_x \; -\frac{1}{2}(e^x)^2 + 8e^x + \frac{7}{2} \quad \text{s.t. } x \le \log 2, \; e^x \le 2$$

$e^x = y$

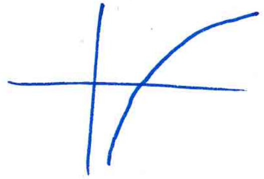
$$\min_y \; -\frac{1}{2}y^2 + 8y + \frac{7}{2} \quad \text{s.t. } 0 \le y \le 2$$

nonconvex QP

②

$$\min_{x} \underbrace{e^{x_1-2x_2+3x_3}}_{\text{convex}} + \underbrace{\log(-x_1+2x_2+5x_3)}_{\text{nonconvex}}$$

$$(x_1+x_2+x_3-7)^5 \le 1024 = 2^{10}$$



$$\min_{x} f_1(x) + f_2(x)$$

~~Max~~ $f_1 \uparrow$

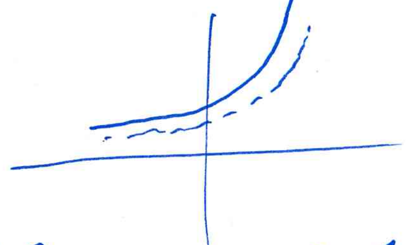
$f_2 \uparrow$

~~$\min_x$ argument$_1$ + argument$_2$~~

if arg$_1$ ≠ arg$_2$ → do not simplify

→ only if argument$_1$ = argument$_2$

$$\min_{x} \underbrace{e^{\boxed{x_1+x_2-3}} + 2^{\boxed{x_1+x_2-3}}}$$



· entire function is increasing

· 1 argument

$e^x \uparrow$, $2^x \uparrow$ → $e^x + 2^x \uparrow$

↓

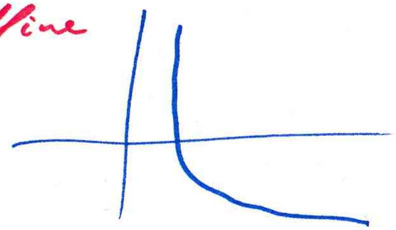
$$\min_{x} x_1+x_2-3$$

(2*)

$$\min_{x} \; e^{x_1 - 2x_2 + 3x_3} - \log(-x_1 + 2x_2 + 5x_3)$$

Convex: · e^x is convex · arg. is affine

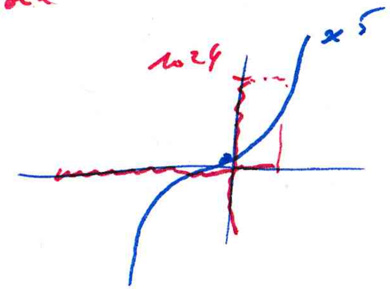
Convex: · $-\log x$ is convex · arg. is affine



s.t. $-x_1 + 2x_2 + 5x_3 \geq 10^{-9}$ linear → convex

$(x_1 + x_2 + x_3 - 7)^5 \leq 2^{10}$

→ $x_1 + x_2 + x_3 - 7 \leq 2^2$ linear → convex



→ increasing

→ Convex problem

set 1

- modified simplex
- interior point ←
- SQP

$$|f(x^*) - f(x_k)| \leq \varepsilon_f$$
$$g(x_k) \leq \varepsilon_g$$

set 2

* simplex
- genetic algorithm → multi-run even if problem convex!
- steepest descent → unconstrained

* barrier function with Powell
- penalty function with line search & BFGS direction → single run is OK as local = global for convex

uses ∇f →

2xx

stopping criterion!

as convex algorithm

$| ~~f(x) - f(x^*)~~ | \le \varepsilon_f$

$~~g(x^*) \le \varepsilon_g~~$

as method used is not convex algo.

$\| \nabla f(x^*) + \nabla g(x^*) \cdot \mu \| \le \varepsilon_1$

$| \mu^T g(x^*) | \le \varepsilon_2$

$\mu \ge -\varepsilon_3$

$g(x^*) \le \varepsilon_4$

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slide 11:

$$\min_x \frac{-x_1 x_2 x_3}{1 + x_1^6 + x_2^4 + x_3^2} \quad , t$$

$$x_3 = 1 - x_1 - x_2$$

$$\min_x \frac{-x_1 x_2 (1 - x_1 - x_2)}{1 + x_1^6 + x_2^4 + (1 - x_1 - x_2)^2}$$

f simplified

→ nonconvex, unconstrainted

$\| \nabla f_{\text{simplified}}(x^*) \| \le \varepsilon$

from set 2: only genetic + steepest descent

↳ uses gradient