1 genetic algorithm

2024-10-24

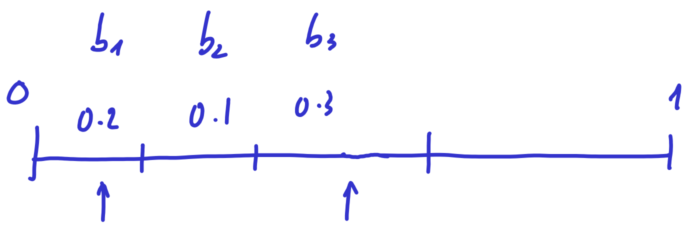
decimal $\leftrightarrow$ binary

$$\overset{2^3}{1}\,\overset{2^2}{0}\,\overset{2^1}{1}\,\overset{2^0}{1} = 1\cdot \overset{2^3}{8} + 0\cdot \overset{2^2}{4} + 1\cdot \overset{2^1}{2} + 1\cdot \overset{2^0}{1} = 11$$

25 = sum of powers of 2

$$= 16 + \underset{8+1}{\underset{\shortparallel}{9}} = 16 + 8 + 1 = 2^4 + 2^3 + \cancel{2^2} + \cancel{2^1} + 2^0$$

$$= 11001$$



next take random uniformly in [0,1]

2 When to use Lagrange

1) equality constraints $\rightarrow \min_x f(x) \text{ s.t. } h(x) = 0$

2) no more elimination possible

linear: $x_1 + x_2 - 3x_3 + x_4 = 5 \rightarrow x_1 = 5 - x_2 + 3x_3 - x_4$

$$e^{x_1} + \cos(x_1 + x_2) + \underbrace{x_3^3} + x_1 x_2 = 1 \rightarrow x_3^3 = 1 - e^{x_1} - \cos(x_1 + x_2) - x_1 x_2$$

$$\rightarrow x_3 = \sqrt[3]{1 - e^{x_1} - \cos(x_1 + x_2) - x_1 x_2}$$

$$x_1 = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

$\rightarrow$ 2 solutions for x_1
fill both out in $f(x)$ + take smallest result

$$x_1^2 + 2x_1 x_2 + e^{x_2 + 2x_3} + \cos(x_2 + x_3) = 1$$ actually quadratic in x_1

$x_1^2 + a \cdot x_1 + b = 0$ with $a = 2x_2$
$b = e^{x_2 + 2x_3} + \cos(x_2 + x_3) - 1$

$$x_1 x_2 e^{x_3} + \cosh(x_1 + 3x_3 x_2) + x_1^2 + x_2^4 + x_3^6 - \sin(x_1 x_2 x_3) = 1$$

$\rightarrow$ not possible to express x_1 or x_2 or x_3 as function of other variables only $\rightarrow$ use Lagrange

write x_1 as function of x_2 and x_3 NOT x_1

$$\cancel{x_1 = \pm\sqrt{\ldots x_1 \ldots}}$$

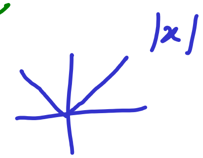
③ When can gradient / Hessian be computed analytically?

$$\sin^8\left(\exp\left(\cosh\left(x_1^2 + x_3^2 + 2^{x_1 + \cos\left(x_1 + \tan^8(x_2 + x_3)\right)^{x_1 + x_3}}\right)\right)\right) + x_2 x_3 x_1$$

→ gradient + Hessian can be computed analytically

$$\cosh\big(\overbrace{|x_1| + |x_2|}^{\geq 0}\big) + |x_1 + x_2 + x_3| + (x_1 + x_2 + 3x_3)^2$$

$|.|$ → derivative does not exist in 0



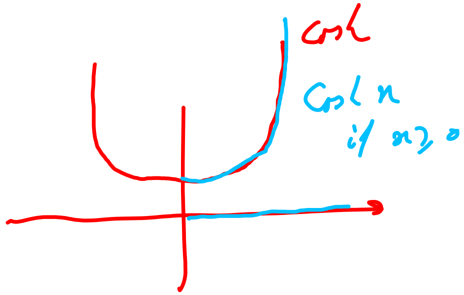
$|.|$ is convex
argument is affine } → convex

$(.)^2$ is convex
argument is affine } → convex

$h(g(x))$

$h = \cosh$ → convex, $h \uparrow$ for argument ≥ 0 } → convex

$g = |x_1| + |x_2|$ → convex



→ convex function → use subgradient

④ When not to use/compute gradient/Hessian analytically/numerically?

$\theta \longrightarrow \boxed{\text{controller } \theta} \longrightarrow J$: performance

$\min_\theta J(\theta)$

θ: tuning parameters

internal structure of controller unknown

experiment takes 10 s

$\theta \in \mathbb{R}^{20}$

2000 $\parallel$

$\nabla f \rightarrow$ 20 function evaluations $\times 100$

$$\frac{\partial f}{\partial x_i}(\bar{x}) \approx \frac{f\left(\bar{x} + \delta \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \leftarrow i\right) - f(\bar{x})}{\delta}$$

$\rightarrow$ don't use ∇f / H here

• different: $\theta \in \mathbb{R}^3$, experiment $= 0.5\,\mu s$ $\rightarrow$ here we can use ∇f, H

$\rightarrow$ look at number of variables + look at evaluation time

$\min_\theta I(\theta)$ $\quad \theta \in \mathbb{R}^{100}$ $\quad I(\theta) = \int_0^1 \int_0^1 \cdots \underline{\cos(\theta_1) + \sin(\theta_1) \cdot t + \cdots} \, dt$

$\rightarrow$ numerical integration $\rightarrow$ multi-dimensional $\rightarrow$ slow

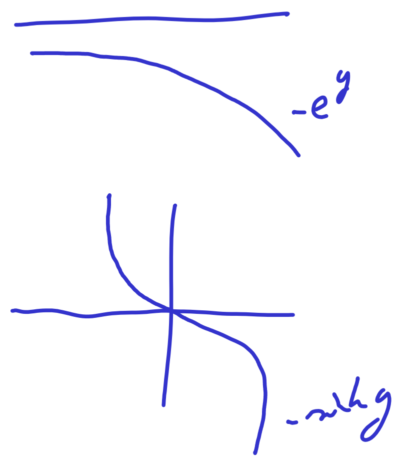
$\rightarrow$ don't use ∇f or H

(5) exam of November 2023

note that arguments of exp and sinh are the same!

P1

$$\min_{x \in \mathbb{Z}^3} \quad \underbrace{-\exp\left(\frac{\cdot}{y}\right) - \sinh\left(\frac{\cdot}{y}\right)}_{\text{decreasing as function of } y}$$



$$\max_{x \in \mathbb{Z}^3} \quad y \to \text{affine in } x \qquad \to \qquad \min_{x \in \mathbb{Z}^3} -y \to \text{affine in } x$$

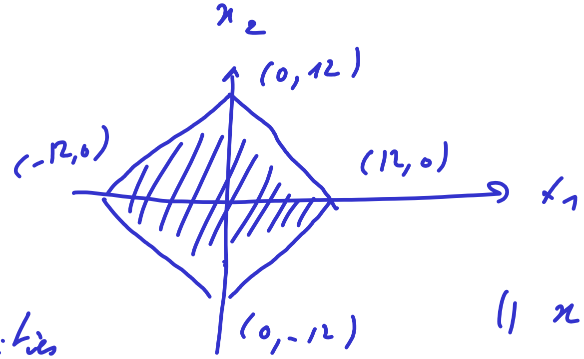
$\to -e^y - \sinh y$ is decreasing in y

$\|x\|_1 \leq 12$

$\downarrow$

$\pm x_1 \pm x_2 \pm x_3 \leq 12$

$\to$ 8 linear inequalities



$\|x\|_1 \leq 12$

$|x_1| + |x_2| \leq 12$

$\pm x_1 \pm x_2 \leq 12$

$(\qquad)^3 \leq 81$

$\underbrace{(\qquad)}_{\text{affine}} \leq \sqrt[3]{81} \to$ linear inequality

$(\cdot)^3$ is increasing

6

$$\max(\underbrace{\cdots\cdots}^{A_1}, \underbrace{\cdots\cdots}^{A_2}, \underbrace{\cdots}^{A_3}) \geq 5$$

$A_1 \geq 5$ <u>or</u> $A_2 \geq 5$ <u>or</u> $A_3 \geq 5$

$\downarrow$

linear ineq.   lin. ineq.   linear ineq.

$\min_{x \in \mathbb{Z}^3}$ affine s.t. affine constr. <u>or</u> $\min_{x \in \mathbb{Z}^3}$ affine s.t. affine constr. <u>or</u> $\min_{x \in \mathbb{Z}^3}$ affine s.t. affine constr.

$MILP_1$ <u>or</u> $MILP_2$ <u>or</u> $MILP_3$

→ branch-and-bound method for MILP

→ stop if entire tree has been explored