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Optimal coordination of variable speed limits to suppress shock waves – Addendum*

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Optimal Coordination of Variable Speed Limits to Suppress Shock Waves – Addendum

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Abstract

The purpose of this document is to give a more detailed explanation of the performance degradation as described in the paper "Optimal coordination of variable speed limits to suppress shock waves" (by A. Hegyi, B. De Schutter, and J. Hellendoorn, *Transportation Research Record*, no. 1852, pp. 167–174, 2004). For the simulation settings in that paper the degradation occurs for the "floor" type of rounding of the real-valued control signal (resulting from the MPC optimization), but does not occur for "round" and "ceil" types of rounding. In this document we show and explain that if the discrete-valued speed limit step size is increased, the degradation occurs in all cases, but for the floor type of rounding first.

In [A1] the performance for the floor type of rounding is poor compared to the other two types of rounding for both the constrained and unconstrained cases (see Table A.1). Therefore, in the remainder of this report we will consider unconstrained cases only.

A Cause of performance degradation

The performance degradation in case of floor can be explained by the relatively slow dynamics of the traffic process and the step size of the discrete speed limits. To explain this we describe the behavior of the closed loop system consisting of the traffic model and the MPC-controller, for the case of an arriving shock wave and discrete-valued MPC speed limit control with the floor type of rounding.

Initially, when the shock wave enters the link, the flow is restricted by low speed limits (here 50 km/h). When the congestion starts resolving, the (optimized, continuous-valued) speed limits will increase to enable the traffic to accelerate. These speed limits will be just above[1] the natural evolution[2] of the speed, (such that they are not limiting anymore) or if necessary (determined by the optimization) below the natural evolution of the speed. This value is rounded downward by floor, and in the next MPC iteration the actual speeds will be lower than the speed limit resulting from the continuous optimization. Since the dynamics of the traffic process is relatively slow, the speeds usually do not increase within the controller sampling time (1 min) with more than 10 km/h. This means that floor will result in the same low value, which keeps the average speed and (out)flow low. This process is repeated for each MPC iteration.

[1] Higher speed limits will not occur, because they do not change the traffic behavior, and consequently do not improve the performance.

[2] We mean by natural evolution of the speed the evolution that would occur if no speed limits were present.

Horizon		Relative improvement (%)							
		unconstrained				constrained			
N_p	N_c	continuous	round	ceil	floor	continuous	round	ceil	floor
9.0	4.0	21.1	20.2	21.5	1.9	20.9	19.4	18.5	3.9
9.0	6.0	20.9	21.1	21.4	3.8	21.1	20.2	21.1	15.1
9.0	8.0	21.1	15.9	21.4	12.1	21.1	20.7	20.4	15.1
10.0	4.0	21.4	20.1	21.5	0.5	21.2	20.0	21.2	0.2
10.0	6.0	21.6	20.5	21.7	14.2	21.4	20.9	21.4	12.7
10.0	8.0	21.5	21.1	21.7	5.2	21.4	20.8	21.4	16.3
11.0	4.0	21.5	19.8	21.5	-2.8	21.3	19.5	21.3	4.2
11.0	6.0	21.6	21.1	21.7	3.7	21.4	21.1	21.4	14.2
11.0	8.0	21.7	21.1	21.7	8.1	21.5	21.0	21.7	13.0
12.0	4.0	21.6	20.3	21.6	-1.1	21.5	19.0	21.5	-0.3
12.0	6.0	21.7	21.3	21.8	7.3	21.5	21.4	21.4	14.2
12.0	8.0	21.7	21.6	21.8	8.9	21.5	21.5	21.4	15.0

Table A.1: The relative improvement of the performance (Total Time Spent) for several combinations of N_p and N_c, and for the continuous-valued speed limits and the three discrete-valued speed limits: round, ceil, and floor.

In Figure A.1 a snapshot of the MPC procedure for speed limits rounded with floor is shown (with $N_p = 11$ min, $N_c = 8$ min). For visibility purposes we show the speed limits for one segment only. The left vertical (green) line is the current time instant, the right vertical line represents the end of the prediction horizon. Between these two lines the speed limit signals are optimized. In case of discrete-valued speed limits the signals between the two vertical lines are approximated by the discrete signals.

Figure A.2 is a zoom-in of Figure A.1, where we can see that the optimal continuous speed limit value in segment 11 at the current time (see the left vertical green line; this speed limit equals approximately 52 km/h) is higher than one time step earlier (50 km/h). However, if floor is used the current speed limit (52 km/h) is rounded to 50 km/h again, because the increase in speed limit is small. Since the continuous optimization rarely results in a speed limit jump (increase) that is larger than the discretization step, floor will tend to round the signals to the same low value.

It is clear that if the step size of the discretized speed limits is smaller then the probability of repeated downward rounding of the speed limits is smaller, and the performance will be better. To verify this explanation we will now investigate whether the performance of floor improves when the step size of the discrete-valued speed limits is reduced.

We compare several speed limit step sizes with $N_p = 11$ min and $N_c = 8$ min, see Table A.2. We can conclude from the table that in general the performance improves if the speed limit step size is decreased, and that the performance of floor breaks down first when the speed limit step size is increased. These findings are in accordance with our expectations[3].

[3] It is remarkable that the performance of floor slightly improves when the speed limit step size is increased from 10 km/h to 15 km/h. This may be caused by the specific traffic scenario, or by the fundamental difference between the MPC optimization (for a horizon of length N_p) and optimization for the whole simulation length.

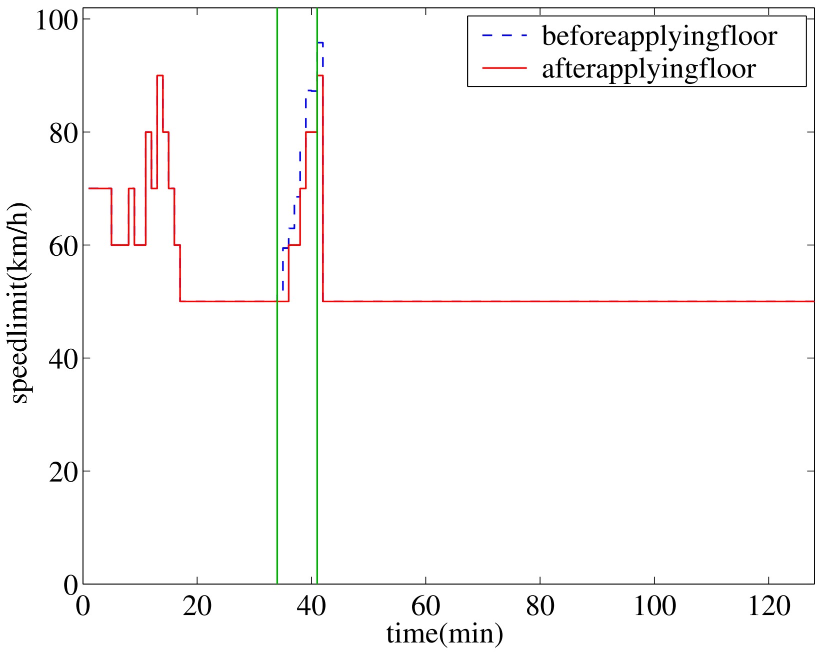

Figure A.1: Snapshot of the speed limit (segment 11) resulting from the MPC procedure rounded with floor. The signal is shown before and after rounding. The left and right vertical (green) lines indicate respectively the current time instant and the end of the prediction horizon.

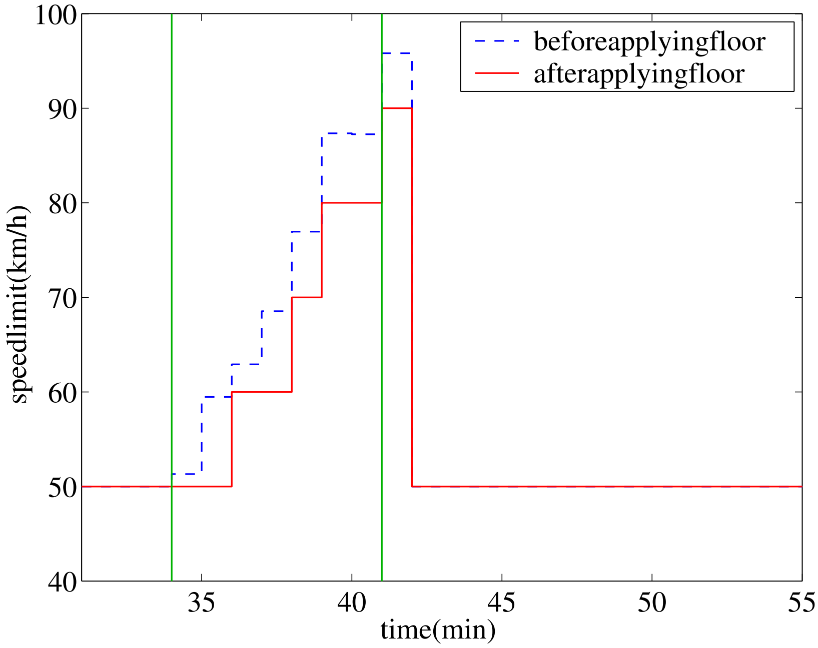

Figure A.2: Zoom-in of Figure A.1

ΔSL (km/h)	Relative improvement (%)		
	round	ceil	floor
0 (continuous)	21.7	21.7	21.7
5	21.7	21.6	18.1
10	20.8	21.7	3.3
15	19.0	2.9	4.5
20	1.6	2.9	-12.8

Table A.2: The relative improvement of the performance (Total Time Spent) for several speed limit step sizes (ΔSL) for the continuous-valued speed limits and the three discrete-valued speed limits: round, ceil, and floor

Additional references

[A1] A. Hegyi, B. De Schutter, and H. Hellendoorn, "Optimal coordination of variable speed limits to suppress shock waves," *Transportation Research Record*, no. 1852, pp. 167–174, 2004.