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Anticipative model predictive control for ramp metering in freeway networks

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Abstract

When implementing ramp metering in the context of freeway networks, traffic can spontaneously re-route due to the applied control actions. Although this re-routing can have an important impact on the resulting traffic situation in the traffic network and on the performance of the traffic network as a whole, re-routing is not automatically included in current model predictive frameworks for freeway traffic control. Therefore, we propose a method to calculate and incorporate the re-routing effects into the model predictive framework. In this way, we realize anticipative model predictive control for ramp metering in freeway networks.

1 Introduction

Advanced traffic management systems are powerful tools in the battle against traffic congestion on freeways. These advanced traffic management systems consist of a controller or a hierarchy of controllers which steer one or more actuators based on traffic measurements. We consider ramp metering and we use model predictive control to determine the control signals. More specifically, we consider network-wide control of freeway networks. Since there are typically multiple routes in a freeway network between a driver's origin and destination, every driver has to choose a route for his trip. This route choice can be modeled as the driver assigning a cost to every possible route and choosing the route with the lowest cost. The cost assignment to each route is a subjective process which is influenced by the estimated travel time of the route, by the total length of the route and even by subjective factors such as, e.g., the driver's dislike of a certain freeway.

When advanced traffic management systems are activated they influence the traffic state on the freeway. This altered traffic state can change the cost that an average driver assigns to a route. This way the traffic management system influences the route choice of the drivers. The model predictive control we discuss in this paper takes the re-routing of traffic due to control actions into account. Hence, we obtain an anticipative control framework.

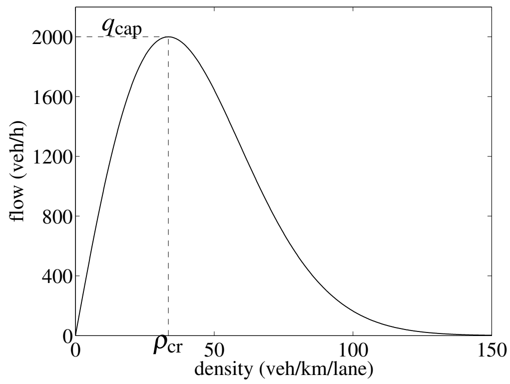

Figure 1: The flow–density relation of the traffic in a freeway section, also known as the fundamental diagram.

2 Ramp metering

Studies of traffic measurements usually show an increasing traffic throughput (number of vehicles per hour) with increasing traffic density (number of vehicles per kilometer) until the critical density $\rho_{\rm cr}$ is reached after which the traffic throughput starts decreasing with increasing traffic density. This phenomenon is known as the fundamental diagram [5] and is presented is Figure 1. The maximal throughput is called the capacity of the freeway $q_{\rm cap}$. Free-flow traffic occurs when the traffic density is lower than or equal to the critical density $\rho_{\rm cr}$. Traffic operation at densities larger than $\rho_{\rm cr}$ corresponds to a congested traffic flow.

Advanced traffic management systems are implemented to control the traffic flows in such a way that the traffic is in the free flow state as much as possible. One technique that is frequently used for these purposes is ramp metering. Ramp metering tries to avoid that the traffic density on the freeway becomes larger than the critical traffic density, thus avoid-

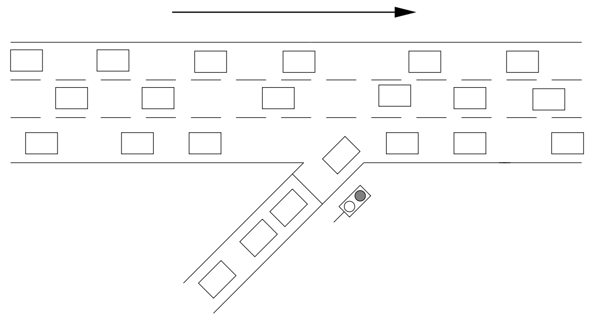

Figure 2: Schematic representation of ramp metering.

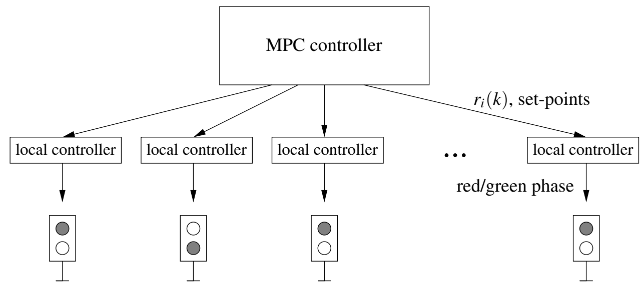

Figure 3: MPC for ramp metering. The MPC controller provides the metering rates as set-points to the local traffic signal controllers, which convert metering rates into appropriate lengths of the red and the green phases.

ing congestion [1, 12]. The way ramp metering limits the traffic density on the freeway is by restricting the number of vehicles that are allowed to enter the freeway through the on-ramp. This can be implemented by installing a traffic signal at the on-ramp as presented in Figure 2. The green period is selected such that only one car is allowed to enter the freeway per red-green cycle.

We use a discrete-time controller, with as control parameter the metering rate, which is defined as follows [3]:

$$r_i(k) = \frac{q_{\max,i}(k)}{Q_{\mathrm{cap},i}} \quad , \tag{1}$$

where k is the sample step, i is the on-ramp index, $q_{\max,i}(k)$ is the maximal number of cars allowed to enter the freeway via on-ramp i, and $Q_{\mathrm{cap},i}$ is the capacity of the ith on-ramp.

Based on the metering rate, a sequence of red and green phases for the traffic signal is calculated. The length of the green phase is selected such that only one vehicle is allowed to enter the freeway during the green phase. The duration of the red phase in between two green phases is adjusted such that the metering rate is met. The calculation of the phase lengths is done by a local controller that uses the metering rate as its set-point and that outputs the red and green phases for the traffic signal as shown in Figure 3.

If the traffic demand $d(k)$ at the on-ramp is larger than the number of cars that is allowed to enter the freeway, a waiting queue occurs at the on-ramp. Hence, we get a trade-off between smooth traffic flows on the freeway and the number of cars that are waiting at the on-ramps. The metering

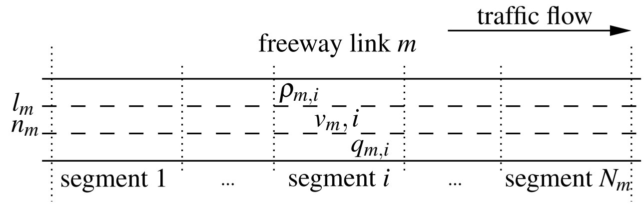

Figure 4: In METANET a link is subdivided into several segments.

rate can be used to optimize the performance of the traffic system under study. A performance measure or cost function that is often used in literature is the total time spent by all the vehicles in the network [12]. This includes the time spent by the vehicles traveling on the mainstream as well as the time spent by the vehicles in the queue at the on-ramp. This cost function makes a trade-off between the waiting time in the queue and the gain in travel time achieved on the mainstream. A more detailed discussion of the choice of the cost function is given in Section 3.2.

3 Model predictive control

We use a model predictive control (MPC) approach [4] to find an optimal metering rate. The optimal metering rates (control signals) are determined by minimizing the cost function (total time spent) over a given prediction horizon, where the predictions of the future behavior of the system are made using a traffic flow model. MPC uses a receding horizon framework in which only the first sample of the calculated metering rates is implemented while the others are discarded and recalculated during the next iteration. Once the first sample is applied to the system, the state (and/or the model parameters) are updated using measurements and next the whole process is repeated with the control and the prediction horizon shifted one sample forward. In this way we obtain an adaptive control strategy that is robust for small changes in the system parameters, noise, and small disturbances and measurement errors.

3.1 Destination dependent traffic flow model

We use the macroscopic traffic flow model for freeway networks as described by Payne [8] and enhanced by Papageorgiou in the METANET model [6, 7]. For the sake of completeness we include a brief description of the METANET model. Note however that the MPC approach is generic so that we could also work with other traffic flow models. In the METANET framework a freeway network consists of links, which join or bifurcate at nodes.

3.1.1 Link equations: A link in a freeway network is a part of the network that connects two nodes. For simulation, link m is subdivided into N_m segments (cf. Figure 4). A typical segment length is 500 meters.

In a freeway network with multiple origins and destinations the traffic in a link can be composed of vehicles with different destinations. In order to be able to pin-point the contributions of the traffic destined to the different destinations we use partial densities and partial flows. The partial density $\rho_{m,i,j}(k)$ is defined as the density induced by the traffic traveling to destination j in the ith segment of link m at simulation step k. The partial flow $q_{m,i,j}(k)$ is defined in a similar way.

The behavior of each of the segments in the freeway links can be described as follows. The law of conservation of the number of vehicles in a segment yields:

$$\rho_{m,i,j}(k+1) = \rho_{m,i,j}(k) + \frac{\Delta T}{n_m l_m}\left[\gamma_{m,i-1,j}(k) q_{m,i-1}(k) - \gamma_{m,i,j}(k) q_{m,i}(k)\right] \quad (2)$$

where $\rho_{m,i,j}(k)$ is the partial density induced by the traffic traveling to destination j in the ith segment of link m at step k, ΔT is the time step, and l_m and n_m are respectively the length and the number of lanes of link m. The composition rate of the density $\gamma_{m,i-1,j}(k)$ (and of the flow) at step k is defined as $\frac{\rho_{m,i,j}(k)}{\rho_{m,i}(k)}$ and $q_{m,i-1}(k)$ is the flow out of segment $i-1$ of link m at step k.

The average speed $v_{m,i}(k+1)$ in segment i of link m at step $k+1$, as described in (3), is the average speed $v_{m,i}(k)$ in the segment altered by three terms accounting for relaxation, convection and anticipation phenomena.

$$\begin{aligned} v_{m,i}(k+1) = {} & v_{m,i}(k) \\ & + \frac{\Delta T}{\tau}\left[V[\rho_{m,i}(k)] - v_{m,i}(k)\right] && \text{Relaxation} \\ & + \frac{\Delta T}{l_j} v_{m,i}(k)\left[v_{m,i-1}(k) - v_{m,i}(k)\right] && \text{Convection} \\ & - \frac{\nu \Delta T[\rho_{m,i+1}(k) - \rho_{m,i}(k)]}{\tau\, l_j[\rho_{m,i}(k) + \kappa]} \quad . && \text{Anticipation} \end{aligned} \quad (3)$$

The relaxation term expresses that vehicles in a freeway segment tend to obtain a desired speed $V[\rho_{m,i}(k)]$. An empirical formula for this average speed-density relationship is

$$V[\rho_{m,i}(k)] = v_{\mathrm{f}} \exp\left(-\frac{1}{a_m}\left(\frac{\rho_{m,i}(k)}{\rho_{\mathrm{cr},j}}\right)^{a_m}\right) \quad , \quad (4)$$

where the free flow speed v_{f} is the average speed the vehicles adopt when the traffic density tends to zero and a_m is a parameter.

The partial flow $q_{m,i,j}(k)$ in segment i of link m induced by traffic for destination j at step k is the product of the traffic density $\rho_{m,i,j}(k)$, the average speed $v_{m,i,j}(k)$ and the number of lanes n_m:

$$q_{m,i,j}(k) = \rho_{m,i,j}(k) v_{m,i,j}(k) n_m. \quad (5)$$

The total flow in a segment is the sum of all partial flows in that segment.

The dynamics of the queue at the on-ramp are described by:

$$w_i(k+1) = w_i(k) + \Delta T \gamma_{0,i}(k)\left[d_i(k) - q_{\mathrm{s},i}(k)\right] \quad (6)$$

where $w_i(k+1)$ is the length of the queue at on-ramp i at step $k+1$, $d_i(k)$ is the demand at the ith on-ramp, $\gamma_{0,i}(k)$ is the composition rate of the demand at the ith on-ramp and $q_{\mathrm{s},i}(k)$ is the service rate of the on-ramp, i.e., the number of cars that is allowed to enter the freeway.

3.1.2 Node equations: Where the link equations describe the state of the traffic in the traffic links, the node equations describe the relations between traffic flows in different links connected to a node. The node equations are important since they describe how traffic flows through the network. In a node, traffic for a destination j will distribute over the links through which destination j can be reached. The splitting rate $\beta^m_{n,j}(k)$ is the fraction of the total traffic flow $Q_{n,j}(k)$ entering node n and destined for j that leaves for destination j through link m.

The following node equations express the relation between the traffic flows in the nodes and describe how the traffic flows are routed through the network.

The total traffic volume entering node n and heading for destination j can be calculated as:

$$Q_{n,j}(k) = \sum_{\mu \in I_n} q_{\mu,N_\mu}(k) \gamma_{\mu,N_\mu,j}(k) \quad , \quad (7)$$

where I_n is the set of all links entering node n, $q_{\mu,N_\mu}(k)$ is the traffic flow leaving the last segment of link μ (i.e., segment N_μ), and $\gamma_{\mu,N_\mu,j}(k)$ is the composition rate.

The total traffic flow leaving node n through link m is given by:

$$q_{m,0}(k) = \sum_{j \in J_m} Q_{n,j}(k) \beta^m_{n,j}(k) \quad , \quad (8)$$

where J_m is the set of reachable destinations through link m, and O_n is the set of links leaving node n.

The composition rate $\gamma_{m,0,j}(k)$ of the inflow into link m is given by:

$$\gamma_{m,0,j}(k) = \beta^m_{n,j}(k) \frac{Q_{n,j}(k)}{q_{m,0}(k)} \quad . \quad (9)$$

At the on-ramp node a special equation holds. The service rate $q_{\mathrm{s},i}(k)$ of the on-ramp is limited by the capacity of the on-ramp $Q_{\mathrm{cap},i}$, by the metering rate $r_i(k)$, and by the traffic density $\rho_{m,1}$ of the first segment on the mainstream. This results in the following equation for the service rate:

$$q_{\mathrm{s},i}(k) = \min\Bigg[d_i(k) + \frac{w_i(k)}{\Delta T}, \; Q_{\mathrm{cap},i} \min\left(r_i(k), \frac{\rho_{\max,m} - \rho_{m,1}(k)}{\rho_{\max,m} - \rho_{\mathrm{cr},m}}\right)\Bigg] \quad , \quad (10)$$

with $\rho_{\max,m}$ the maximal possible density in link m. The metering rate $r_i(k)$ is contained to the interval $[r_{\min,i}, 1]$, where $r_{\min,i} > 0$ is the minimal metering rate for on-ramp i.

The system of equations (2)–(10) allows for a simulation of the freeway network.

3.2 Objective function

The objective function, or the cost function, defines a measure of the performance of a traffic situation in the freeway network. A cost function that is often used in literature is the total time spent (TTS) by all vehicles in the network during a certain period [2, 3]. In this paper we consider the following cost function at sample step k_0:

$$J(k_0) = \sum_{k=k_0}^{k_0+N_\mathrm{p}-1} \Bigg[\sum_{(m,i)\in\mathscr{I}_\mathrm{s}} \rho_{m,i}(k) l_m n_m + \alpha \sum_{o\in\mathscr{I}_\mathrm{o}} w_o(k) + \alpha_\mathrm{ramp} \sum_{i\in I_\mathrm{c}} \big(r_i(k) - r_i(k-1)\big)^2 \Bigg] \Delta T \qquad (11)$$

with $\mathscr{I}_\mathrm{s}$ the set of pairs of indices of the freeway segments and links, $\mathscr{I}_\mathrm{o}$ the set of indices of the on-ramps, and I_c the set of indices of the controlled on-ramps. The cost is calculated over the period $[k_0\Delta T, (k_0+N_\mathrm{p})\Delta T)$ where N_p is the prediction horizon. The exact meaning of the parameter N_p will become clear during the discussion of the computation of the control signals in the next section.

The parameter α in (11) determines the impact of the time spent by vehicles in the queues at the on-ramps. By decreasing α we decrease the contribution of vehicles in on-ramp queues to the total cost $J(k_0)$, thus favoring the vehicles on the mainstream. When optimizing the TTS, the resulting control signal may oscillate. In order to suppress the oscillations, a term proportional to the control variation is added to (11). The parameter α_ramp needs to be tuned such that the oscillations are suppressed to an acceptable level.

3.3 Computing the control signals

When minimizing the cost function, a trade-off between the time spent on the mainstream and the time spent in the queues at the on-ramp is found. The straightforward optimization does not impose a maximal queue length on the queue at the on-ramp. Since the available capacity to store vehicles at the on-ramp is limited and since we want to prevent the on-ramp queue spilling back into the underlying traffic network, we add a constraint on the queue length to the optimization process. The resulting optimization problem to be solved is nonlinear, non-convex and has constraints. This problem can be solved using a (multi-start[1]) sequential quadratic programming method.

In a receding horizon framework, controller performance can be tuned using the prediction horizon and the control horizon. The larger the prediction horizon N_p, the further the controller looks ahead. This allows the controller to foresee certain events (such as an increase or decrease in traffic demand), but it also increases the computational complexity. Since we want to implement ramp metering in a real-time framework, N_p is bounded from above. The control horizon N_c ($N_\mathrm{c} \le N_\mathrm{p}$) determines the time period during which the control signal is allowed to change. After the control horizon, the control signal is taken to be constant. The number of parameters to be optimized is proportional to N_c. Since the computational complexity of the optimization problem increases strongly with the number of parameters to be optimized, N_c is bounded from above as well. The choice of N_p and N_c is based on a trade-off between controller performance and computational complexity.

[1] As MPC uses a receding horizon approach we can use the optimal solution of the previous step (i.e., step k_0-1) as a good initial guess for the optimization of the current step (i.e., step k_0).

4 Anticipative traffic assignment

In a freeway network with multiple routes from the origins to the destinations, drivers have to choose their route in the network. Given the traffic demands for each origin-destination pair and the topology of the network, the traffic has to be assigned to the routes before a simulation can be performed. First, we present a static equilibrium traffic assignment algorithm based on the collective behavior of drivers that assigns the traffic to the different routes. Next, we propose an approach to incorporate the spontaneous rerouting of drivers due to control measures in the simulation.

4.1 Static equilibrium traffic assignment

When traveling in a freeway network, drivers try to find the route that is optimal for themselves. In fact, it seems as if every driver assigns a cost to every alternative route leading to his destination and chooses the one with the smallest cost. Two factors are of importance when deciding which route to choose: the travel time along the route and the length of the route. The importance a driver assigns to these and other components can vary from driver to driver. In what follows, we use the travel time of the route as the cost assigned to that route. Since a route consists of consecutive links connecting the origin with the destination, the cost assigned to a route can be calculated by adding the link costs. Similarly, the link cost $c_m(q_m)$ can be calculated as

$$c_m(q_m) = \sum_{i\in I_m} \frac{l_{m,i}}{v_{m,i}} \qquad (12)$$

where I_m is the set of segments in link m. According to (4) and (12) we see that the link cost is dependent on the traffic density on the freeway. Indeed, when more drivers use a link, the densities in the segments of that link increase and the desired average speeds in the corresponding segments decrease, increasing the travel time (cost) of the link. In order to be able to compute the link costs for a whole freeway network using (12), we need to be able to calculate the average link speeds given a set of link flows. Since static equilibrium traffic assignment implies that the traffic flows in the network are invariant in time, we assume that the traffic flows in the links are in equilibrium. By consequence, we can compute the equilibrium average segment speeds by using equations (2), (3) and (5) of the METANET model with $\rho_{m,i,j}(k+1) = \rho_{m,i,j}(k)$ and $v_{m,i}(k+1) = v_{m,i}(k)$.

Wardrop stated in 1952 [11] that the traffic in a network distributes over the links in such a way that an equilibrium occurs where no individual driver can lower his travel cost by changing routes. In equilibrium all used routes between an origin-destination pair have the same travel cost and nonused routes have a higher travel cost. The resulting equilibrium is called the user optimal equilibrium since it occurs when every driver individually optimizes his route.

There exist several methods to compute the user optimum as defined by Wardrop's principle such as, e.g., the Frank-Wolfe algorithm [10] and the method of the successive averages [9]. We describe the method of the successive averages (MSA) here. The MSA is an iterative static equilibrium traffic assignment method that takes the impact of vehicle densities on the link costs into account through the cost function (12). The algorithm uses link flow vectors $q_{\mathrm{lf}}^{(i)}$, which contain the link flows for all the links in the network at iteration i. The algorithm contains the following steps:

Initialization: $i = 1$, $q_{\mathrm{lf}}^{(i)} = 0$, $\phi = 1$
repeat
Step 1: Calculate costs $c_{\mathrm{lc}}^{(i)}(q_{\mathrm{lf}}^{(i)})$ according to (12)
Step 2: Determine $q_{\mathrm{lf,AON}}^{(i)}$ by the all-or-nothing assignment
Step 3: $q_{\mathrm{lf}}^{(i+1)} = (1-\phi)q_{\mathrm{lf}}^{(i)} + \phi q_{\mathrm{lf,AON}}^{(i)}$
Step 4: $i = i+1$, $\phi = \frac{1}{i}$
until the stopping criterion is reached

In Step 1, the vector $c_{\mathrm{lc}}^{(i)}(q_{\mathrm{lf}}^{(i)})$ containing the links costs associated with the traffic assignment in iteration i is calculated. The link costs are computed based on the vector with the link flows $q_{\mathrm{lf}}^{(i)}$ and equations (2), (3) and (5) of the METANET model as described before. In Step 2, we assign for every origin-destination pair all the traffic to the cheapest route. This is called an all-or-nothing assignment. During the ith iteration we search for every origin-destination pair the cheapest route based on the link cost vector $c_{\mathrm{lc}}^{(i)}(q_{\mathrm{lf}}^{(i)})$. For smaller networks, the cheapest route can be searched exhaustively but for larger networks a more advanced method like Dijkstra's shortest path algorithm is needed. The flows in the links caused by the all-or-nothing assignments for all origin-destination pairs lead to a link flow vector $q_{\mathrm{lf,AON}}^{(i)}$. In Step 3 we calculate a new link flow vector $q_{\mathrm{lf}}^{(i+1)}$ as a linear combination of the previous link flow vector and the link flow vector $q_{\mathrm{lf,AON}}^{(i)}$ resulting from the all-or-nothing assignments in Step 2. The meaning of the parameter ϕ is the following: In iteration i, the value of ϕ is such that the new link flow vector we find is the average of all i link flow vectors — hence the name: the method of successive averages. For the MSA the stopping criterion is in general a maximal number of iterations. It can be proved that the MSA converges to a solution.

4.2 Anticipative traffic assignment

In the previous section we dealt with static equilibrium traffic assignment where the traffic demands were invariant in time. In this section we present anticipative traffic assignment that takes the drivers' experience of the traffic state and their responses to this state into account. This method only uses the experienced traffic state and does not require the traffic demands to be constant. The anticipative traffic assignment is incorporated into the MPC framework for ramp metering in order to allow for the design of control signals that anticipate the drivers' response to the control actions.

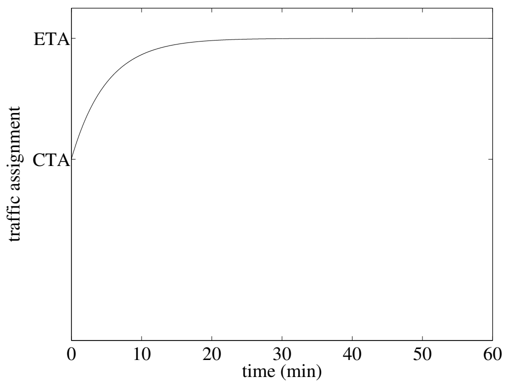

Figure 5: Exponential evolution from the current traffic assignment (CTA) to the equilibrium traffic assignment (ETA) for $\tau_{\mathrm{evol}} = 5$ min.

During their trip in the freeway network, drivers experience the traffic state of the network. Based on the information they gather on the global state of the traffic network, drivers will determine their optimal route. The current state assessment, which is used to determine the optimal route, is based on information on the traffic state in the near past. This process of gathering traffic state information can be modeled as follows: The traffic state in the near past results from the traffic demands and the metering rates in the near past. The fact that the gathering of traffic state information takes some time is modeled by averaging the traffic demands and the metering rates over a time horizon τ_{info}. This way we obtain the information about the traffic state as it is perceived and used by the drivers to determine their optimal route. The parameter τ_{info} needs to be tuned and is influenced by the network dimensions and topology, the availability of information (e.g., radio bulletins) and so on. E.g., a value of 15 minutes could be chosen for τ_{info}.

Drivers use the gathered traffic state information to optimize their route. Using the static equilibrium assignment method presented in Section 4.1 combined with the average values of traffic demands and metering rates, we can compute the equilibrium traffic assignment (ETA) associated with the traffic situation experienced by the drivers. Since we use the average traffic demand, the static equilibrium traffic assignment method will yield acceptable results even in the case of time varying traffic demands. Based on the assignment of the flows to the routes we can compute the splitting rates needed for simulation of the METANET model.

It takes some time before the traffic flows in the freeway network will reorganize according to the ETA. This due to the fact that not all drivers will decide to use the new route immediately. We model this evolution from the current traffic assignment (CTA) to the ETA as an exponential evolution of the splitting rates as shown in Figure 5. The time constant τ_{evol} needs to be tuned such that the evolution from the CTA to the ETA occurs in a realistic time frame. In Figure 5 we see that for $\tau_{\mathrm{evol}} = 5$ min it takes about 30 min for the splitting rates to evolve from their values corresponding to the CTA to the values corresponding to the ETA.

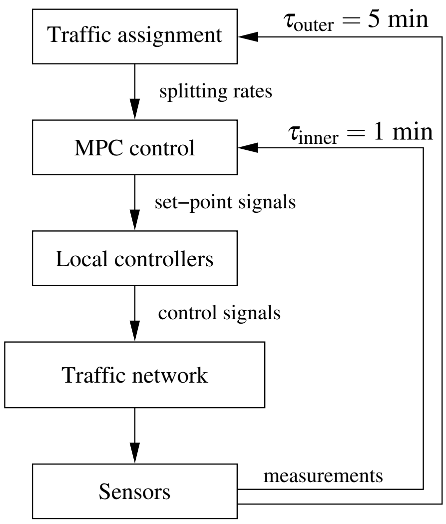

Figure 6: Representation of the overall control strategy.

5 The overall control strategy

In order to obtain an adaptive and anticipative control framework, we now propose an overall control strategy that combines MPC and anticipative traffic assignment. A schematic representation of the overall control strategy is presented in Figure 6. There are two loops in the scheme: an inner loop in which the MPC control signals are computed and applied, and an outer loop in which the anticipative traffic assignment takes place. We now discuss these loops in more detail.

The MPC control module produces control signals in the form of metering rates which are applied to the traffic system. The state of the traffic situation is measured (e.g., every $\tau_{\text{inner}} = 1$ min) and fed back to the MPC module. These measurement are used to update the state and to subsequently start a new optimization over the prediction horizon the traffic states.

During traffic assignment, a prediction of the ETA is made using information of the traffic situation experienced by the drivers. This experienced traffic situation is computed based on the traffic demands and the metering rates as described in Section 4.2. The ETA and the CTA are combined to provide us with a description of the evolution of the splitting rates in time. The evolution of the splitting rates in time is fed to the MPC control module in order to be used during the prediction horizon. By supplying the MPC module with the evolution of the splitting rates from CTA to ETA, the controller is able to take the re-routing behavior of the drivers into account. Since the dynamics of the re-routing are slower than the dynamics of the traffic system near the on-ramps, it suffices to update the traffic assignment at a slower pace than the traffic states (e.g., every $\tau_{\text{outer}} = 5$ min).

6 Conclusions

We have presented ramp metering combined with the MPC framework. As drivers tend to use the cheapest route available, a control action can cause a re-routing of traffic. Hence, an adequate control strategy has to take this re-routing into account. We have proposed a method to take this re-routing into account within the model predictive control framework. In this way, a controller can avoid unwanted re-routing of traffic as a response to control actions. This results in an adaptive, anticipative traffic control approach.

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