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Technical report 03-002

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If you want to cite this report, please use the following reference instead: M. van den Berg, A. Hegyi, B. De Schutter, and J. Hellendoorn, "A macroscopic traffic flow model for integrated control of freeway and urban traffic networks," *Proceedings* of the 42nd IEEE Conference on Decision and Control, Maui, Hawaii, pp. 2774–2779, Dec. 2003.

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* This report can also be downloaded via https://pub.bartdeschutter.org/abs/03_002.html

A Macroscopic Traffic Flow Model for Integrated Control of Freeway and Urban Traffic Networks

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Abstract— We develop a macroscopic model for mixed urban and freeway traffic networks that is particularly suited for control purposes. In particular, we use an extended version of the METANET traffic flow model to describe the evolution of the traffic flows in the freeway part of the network. For the urban network we propose a new model that is based on the Kashani model. Furthermore, we also describe the interface between the urban and the freeway model. This results in an integrated model for mixed freeway and urban traffic networks. This model is especially suited for use in a model predictive traffic control approach.

I. INTRODUCTION

As the number of vehicles grows and the need for mobility increases on a world-wide scale, the frequency and duration of traffic jams in and around major cities and on major freeways increase. In the short term the most effective measures to deal with traffic congestion seem to be a selective construction of new roads — an option which is often not viable due to lack of space and/or budgetary means, or due to environmental or societal requirements, — and a more efficient use of the existing infrastructure and capacity through dynamic traffic management and control. In this paper we will concentrate on the latter option.

For urban traffic networks systems such as UTOPIA-SPOT [1], and SCOOT [2] use an integrated approach that coordinates the operation of several traffic signal set-ups in a city to obtain a smoother flow and/or a better circulation. For freeway traffic networks several authors [3], [4], [5] have considered a coordinated approach in which many different control measures (such as ramp metering, route guidance, variable speed limits, ...) are coordinated on a larger scale, which results in a better overall performance. However, up to now little attention has been paid to integrated control of networks consisting of both urban and freeway roads.

Freeway traffic control measures such as ramp metering often allow a better flow, higher speeds, and a larger throughput on the freeway at the cost of queues at the on-ramp, which may spill back and block urban roads. Vice versa, many cities try to get the vehicles out of the urban road network as soon as possible, thereby displacing the congestion to the neighboring ring roads and freeways. This shift of congestion between urban and freeways and vice versa is often made worse by the fact that in many countries urban, regional, and freeway roads are managed and controlled by different traffic management bodies, each with their own traffic policies and objectives. This situation is certainly not optimal. By considering an integrated approach the performance of the overall network (taking into account the trade-off between the often conflicting objectives and interests of different traffic management bodies) can be significantly improved. Therefore, our goal is to develop an integrated traffic control approach for coordinated control of mixed urban and freeway traffic networks that makes an appropriate trade-off between the performance of the urban and freeway traffic operations and that prevents a shift of problems from urban roads to freeways, and vice versa.

We propose to use a model predictive control (MPC) approach [6], which has already been applied successfully to coordinated control of freeway networks [7], [8]. As MPC uses a model to predict the future evolution of the traffic flow, an important requisite for an MPC-based traffic control approach for mixed urban/freeway traffic networks is a model that describes the evolution of the traffic in the network. Furthermore, the model should present a well-balanced tradeoff between accuracy and computational complexity, as MPC is an on-line control strategy. Since to the authors' best knowledge such models are not yet available, we will develop a macroscopic traffic model for mixed urban/freeway traffic network in this paper. We opt for a macroscopic model that yields a sufficiently accurate description of the evolution of the traffic flows for given traffic demands, traffic conditions, and output restrictions on the one hand, and that can be simulated sufficiently fast on the other hand, so that it can be used in on-line traffic control. In particular, we use an extended version of the METANET [9], [10] traffic flow model to model the freeway traffic. For the urban network we use a modified and extended model that is based on the Kashani model [11]. Furthermore, we also model the interface between the urban and the freeway model. This results in an integrated model for mixed freeway and urban traffic networks. This model is especially suited for use in an MPC-based traffic control approach.

Remark 1.1 As we will explicitly make a difference between the simulation time step T_f for the freeway part of the network, the simulation time step T_u for the urban part of the network, and the controller sample time T_c , we will also use three different counters for the freeway network model

	f	reeway link i	 traffic flow	
			 :	
segment 1	•••	segment <i>i</i>	 segment N _m	

Fig. 1. In the METANET model a freeway link is divided into segments.

(*k*), the urban model (*l*), and the controller (*n*). For the sake of simplicity we assume that T_u is an integer divisor of T_f , and that T_f is an integer divisor of T_c :

$$T_{\rm f} = K T_{\rm u}, \quad T_{\rm c} = L T_{\rm f} = L K T_{\rm u}$$

II. FREEWAY TRAFFIC MODEL

In order to model the traffic flows on the freeway part of the network, we use a slightly modified version of the (destination-independent) METANET model [9], [10], [12]. For the sake of completeness, we will first give a concise description of the METANET model, and afterwards present the extensions.

A. Basic METANET model

The METANET model [9], [10], [12] represents a freeway network as a directed graph with the links corresponding to freeway stretches. Each link has uniform characteristics, i.e. no on-ramps or off-ramps and no major changes in geometry. Where a major change occur in the characteristics of the freeway stretch or in the road geometry (e.g., on-ramp or off-ramp), a node is placed. Each link *m* is divided into N_m segments of length L_m (see Figure 1). A typical link length is 500 m. The state of the traffic in segment *i* of link *m* at time $t = kT_f$ is characterized by

- the traffic density $\rho_{m,i}(k)$ (i.e., the number of vehicles in the segment per lane and per length unit),
- the mean speed $v_{m,i}(k)$,
- the outflow $q_{m,i}(k)$ (i.e., the number of vehicles that leave the segment per time unit),

where k denotes the (freeway) simulation step counter, and $T_{\rm f}$ the time step used for the simulation of the (freeway) traffic flows. A typical value for $T_{\rm f}$ is 10 s [9], [10]. This value should be selected such that $T_{\rm f}$ is smaller than the time it takes a vehicle driving at maximal speed to cross one segment.

The outflow of segment i of link m at simulation step k is

$$q_{m,i}(k) = \rho_{m,i}(k) v_{m,i}(k) \lambda_m \quad . \tag{1}$$

where λ_m denotes the number of lanes of link *m*. The law of conservation of vehicles results in:

$$\rho_{m,i}(k+1) = \rho_{m,i}(k) + \frac{T_{\rm f}}{L_m \lambda_m} (q_{m,i-1}(k) - q_{m,i}(k))$$

The speed update equation contains a relaxation term that expresses that the drivers try to achieve a desired speed $V(\rho)$, a convection term that expresses the speed increase (or decrease) caused by the inflow of vehicles, and an anticipation

term that expresses the speed decrease (increase) as drivers experience a density increase (decrease) downstream:

$$v_{m,i}(k+1) = v_{m,i}(k) + \frac{T_{\rm f}}{\tau} \Big(V \big(\rho_{m,i}(k) \big) - v_{m,i}(k) \Big) + \frac{T_{\rm f}}{L_m} v_{m,i}(k) \big(v_{m,i-1}(k) - v_{m,i}(k) \big) - \frac{\eta T_{\rm f}}{\tau L_m} \frac{\rho_{m,i+1}(k) - \rho_{m,i}(k)}{\rho_{m,i}(k) + \kappa} , \qquad (2)$$

where τ , η and κ are model parameters, and

$$V(\rho_{m,i}(k)) = v_{\text{free},m} \exp\left[-\frac{1}{a_m} \left(\frac{\rho_{m,i}(k)}{\rho_{\text{crit},m}}\right)^{a_m}\right] \quad , \qquad (3)$$

with a_m a model parameter, and where the free-flow speed $v_{\text{free},m}$ is the average speed that drivers assume if traffic is flowing freely, and the critical density $\rho_{\text{crit},m}$ is the density at which the traffic flow becomes unstable.

Origins are modeled with a simple queue model:

$$w_o(k+1) = w_o(k) + T_f(d_o(k) - q_o(k)) \quad , \tag{4}$$

where $w_o(k)$ denotes the queue length at origin o at simulation step k, $d_o(k)$ the demand, and $q_o(k)$ the outflow, which depends on the traffic conditions on the freeway, the capacity of the origin, and the ramp metering rate $r_o(k)^1$:

$$q_o(k) = \min\left[d_o(k) + \frac{w_o(k)}{T_{\rm f}}, Q_{\rm free,o} \min\left(r_o(k), \frac{\rho_{\rm max} - \rho_{\mu,1}(k)}{\rho_{\rm max} - \rho_{\rm crit,\mu}}\right)\right], \quad (5)$$

where $Q_{\text{free},o}$ is the free-flow on-ramp capacity, i.e., the maximal number of vehicles per time unit that can pass the on-ramp under free-flow conditions, ρ_{max} is the maximum density, and μ the index of the freeway link to which the on-ramp is connected.

B. Extensions

In [7], [8] we have proposed some modifications and extensions to the basic METANET model. Since the basic METANET model does not describe the effect of speed limits, we have slightly modified the equation for the desired speed (3) to incorporate speed limits (including a certain degree of non-compliance). The second extension regards the modeling of a mainstream origin, which has a different nature than an on-ramp origin. The third extension describes the different effects of a positive or negative downstream density gradient on the speed (cf. the anticipation term in (2)). For a description of these extensions we refer to the companion paper [13], which can also be found in these proceedings.

Now we introduce another, new extension in order to more accurately model the behavior of the off-ramp outflow

¹The metering rate $r_o(k) \in [0, 1]$ determines the fraction of the flow that is allowed to enter the freeway in simulation step k. The absence of ramp metering corresponds to $r_o(k) = 1$.

when blocking occurs as the usual boundary conditions used in the METANET model do not allow us to describe this phenomenon in a completely satisfactory and consistent way. For the boundary condition at (the last segment of) the offramp link in case of congestion-free operation the following assumption is made for the downstream density:

$$\rho_{\mu,N_{\mu}+1}(k) = \min\left(\rho_{\mu,N_{\mu}}(k), \rho_{\operatorname{crit},\mu}\right)$$

where μ is index of the off-ramp link. In case of congestion the outflow is limited by the maximal outflow $Q_{\max,\mu}(k)$ (this is a boundary condition, see also Section IV-B):

$$q_{\mu,N_{\mu}}(k) = \min\left(q_{\text{basic},\mu,N_{\mu}}(k), Q_{\max,\mu}(k)\right) \quad (6)$$

where the q_{basic} denotes the outflow obtained using the basic METANET equations (cf. Section II-A). When the outflow is limited by congestion the speed of the last segment needs to be recalculated as follows.

$$\begin{split} v_{\mu,N_{\mu}}(k) &= \\ \begin{cases} v_{\text{basic},\mu,N\mu}(k) & \text{if } q_{\text{basic},\mu,N_{\mu}}(k) < Q_{\max,\mu}(k) \\ v_{\text{basic},\mu,N_{\mu}}(k) \frac{Q_{\max,\mu}(k)}{q_{\text{basic},\mu,N_{\mu}}(k)} & \text{otherwise}, \end{cases} \end{split}$$

where v_{basic} denotes the speed obtained using the basic METANET equations.

III. URBAN TRAFFIC MODEL

Several authors have already developed models to describe traffic flows in urban traffic networks [11], [14], [15]. Recall that we will use the model for on-line traffic control, and that we have to select a model that offers an appropriate trade-off between accuracy and computational complexity. Therefore, we opt for a macroscopic urban traffic model instead of a microscopic model (in which the trajectory of each vehicle in the network is modeled individually). Furthermore, the model should satisfy the following requirements:

- it should be able to model both light and heavy traffic,
- it should contain horizontal queues: Congestion in urban areas can result in long queues. Often these queues become long compared with the buffer capacities (i.e., the lengths) of the streets that connect the intersections, so that they cannot be neglected as they influence the time it takes to travel from one intersection to the next, and as they can even block the upstream intersection.
- it should describe blocking effects: When a queue blocks an intersection, the traffic is not able to cross it. This effect should be described by the model in order to get an accurate estimation of the evolution of the traffic flows.

The first and the second feature are present in many macroscopic urban traffic models such as the cell transmission model [15], the Kashani model [11], and the INTUC model [14]. As the cell transmission model is too computationally intensive for on-line control and describes too many functions which are not needed, we will base our model on the other two models, and in particular, the Kashani model.

Our model is based on the Kashani model, but has the following extensions:

- First of all, we use horizontal queues.
- We take into account the blocking effect that arises when cars are waiting before an intersection. This results in a constraint on the number of cars that can enter the given lane, i.e., it limits the number of cars that can depart from the upstream intersection. This constraint depends on the length of the link and the queue which is already waiting in the link.
- The Kashani model uses the cycle time of the traffic signal set-up as the simulation time step. Such a large simulation time step poses problems when we want to model the blocking effect accurately. Furthermore, we also want to allow different cycle times for different traffic signal installations (as this results in more degrees of freedom for the controller), we will use a fixed simulation step T_u for the urban network that is independent of the cycle times of the traffic signal installations². The urban simulation step T_u should be selected such that a vehicles driving at maximal speed cannot drive through a link within T_u time units. As the urban traffic links are usually shorter than the freeway traffic links, this results in values for T_u of 2 to 5 s.

In the traffic flow model the state variables are the length of the queues that are waiting at an intersection or any other flow restriction, such as a reduction in the number of lanes, an obstacle, a pedestrian crossing, etc. For the sake of conciseness, we will only explicitly describe the evolution of the queues at controlled intersections, which yield the most complex situations in an urban traffic network. The equations for the other cases (such as uncontrolled intersections and flow restrictions) can easily be derived from the controlledintersection equations.

A queue is modeled as follows. For the sake of simplicity, we assume that at an intersection the cars going to the same destination move into the correct lane, so that they do not block the traffic flows going to other destinations³. For each lane (or destination) we will construct a separate queue (with a queue length denoted by *x*). Furthermore, we assume that T_u is selected such that it takes a car driving at maximal speed more than at T_u time units to drive from one intersection to another (see also (7) below). However, cars arriving at the end of a queue in simulation period $[lT_u, (l+1)T_u)$ are allowed to cross the intersection in the same period (provided that they

²In fact, the cycle times will be integer multiples of $T_{\rm u}$. Furthermore, as we have to couple the urban traffic model with the freeway traffic model, $T_{\rm u}$ has to be an integer divisor of the freeway simulation step $T_{\rm f}$.

 $^{^{3}}$ If there is only one lane, the blocking could be modeled by assuming that the cars going to different destinations depart as long as there is space in all the destination links and that as soon as one destination link is full the entire flow stops.

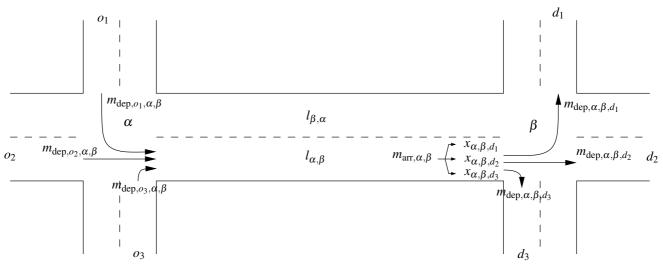


Fig. 2. A link connecting two traffic-signal-controlled intersections.

have green, that there is enough space in the destination link, and that there are no other restrictions).

In order to describe the evolution of the queue lengths, we first define some extra variables (see also Figure 2):

- l : simulation step counter for the urban trafficmodel, $<math display="block">O_{\sigma} : set of origins of intersection \sigma,$ $D_{\sigma} : set of destinations of intersection \sigma,$ $l_{o,d} : link connecting origin o and destination d,$ $x_{o,\sigma,d}(l) : queue length at simulation step l of the queue$ $at intersection <math>\sigma$ for the traffic going from origin o to destination d,
- $\beta_{o,\sigma,d}(l)$: relative fraction of the traffic arriving from origin *o* at intersection σ that wants to go to destination *d*,
- $m_{\operatorname{arr},o,\sigma}(l)$: number of cars arriving at the (end of the) queue in link $l_{o,\sigma}$, during $[lT_{\mathrm{u}},(l+1)T_{\mathrm{u}}),$
- $m_{\operatorname{arr},o,\sigma,d}(l)$: number of cars arriving at the (end of the) queue in link $l_{o,\sigma}$ and going to destination d,
- $m_{\text{dep},o,\sigma,d}(l)$: number of cars departing from link $l_{o,\sigma}$ and going to destination d,
- $S_{o,d}(l)$: available storage space of link $l_{o,d}$ at time $t = lT_u$ (i.e., the buffer capacity of the link minus the number of cars that are already present at time $t = lT_u$),
- $g_{o,\sigma,d}(l)$: boolean value indicating whether the traffic signals at intersection σ for the traffic going from *o* to *d* are green (1) or red (0)⁴ in the period $[lT_c, (l+1)T_c)$.

Consider link $l_{\alpha,\beta}$ (cf. Figure 2). For each $d_i \in D_\beta$ the number of cars leaving link $l_{\alpha,\beta}$ for destination d_i in the period $[lT_u, (l+1)T_u)$ is given by

$$m_{\text{dep},\alpha,\beta,d_i}(l) =$$

$$\begin{cases} 0 & \text{if } g_{\alpha,\beta,d_i}(l) = 0\\ \max\left(0,\min(x_{\alpha,\beta,d_i}(l) + m_{\operatorname{arr},\alpha,\beta,d_i}(l), \\ S_{\beta,d_i}(l), T_f Q_{\operatorname{free},\alpha,\beta,d_i})\right) & \text{if } g_{\alpha,\beta,d_i}(l) = 1, \end{cases}$$

where $Q_{\text{free},\alpha,\beta,d_i}$ denotes the number of cars wanting to go from *o* to *d* via σ that can pass the intersection per time unit under free-flow conditions.

The traffic arriving at the end of the queue in link $l_{\alpha,\beta}$ is given by the traffic entering the link via the upstream intersection delayed by the time needed to drive from the upstream intersection to the end of the queue in the link; to this extent $m_{\text{arr},\alpha,\beta}$ is updated as follows:

$$m_{\operatorname{arr},\alpha,\beta}^{\operatorname{new}}(l+\delta_{\alpha,\beta}(l)) = m_{\operatorname{arr},\alpha,\beta}^{\operatorname{old}}(l+\delta_{\alpha,\beta}(l)) + \sum_{o_i \in O_{\alpha}} m_{\operatorname{dep},o_i,\alpha,\beta}(l)$$

where before starting the simulation all values $m_{arr,\alpha,\beta}^{old}(\cdot)$ are initialized to 0, and where⁵

$$\delta_{\alpha,\beta}(l) = \operatorname{ceil}\left(\frac{S_{\alpha,\beta}(l)L_{\mathrm{av}}}{v_{\mathrm{free},\alpha,\beta}}\right) \quad , \tag{7}$$

with $v_{\text{free},\alpha,\beta}$ the average speed of the traffic in link $l_{\alpha,\beta}$ in free-flow conditions, L_{av} the average vehicle length, and where ceil(*r*) with *r* a real number denotes the smallest integer larger than or equal to *r*. The fraction of the arriving traffic in link $l_{\alpha,\beta}$ destined for destination $d_i \in D_\beta$ is

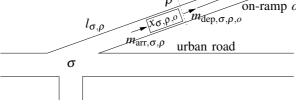
$$m_{\operatorname{arr},\alpha,\beta,d_i}(l) = \beta_{\alpha,\beta,d_i}(l) m_{\operatorname{arr},\alpha,\beta}(l)$$
.

The new queue lengths are given by the old queue lengths plus the arriving traffic minus the departing traffic

$$x_{\alpha,\beta,d_i}(l+1) = x_{\alpha,\beta,d_i}(l) + m_{\operatorname{arr},\alpha,\beta,d_i}(l) - m_{\operatorname{dep},\alpha,\beta,d_i}(l)$$

⁵As we assume that it takes a car driving at maximal speed more than at least one simulation time step $T_{\rm u}$ to drive from one intersection to another we have to round up the value so that $\delta_{\alpha,\beta}(l)$ is always larger than 0.

⁴The amber phase can be modeled as being partially green and red.



freeway

Fig. 3. Overview of an on-ramp.

for each $d_i \in D_\beta$. The new available storage stage depends on the number of cars that enter and leave the link in the period $[lT_u, (l+1)T_u)$:

$$S_{\alpha,\beta}(l+1) = S_{\alpha,\beta}(l) - \sum_{o_i \in O_{\alpha}} m_{\text{dep},o_i,\alpha,\beta}(l) + \sum_{d_i \in D_{\beta}} m_{\text{dep},\alpha,\beta,d_i}(l) .$$

IV. INTERFACE BETWEEN FREEWAY AND URBAN MODEL

The urban part and the freeway part of the network are coupled via on-ramps and off-ramps. Now we present the formulas that describe the interaction between the urban traffic model and the freeway traffic model at these ramps. The two main problems that have to be dealt with are the different simulation time steps for the urban and the freeway model, and the transformation of urban traffic flows into demands and boundary conditions for the freeway model.

A. On-ramps

Let o be the METANET index of an on-ramp, and let σ be the index of the urban intersection to which the on-ramp is connected (cf. Figure 3). As far as the urban model is concerned the on-ramp imposes a flow restriction that results in a queue that builds up. Let ρ be the index of the flow restriction in the urban network. The arrivals at the queue are given by the urban model but the departures are determined by the METANET model. Recall that we have assumed that the METANET simulation time $T_{\rm f}$ is chosen small enough to make sure that no car can enter and leave the link in the same simulation step, so the departures depend on the available vehicles on the on-ramp and the density on the freeway.

Consider the freeway simulation step *k* and the corresponding urban simulation step $l = \frac{T_{\rm f}}{T_{\rm u}}k = Kk$. The evolution of the METANET queue length $w_o(k)$ and the urban queue length $x_{\sigma,\rho,o}(l)$ can be determined as follows:

- First, we determine the on-ramp departure flow $q_o(k)$ in the period $[kT_f, (k+1)T_f)$ using formula (5).
- Next, we assume that these departures are spread out evenly over the equivalent urban simulation period $[lT_u, (l+K)T_u)$. Hence, for each $\ell \in \{l, \dots, l+K-1\}$ we have $m_{\text{dep},\sigma,\rho,o}(\ell) = \frac{q_o(k)}{K}$.

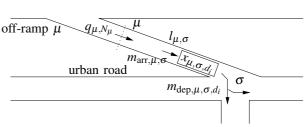


Fig. 4. Overview of an off-ramp.

- The arrivals and the queue lengths at link $l_{\sigma,\rho}$ can now be computed using the equations for the urban traffic model given in Section III.
- Once the queue length $x_{\sigma,\rho,o}(l+K)$ has been computed we set $w_o(k+1) = x_{\sigma,\rho,o}(l+K)$. It is easy to verify that this is equivalent to (4).

B. Off-ramps

Consider an off-ramp link μ in the METANET model, and assume that the off-ramp connects to intersection σ in the urban network (cf. Figure 4). The flow which wants to leave the freeway off-ramp is limited by the available space in the (urban) link $l_{\mu,\sigma}$. This available space depends on the length of the link, the queue waiting on it, and the traffic that is going to leave the link during the next freeway simulation period. Now we assume either that the link $l_{\mu,\sigma}$ is sufficiently long or that $T_{\rm f}$ is sufficiently small so that no car can enter and leave the off-ramp link in one freeway simulation period. Hence, the number of cars departing from the (urban) link $l_{\mu,\sigma}$ in one freeway simulation period does not directly depend on the number of cars entering the link $l_{\mu,\sigma}$ during that period. Therefore, the number of cars departing from intersection σ can be computed first, and afterwards the number of cars entering the link $l_{\mu,\sigma}$ can be computed.

Consider the freeway simulation step k and the corresponding urban simulation step $l = \frac{T_f}{T_u}k = Kk$. In order to describe the evolution of the queue lengths in link $l_{\mu,\sigma}$ we use the following procedure.

- First, we determine the departures from link $l_{\mu,\sigma}$ at the intersection σ in the period $[lT_u, (l+K)T_u)$ using the urban traffic flow model of Section III.
- Next, we determine the maximal allowed off-ramp outflow $Q_{\max,\mu}(k)$ in $[kT_{\rm f}, (k+1)T_{\rm f})$ based on the available storage space in the link $l_{\mu,\sigma}$ at the end of the period, i.e., at time $t = (k+1)T_{\rm f} = (l+K)T_{\rm u}$. We have⁶

$$Q_{\max,\mu}(k) = \frac{S_{\mu,\sigma}(l) + \sum_{\ell=l}^{l+K-1} \sum_{d_i \in D_{\sigma}} m_{\deg,\mu,\sigma,d_i}(\ell)}{T_{\mathrm{f}}}$$

⁶Note that at this stage we have not yet computed the number of arrivals in the link $l_{\mu,\sigma}$ for the urban simulation steps $\ell = l, ..., l + K - 1$ so that the value of $S_{\mu,\sigma}(l+K)$ is also not known yet.

The effective outflow $q_{\mu,N_{\mu}}(k)$ of the off-ramp is then given by (6).

- Now the METANET model can be updated for simulation step *k*+1.
- For the urban traffic model, we assume that the outflow of the off-ramp is distributed evenly over the period $[kT_{\rm f}, (k+1)T_{\rm f}) = [lT_{\rm u}, (l+K)T_{\rm u})$ such that

$$m_{\operatorname{arr},\mu,\sigma}(\ell+\delta_{\mu,\sigma}(\ell))=rac{q_{\mu,N_{\mu}}(k)}{K} ext{ for } \ell=l,\ldots,l+K-1.$$

Now we can also compute the corresponding urban queue lengths $x_{\mu,\sigma,d_i}(\ell)$ for $\ell = l+1, \ldots, l+K$ and each $d_i \in D_{\sigma}$ using the urban traffic model of Section III.

V. OVERALL MODEL FOR MIXED FREEWAY/URBAN TRAFFIC NETWORKS

If we combine the model equations presented in Sections II–IV for the freeway network, the urban network, and their interface respectively, we get a model for the mixed urban and freeway network. This model can be used as the prediction model for a model predictive control (MPC) scheme [6] for integrated and coordinated traffic flow control for mixed urban and freeway networks.

VI. CONCLUSIONS

We have proposed a new integrated model for mixed urban and freeway traffic networks. The model has been developed for use in a model predictive control approach, and offers an appropriate trade-off between accuracy and computational complexity (as it has to be used for on-line and faster-thanreal-time prediction). For the freeway part of the network we use an extended version of the macroscopic METANET traffic flow model. For the urban part we have proposed a new model, which has the Kashani model at its core. Furthermore, we have also described the coupling between the freeway and the urban part of the model.

Topics for future research include: thorough assessment of the trade-off between accuracy and computational complexity of the model for various set-ups and various choices of simulation steps, calibration of the model parameters based on measured traffic data, application of the proposed approach to real-life case-studies, and refinement of the models.

ACKNOWLEDGMENTS

Research supported by the Traffic Research Centre (AVV) of the Dutch Ministry of Transport, Public Works and Water Management, and the NWO-CONNEKT project AMICI.

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