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Model Predictive Control for Mixed Urban and Freeway Networks

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Abstract. We consider coordinated traffic control for networks consisting of both urban roads and freeways. One of the main problems that has to be addressed when designing traffic control strategies for such networks is that we should prevent a shift of problems from the urban network to the freeway network (or vice versa) due to the applied control strategy.

First, we develop an integrated model to describe the evolution of the traffic situation in mixed urban and freeway networks. For the freeways we use the METANET macroscopic traffic flow model. For the urban traffic a new model is developed that is based on an earlier model by Kashani. Furthermore, we also provide model equations that describe the connection between the two models via on-ramps and off-ramps. Next, we present a model predictive control framework for mixed urban and freeway networks. The control objective used in this paper is the total time spent by all vehicles in the network, and the control measures are the urban traffic signals (but the method can easily be extended to include other objectives and/or control measures). Finally, we illustrate our approach with a synthetic case study that captures the essential elements of a mixed urban and freeway network.

1 INTRODUCTION

In this paper we consider traffic control for networks containing both urban roads and freeways.

Freeway traffic control measures such as ramp metering often allow a better flow, and higher speeds and throughput on the freeway at the cost of queues at the on-ramp, which may spill back and block urban roads. On the other hand, many cities try to get the vehicles out of the urban road network as soon as possible, thereby displacing the congestion to the neighboring ring roads and freeways. This shift of congestion between urban and freeways, and vice versa, is often made worse by the fact that in many countries urban, regional, and freeway roads are managed and controlled by different traffic management bodies, each with their own traffic policies and objectives. However, the situation sketched above is certainly not optimal. By considering an integrated and coordinated approach, the performance (taking into account the trade-off between the often conflicting objectives and interests of different traffic management bodies) of the overall network can be significantly improved. Therefore, our goal is to develop an integrated traffic control approach for coordinated control of mixed urban and freeway traffic networks that makes an appropriate trade-off between the performance of the urban and freeway traffic operations and that prevents a shift a problems from urban roads to freeways, and vice versa.

For urban traffic networks, systems such as UTOPIA/SPOT and SCOOT use an integrated approach that coordinates the operation of several traffic signal set-ups in a city to obtain a smoother flow and/or a better circulation. For freeway traffic networks several authors (*4, 7, 11, 12*) have considered a coordinated approach in which many different control measures (such as ramp metering, route guidance, variable speed limits, etc.) are coordinated on a larger scale, which results in a better overall performance. However, up to now little attention has been paid to integrated control of networks consisting of both urban and freeway roads.

We propose to use a model predictive control (MPC) approach (*2, 9*), which has already been successfully applied to coordinated control of freeway networks (*1, 5*). As MPC requires a model to predict the future evolution of the traffic flow, a first requirement is a model that describes the evolution of the traffic in a mixed urban/freeway traffic network. In this paper we will develop a macroscopic traffic model for networks containing both urban roads and freeways. We opt for a macroscopic model that yields a sufficiently accurate description of the evolution of the traffic flows for given traffic demands, traffic conditions, and output restrictions on the one hand, and that can be simulated sufficiently fast — so that it can be used in on-line traffic control — on the other hand. In particular, we use an extended version of the METANET (*10, 13*) traffic flow model to model the freeway traffic. For the urban network we use a modified and extended version of the Kashani model (*6*). Furthermore, we also model the interface between the urban and the freeway model. This results in an integrated model for mixed freeway and urban traffic networks, which is especially suited for use in a model predictive traffic control approach. We propose such an approach, and we illustrate it using a simple synthetic case study.

2 MODEL FOR FREEWAY TRAFFIC: METANET

To model the evolution of the traffic flows on the freeway, we have selected the METANET macroscopic traffic flow model developed by Messmer and Papageorgiou (*10, 13*). This model provides a balanced trade-off between accuracy and computational complexity (Note that as the MPC strategy discussed in Section 6 involves on-line simulation and optimization, we require a model that can be simulated sufficiently fast).

For the sake of completeness, we will briefly present the (destination-independent) METANET model below, with a focus on the parts of the model that are required to model the case study of Section 7 and to interpret the simulation results. For a more detailed explanation we refer to (*10, 13*) and (*1, 5*), where some modifications to the METANET model are proposed. Note that for simplicity we assume that the control measures do not influence the route choice of the drivers such that we can to use the destination-independent model. However, the approach can also be extended so as to include route choice.

The METANET model describes the network as a directed graph, with links and nodes. A link corresponds to (parts of) freeway stretches that have uniform characteristics, like maximum capacity and number of lanes. Each link f is divided into N_f segments with length L_f. In the METANET model the following variables are used:

f	:	index of the freeway link
m	:	segment index
T_f	:	time step of the freeway simulation (h; a typical value is about $10\,\mathrm{s} \approx 0.0028\,\mathrm{h}$)
k	:	freeway time step counter
N_f	:	number of segments in freeway link f
λ_f	:	number of lanes in freeway link f
L_f	:	length of the segments in link f (km)
τ, κ, a_f, η	:	constant parameters reflecting street geometry, vehicle characteristics, drivers' behavior, etc.
$\rho_{\mathrm{crit},f}f$	:	critical density (veh/km/lane) in link f
ρ_max	:	maximum density (veh/km/lane)
$v_{\mathrm{free},f}f$	:	speed that the vehicles tend to drive at under free flow conditions (km/h/lane) in link f
Q_f	:	capacity of the segments of freeway link f (veh/h)
$\rho_{f,m}(k)$	:	density of segment m of freeway link f at time $t = kT_\mathrm{f}$ (veh/km/lane)
$v_{f,m}(k)$	:	speed in segment m of freeway link f at time $t = kT_\mathrm{f}$ (km/h)
$w_o(k)$	:	length of the queue waiting on on-ramp o at time $t = kT_\mathrm{f}$ (veh)
$q_{\mathrm{arr,orig},f}(k)$	:	flow that enters freeway link f on the mainstream origin in the time interval $[kT_\mathrm{f}, (k+1)T_\mathrm{f})$ (veh/h)
$q_{\mathrm{dep},f,m}(k)$	:	flow leaving segment m of freeway link f in $[kT_\mathrm{f}, (k+1)T_\mathrm{f})$ (veh/h)
$d_{\mathrm{origin},f}(k)$	:	flow arriving at the origin of freeway link f in $[kT_\mathrm{f}, (k+1)T_\mathrm{f})$ (veh/h).

Remark 1: As we will explicitly make a difference between the simulation time step T_f for the freeway part of the network, the simulation time step T_u for the urban part of the network, and the controller sample time T_c, we will also use three different counters for the freeway network model (k), the urban model (l), and the controller (z). For the sake of simplicity, we assume that T_u is an integer divisor of T_f, and that T_f is an integer divisor of T_c:

$$T_\mathrm{f} = T\,T_\mathrm{u}, \quad T_\mathrm{c} = L\,T_\mathrm{f} = LT\,T_\mathrm{u} \ ,$$

with T and L integers. $\diamond$

In the METANET model the traffic state in segment m of link f at time $t = kT_\mathrm{f}$ is described with the macroscopic variables density $\rho_{f,m}(k)$, speed $v_{f,m}(k)$, and flow $q_{\mathrm{dep},f,m}(k)$. These variables evolve as follows:

$$\rho_{f,m}(k+1) = \rho_{f,m}(k) + \frac{T_\mathrm{f}}{L_f \lambda_{f,m}} \left[q_{\mathrm{dep},f,m-1}(k) - q_{\mathrm{dep},f,m}(k)\right] \tag{1}$$

$$q_{\mathrm{dep},f,m}(k) = \rho_{f,m}(k) v_{f,m}(k) \lambda_f \tag{2}$$

$$v_{f,m}(k+1) = v_{f,m}(k) + \underbrace{\frac{T_\mathrm{f}}{\tau}\left(V(\rho_{f,m}(k)) - v_{f,m}(k)\right)}_{\text{relaxation}} + \underbrace{\frac{T_\mathrm{f}}{L_f} v_{f,m}(k)\left[v_{f,m-1}(k) - v_{f,m}(k)\right]}_{\text{convection}} - \underbrace{\frac{\eta T_\mathrm{f}[\rho_{f,m+1}(k) - \rho_{f,m}(k)]}{\tau L_f [\rho_{f,m}(k) + \kappa]}}_{\text{anticipation}} \ , \tag{3}$$

where $V(\rho_{f,m}(k))$ is given by

$$V(\rho_{m,f}(k)) = v_{\mathrm{free},f} \exp\left[-\frac{1}{a_f}\left(\frac{\rho_{f,m}(k)}{\rho_{\mathrm{crit},f}}\right)^{a_f}\right] \ .$$

Origins are described with a flow that enters and a queue waiting to enter:

$$q_{\mathrm{dep},f,\mathrm{origin}}(k) = \min\left[d_{\mathrm{origin},f}(k) + \frac{w_f(k)}{T_\mathrm{f}},\ Q_f \frac{\rho_{\max} - \rho_{f,1}(k)}{\rho_{\max} - \rho_{\mathrm{crit},f}}\right]$$
$$w_f(k+1) = w_f(k) + T_\mathrm{f}(d_{\mathrm{origin},f}(k) - q_{\mathrm{arr,orig},f}(k)) \ .$$

Now we describe the node equations. Traffic enters a node n from freeway links $f \in O_n$ according to

$$q_{\mathrm{arr},n}(k) = \sum_{f \in O_n} q_{\mathrm{dep},f,N_f}(k) \ ,$$

where O_n is the set of freeway links entering node n, and $q_{\mathrm{arr},n}(k)$ the total flow arriving at the node. The traffic leaves a node n towards freeway links $f \in D_n$ according to

$$q_{\mathrm{dep},n,f}(k) = \beta_{n,f}(k) q_{\mathrm{arr},n}(k) \ ,$$

with D_n the set of freeway links leaving the node, and $\beta_{n,f}(k)$ the turning rate from node n towards link f in the period $[kT_\mathrm{f}, (k+1)T_\mathrm{f})$.

3 URBAN MODEL

Several authors have already developed models to describe traffic flows in urban traffic networks (*3, 6, 8*). Recall that we will use the model for on-line traffic control, and that we have to select a model that offers an appropriate trade-off between accuracy and computational complexity.

Our model to describe the traffic in the urban parts of the network is based on the Kashani model (*6*), but it has the following extensions:

- We use horizontal queues, which allows us to take into account the blocking effect that arises when a link is full of vehicles and other vehicles from an upstream intersection cannot enter anymore.
- We use turning-direction-dependent queues, which correctly model the queue dynamics if one turning direction is blocked and the other directions are free.
- The Kashani model uses the cycle time of the traffic signal set-up as the simulation time step. Such a large simulation time step poses problems when we want to model the blocking effect accurately. Furthermore, as we also want to allow different cycle times for different traffic signal installations, we will use a fixed simulation step T_u (typically 1 to 5 s) for the urban network that is independent of the cycle times of the traffic signal installations.

The new model is described using the following parameters (see also Figure 1):

s, σ	:	index of the intersection
T_u	:	time step used for the urban simulation (h)
l	:	urban time step counter
$o_{s,i}$	:	origin number i of intersection s
O_s	:	set of origins of intersection s

$d_{s,j}$	:	destination number j of intersection s
D_s	:	set of destinations of intersection s
$x_{o_i,s,d_j}(l)$	:	queue length at time $t = lT_\mathrm{u}$ in number of vehicles of the queue at intersection s, for traffic that goes from origin o_i to destination d_j
$l_{s,\sigma}$	:	link connecting intersections s and σ
$\beta_{o_i,s,d_j}(l)$	:	relative fraction of the traffic arriving from origin o_i on intersection s that wants to go to destination d_j in the time interval $[lT_\mathrm{u}, (l+1)T_\mathrm{u})$
$L_{s,\sigma}$	:	length of link $l_{s,\sigma}$ in number of vehicles
$L_{\mathrm{km},s,\sigma}$	:	length of link $l_{s,\sigma}$ (km)
$S_{s,\sigma}(l)$	:	available free space of link $l_{s,\sigma}$ at time $t = lT_\mathrm{u}$ in number of vehicles (i.e., the buffer capacity $L_{s,\sigma}$ minus the number of vehicles that are already present at time $t = lT_\mathrm{u}$)
$m_{\mathrm{arr},s,d_j}(l)$	:	number of vehicles arriving at the tail of the queue in link l_{s,d_j} during $[lT_\mathrm{u}, (l+1)T_\mathrm{u})$
$m_{\mathrm{arr},o_i,s,d_j}(l)$	:	number of vehicles coming from intersection o_i arriving at the tail of the queue in link l_{s,d_j} during $[lT_\mathrm{u}, (l+1)T_\mathrm{u})$
$m_{\mathrm{dep},o_i,s,d_j}(l)$	:	number of vehicles departing from link $l_{o_i,s}$ towards destination d_j in $[lT_\mathrm{u}, (l+1)T_\mathrm{u})$
$m_{\mathrm{dep},s,d_j}(l)$	:	number of vehicles departing from intersection s towards link l_{s,d_j} in $[lT_\mathrm{u}, (l+1)T_\mathrm{u})$
$g_{o_i,s,d_j}(l)$	:	indicates whether the traffic sign at intersection s for the traffic going from o_i to d_j is green (1) or red (0) during $[lT_\mathrm{u}, (l+1)T_\mathrm{u})$
Q_{o_i,s,d_j}	:	capacity of intersection s for traffic arriving from o_i and turning to d_j (veh/s)
$v_{s,\sigma}$	:	mean speed for the urban traffic in link $l_{s,\sigma}$ (km/h)
$\delta_{s,\sigma}(l)$	:	time required to reach the end of the queue waiting in link $l_{s,\sigma}$ at time $t = lT_\mathrm{u}$
L_vehicle	:	average length of the vehicles (m)
o	:	index of freeway node connected to on-ramp or off-ramp
$w_{s,o,n}(l)$	:	queue length in number of vehicles on on-ramp o coming from intersection s waiting to depart towards freeway node n at time $t = lT_\mathrm{u}$.

The new model is formulated as follows. The traffic leaving the link $l_{o_i,s}$ towards destination d_j is given by

$$m_{\mathrm{dep},o_i,s,d_j}(l) = \begin{cases} 0 & \text{if } g_{o_i,s,d_j}(l) = 0, \\ \min\left(x_{o_i,s,d_j}(l) + m_{\mathrm{arr},o_i,s,d_j}(l), S_{s,d_j}(l), T_\mathrm{u} Q_{o_i,s,d_j}\right) & \text{if } g_{o_i,s,d_j}(l) = 1. \end{cases}$$

The free space S_{s,d_j} in link l_{s,d_j} is equal to the number of vehicles that can enter the link. The free space is an implicit constraint on the number of vehicles that can depart towards each link, $m_{\mathrm{dep},o_i,s,d_j}$, and it can never be larger than the link length. It is computed as

$$S_{s,d_j}(l+1) = S_{s,d_j}(l) - m_{\mathrm{dep},s,d_j}(l) + \sum_{o_i \in O_s} m_{\mathrm{dep},o_i,s,d_j}(l) \ . \tag{4}$$

The total flow towards one destination consists of several flows from different origins. These different flows do not have to have the same value because not all the queues from which they are coming have the same length and not all incoming flows have the same priority to enter the link. To illustrate how the effective values of $m_{\mathrm{dep},o_i,s,d_j}(l)$ can be computed we assume there are two origins, and so two queues from which vehicles want to drive into the same link. Let $m_{\mathrm{dep,int},1}(l)$ and $m_{\mathrm{dep,int},2}(l)$ denote the number of vehicles wishing to enter the link l_{s,d_j} from respectively origin 1 and origin 2. If we assume without loss of generality that $m_{\mathrm{dep,int},1}(l) \leq m_{\mathrm{dep,int},2}(l)$, then the effective values for $m_{\mathrm{dep},1}(l)$ and $m_{\mathrm{dep},2}(l)$ can be computed as follows:

- if $m_{\mathrm{dep,int},1}(l) + m_{\mathrm{dep,int},2}(l) \leq S_{s,d_j}(l)$, then

$$m_{\mathrm{dep},1}(l) = m_{\mathrm{dep,int},1}(l) \quad \text{and} \quad m_{\mathrm{dep},2}(l) = m_{\mathrm{dep,int},2}(l) \ .$$

- if $m_{\text{dep,int},1}(l) + m_{\text{dep,int},2}(l) \geq S_{s,d_j}(l)$, then

$$\begin{cases} m_{\text{dep},1}(l) = m_{\text{dep,int},1}(l) \text{ and } m_{\text{dep},2}(l) = S_{s,d_j}(l) - m_{\text{dep,int},1}(l) & \text{if } m_{\text{dep,int},1}(l) \leq \frac{1}{2}S_{s,d_j}(l), \\ m_{\text{dep},1}(l) = m_{\text{dep},2}(l) = \frac{1}{2}S_{s,d_j}(l) & \text{if } m_{\text{dep,int},1}(l) \geq \frac{1}{2}S_{s,d_j}(l). \end{cases}$$

The extension to a link with more queues with vehicles waiting to enter it is straightforward.
The traffic arriving at link l_{s,d_j} can be computed as

$$m_{\text{dep},s,d_j}(l) = \sum_{o_i \in O_s} m_{\text{dep},o_i,s,d_j}(l) \ .$$

These vehicles drive from the beginning of the link l_{s,d_j} towards the tail of the queue waiting on the link. This gives a time delay $\delta_{s,d_j}(l)$:

$$\delta_{s,d_j}(l) = \text{ceil}\left(\frac{S_{s,d_j}(l)\, L_{\text{vehicle}}}{v_{s,d_j}}\right) \ , \tag{5}$$

where $\text{ceil}(x)$ with x a real number denotes the smallest integer larger than or equal to x. The traffic arriving at the tail of the queue should be added to the traffic that arrived in the link in earlier time steps but needed more time to reach the tail of the queue. This results in:

$$m_{\text{arr},s,d_j}(l + \delta_{s,d_j}(l))_{\text{new}} = m_{\text{arr},s,d_j}(l + \delta_{s,d_j}(l))_{\text{old}} + m_{\text{dep},s,d_j}(l) \ .$$

The traffic reaching the tail of the queue in link l_{s,d_j} divides itself over the sub-queues according to the turning rates $\beta_{o_i,s,d_j}(l)$. The number of vehicles arriving at the end of each turning-direction-dependent sub-queue is then given by:

$$m_{\text{arr},o_i,s,d_j}(l) = \beta_{o_i,s,d_j}(l) m_{\text{arr},o_i,s}(l) \ .$$

Finally, the sub-queue lengths are updated as follows:

$$x_{o_i,s,d_j}(l+1) = x_{o_i,s,d_j}(l) + m_{\text{arr},o_i,s,d_j}(l) - m_{\text{dep},o_i,s,d_j}(l) \ .$$

4 ON-RAMPS AND OFF-RAMPS

Both the urban model and the freeway model have now been presented. The next step is to make the connection between the two models. This connection consists of on-ramps and off-ramps.

Recall that we assume that the freeway time step is an integer multiple of the urban time step (cf. Remark 1):

$$T_{\text{f}}/T_{\text{u}} = T \ ,$$

with T is a positive integer. Hence, at a given time $t = lT_{\text{u}} = kT_{\text{f}}$ the urban time counter l and the freeway counter are related by $l = Tk$.

4.1 On-ramps

Consider an on-ramp o that connects intersection s of the urban network to node n of the freeway network. The traffic that enters the on-ramp from the urban network is given by $m_{\text{dep},s,n}(l)$. This traffic has a delay given by $\delta_{s,n}(l)$, and is determined similarly as in equation (5).
The vehicles arrive at the tail of the on-ramp queue. The queue length $w_{s,o,n}(l)$ is computed as

$$w_{s,o,n}(l+1) = w_{s,o,n}(l) + m_{\text{arr},s,o,n}(l) - m_{\text{dep},s,o,n}(l) \ .$$

The number of departures at the front of the on-ramp queue depends on the available space on the freeway, which space depends on the density on the first segment of freeway f downstream of node n. This results in a maximum flow that can leave the on-ramp:

$$q_{\text{dep},\max,o,n}(k) = \begin{cases} Q_f \left(1 - \dfrac{\rho_{f,1}(k) - \rho_{\text{crit},f}}{\rho_{\max} - \rho_{\text{crit},f}}\right) & \text{if } \rho_{f,1}(k) > \rho_{\text{crit},f} \text{ ,} \\ Q_f & \text{otherwise.} \end{cases}$$

The flow $q_{\text{dep},o,n}(k)$ that enters the freeway is then given by

$$q_{\text{dep},o,n}(k) = \min\left(\frac{1}{T_\text{f}}\left[w_{s,o,n}(kT) + \sum_{l=kT}^{(k+1)T-1} m_{\text{arr},s,o,n}(l)\right],\ q_{\text{dep},\max,o,n}(k)\right) \ .$$

This flow should be translated into the number of vehicles that leaves the on-ramp. This is done by distributing the flow equally over the urban time step:

$$m_{\text{dep},s,o,n}(l) = \frac{q_{\text{dep},o,n}(k)T_\text{f}}{T} \qquad \text{for } l = Tk, ..., T(k+1) - 1 \ .$$

The free space $S_{s,o}(l)$ is also computed using equation (4).

4.2 Off-ramps

Consider the off-ramp o that connects freeway node n to urban intersection s. When it is assumed that no vehicle can enter and leave the link in one time freeway step, the departing traffic does not depend on the arriving traffic. Therefore, the departing traffic from intersection s can be computed first, and afterwards the traffic entering the link $l_{o,s}$ can be computed.

The flow leaving the freeway cannot be larger than allowed by the free space on the off-ramp. This free space depends on the length of the off-ramp, on the queue currently waiting on it, and on the traffic that is going to leave the link $l_{o,s}$ during the period $[kT_\text{f}, (k+1)T_\text{f})$. The flow that wants to enter the off-ramp is a fraction of the flow on the freeway:

$$q_{\text{dep},\text{demand},n,s}(k) = \beta_{n,s}(k) q_{\text{dep},f,N_f}(k) \ ,$$

where $\beta_{n,s}(k)$ denotes the turning fraction. This flow is not always able to enter the off-ramp, due to the maximum capacity of the off-ramp, $Q_{n,o,s}$, and the free space on the off-ramp, $S_{o,s}$. This free space in fact varies over the time interval $[kT_\text{f}, (k+1)T_\text{f})$, as vehicles are leaving at the front of the queue during the time interval, and so the free space grows. This results in the following expression for the actual flow that arrives at the off-ramp from the freeway:

$$q_{\text{dep},n,o}(k) = \min\left(q_{\text{dep},\text{demand},n,s}(k), Q_{n,o,s}, \frac{1}{T_\text{f}}\left[S_{o,s}(l) + \sum_{\ell=l}^{l+T-1} \sum_{d_j \in D_s} m_{\text{dep},o,s,d_j}(\ell)\right]\right) \ .$$

The flow entering the off-ramp is translated into the number of vehicles per urban time step:

$$m_{\text{dep},o,s}(l) = \frac{q_{\text{dep},n,o}(k)T_\text{f}}{T} \qquad \text{for } l = kT + 1, \ldots, (k+1)T \, .$$

This traffic undergoes a delay $\delta_{o,s}(l)$ and then arrives at the tail of the queue in the urban network.

A constraint for the leaving flow in METANET can be implemented by adjusting the flow of the incoming traffic. The flow is computed using equation (2). A way to influence the flow is changing the speed that is used to compute this flow. This speed can be adapted as follows:

$$v_{f,N_f}(k)_{\text{new}} = \begin{cases} v_{f,N_f}(k)_{\text{old}} & \text{if } q_{\text{dep,demand},n,s}(k) \leq q_{\text{dep},n,o}(k), \\ v_{f,N_f}(k)_{\text{old}} \dfrac{q_{\text{dep},n,o}(k)}{q_{\text{dep,demand},n,s}(k)} & \text{otherwise,} \end{cases}$$

where $v_{f,N_f}(k)_{\text{old}}$ is the value originally computed using equation (3). The density of the off-ramp is computed with:

$$\rho_{\text{off},o}(k) = \frac{L_{n,s} - S_{n,s}((k+1)T - 1)}{L_{\text{km},n,s}} \ . \tag{6}$$

5 OVERALL MODEL

If we combine the model equations presented in Sections 2–4 for the freeway network, the urban network, and their interface respectively, we get a model for the mixed urban and freeway network.

Note that to be able to compute all the variables, some attention should be payed to the order in which they are determined. We will now briefly discuss the order in which the equations should be processed. For the current time $t = kT_{\text{f}} = lT_{\text{u}}$ all the variables are assumed to be known. These variables are: density, speed, flows, and origin queue lengths for the freeways, queue lengths, free space and arriving vehicles for the urban network, and queue lengths and free space for the ramps. To compute the values of the variables for the next time step, we apply the following computation order:

1. Simulate the urban traffic (with the on-ramp outflows and off-ramp inflows excluded) for the urban time steps $l, ..., l + T - 1$. This also gives the arrivals on the on-ramps and the traffic leaving the off-ramps, which makes it possible to compute the free space on the off-ramps.

2. Compute the on-ramp traffic. The amount of traffic that will enter the freeway from the on-ramps, $q_{\text{dep},o,n}(k)$ is distributed evenly over the whole freeway time step, and used to compute the evolution of the queue length on the on-ramp.

3. Compute the off-ramp traffic. The traffic that is able to enter the off-ramp is computed based on the traffic that wants to enter it and the amount of free space that is available at the end of the period, i.e., at urban time step $l = (k + 1)T - 1$.

4. Now the freeway traffic can be simulated. For the next time step (i.e., the $(k + 1)$st) the densities and speeds are determined. The flow $q_{\text{dep},n,o}(k + 1)$ that wants to enter the off-ramp is computed.

This order of computing makes it possible to simulate the whole network without redundant computations and predictions.

6 CONTROL STRATEGY

6.1 Model predictive control

We will apply a model predictive control (MPC) strategy for coordinated traffic control of mixed urban and freeway networks. MPC (*2, 9*) is an on-line optimization-based controller design procedure that has its roots in the process industry. In general, MPC offers the following features and advantages, which are also relevant for traffic control:

- MPC is a model-based controller design procedure that can easily handle multi-input multi-output processes.
- It is an easy-to-tune method: in principle only a few parameters have to be tuned.
- It can handle constraints on the inputs and the outputs of the process in a systematic way during the design and the implementation of the controller.
- MPC can handle structural changes, such as sensor or actuator failures, and changes in system parameters or structure, by adapting the model and by using a moving horizon approach, in which the model and the control strategy are regularly updated.

In (*1, 5*), we have already extended MPC to coordinated control of freeway networks while retaining the advantages mentioned above. Below we will describe how MPC can be used for the integrated control of mixed urban/freeway traffic networks using the integrated urban/freeway traffic model of Sections 2–4 as the prediction model.

The goal of MPC is to find the control signal that minimizes the cost function over a given prediction period. The cost function should give an indication for the performance of the system. To compute the control signal that minimizes the cost function, the MPC approach uses a model to predict of the behavior of the traffic. We use the model described in Sections 2–4 as the prediction model. Note, however, that the MPC approach is generic so that we could also work with other traffic flow models.

The MPC approach proceeds as follows. Assume we are at time $t = zT_\mathrm{c}$ where T_c is the controller time step (typically 1 to 5 min). Now a model of the system is used to predict the behavior of the system for the period $t = [zT_\mathrm{c}, (z + N_\mathrm{p})T_\mathrm{c})$ defined by the prediction horizon N_p. When the optimal control signal is determined, the first step of it is applied to the system. Then a new situation arises, and the whole process is started again, with the horizons shifted one step ahead. This is called the receding horizon principle.

6.2 Control signal, cost function, and constraints

The control signal contains the offsets of the phases of each intersection, the durations of the green times, and the cycle time. In a separate control module the offsets and durations of the green times will be translated into the binary signals g_{o_i,s,d_j}.

We choose the total time spent (TTS) as cost function because it can easily be computed for the urban part as well as for the freeway part. Note, however, that the MPC approach works equally well for other objective functions (or (weighted) combinations of objective functions). To compute the TTS the number of vehicles in each link, $n_{\mathrm{vehicles},s,\sigma}$, is required:

$$n_{\mathrm{vehicles},s,\sigma}(l) = L_{s,\sigma} - S_{s,\sigma}(l) \;\;,$$

where σ can be any intersection connected to intersection s. The number of vehicles must be computed for all the urban links, on-ramps and off-ramps.

Assume we are at time $t = z_0T_\mathrm{c}$. The TTS will be computed over a period $[z_0T_\mathrm{c}, (z_0 + N_\mathrm{p})T_\mathrm{c})$. Define l_0 and k_0 such that $z_0T_\mathrm{c} = l_0T_\mathrm{u} = k_0T_\mathrm{f}$, and define $l_{0,\mathrm{end}}$ and $k_{0,\mathrm{end}}$ such that $(z_0 + N_\mathrm{p})T_\mathrm{c} = (l_{0,\mathrm{end}} + 1)T_\mathrm{u} = (k_{0,\mathrm{end}} + 1)T_\mathrm{f}$.

The TTS in the urban network in the period $[z_0T_\mathrm{c}, (z_0 + N_\mathrm{p})T_\mathrm{c})$ is then given by:

$$\mathrm{TTS}_\mathrm{urban}(z_0) = T_\mathrm{u} \sum_{l=l_0}^{l_{0,\mathrm{end}}} \left(\sum_{(s,\sigma)\in I} n_{\mathrm{vehicles},s,\sigma}(l) \;+ \sum_{(s,o)\in R_\mathrm{on}} n_{\mathrm{vehicles},s,o}(l) \;+ \sum_{o\in O} n_{\mathrm{vehicles,origins},o,}(l) \right) +$$

$$T_{\mathrm{f}} \sum_{k=k_0}^{k_{0,\mathrm{end}}} \sum_{(o,s)\in R_{\mathrm{off}}} n_{\mathrm{vehicles},o,s}(k) \ ,$$

where I the set of all links (s,σ) in the urban network, R_{on} the set of links (s,o) connected to the on-ramps, R_{off} the set of links (o,s) connected to the off-ramps, and O the set of all origins in the urban network.

The TTS in the freeway part of the network is computed using the density of the segments:

$$\mathrm{TTS}_{\mathrm{freeway}}(z_0) = T_{\mathrm{f}} \sum_{k=k_0}^{k_{0,\mathrm{end}}} \sum_{f\in F} \left(L_f \lambda_f \sum_{m\in M_f} \rho_{f,m}(k) + \sum_{o\in O_f} n_{\mathrm{vehicles,origin},f,o}(k) \right) \ ,$$

where F is the set of freeway links in the network, M_f the set of segments of freeway link f, and O_f the set of origins of freeway link f.

The two above formulas together give the TTS for the entire network. Two positive weighting factors α_1, α_2 are added to give more or less importance to one of the two parts:

$$\mathrm{TTS}(z_0) = \alpha_1 \mathrm{TTS}_{\mathrm{freeway}}(z_0) + \alpha_2 \mathrm{TTS}_{\mathrm{urban}}(z_0) \ .$$

Furthermore, we can impose constraints such as maximum queue lengths at intersections, on-ramps or off-ramps, minimum and maximum green times, etc.

7 CASE STUDY

In order to illustrate the model and the MPC control approach presented above we have selected a simple test network (see Figure 2), that contains some essential elements of mixed urban and freeway networks. The test network consists of a two-way freeway with two on-ramps and two off-ramps. Furthermore, there are two urban intersections, which are connected to the freeway and to each other. Between these intersections and the freeways there are some crossing roads, where there is only crossing that does not turn into other directions (e.g., pedestrian traffic, bicycles, etc.).

The traffic that arrives at the origins is given as a flow, with a demand $d_{\mathrm{origin},s}(k) = 1000$ veh/h for urban origins and $d_{\mathrm{origin},f}(k) = 3600$ veh/h for freeway origins. At the beginning of the simulation the network is completely empty, so all the traffic enters it from the origins.

In order to obtain a preliminary assessment of the MPC approach, two simulations are considered:

1. A simulation using a fixed control scheme, in which the offset is zero and the green percentage of all urban intersections is 50%.

2. A simulation using MPC. To show the effect of MPC, the results will be compared with the results of the first simulation.

7.1 Fixed-time control

The first simulation is done with a fixed-time controller. This simulation will be used to illustrate the features of the extended urban traffic model and of the overall model.

The results for the freeway are shown in Figure 3. Figure 3(a) shows the density on one of the freeways. At the beginning the freeway is empty. During the first 400 s the freeway fills up slowly. This causes the triangle at the left-hand side of Figure 3(a). Then a regular density is reached on the whole freeway, until at 800 s an off-ramp connected to the third segment becomes full. Some traffic that wants to turn into the city stays on the freeway, causing a higher density. When some vehicles leave the off-ramp, a few vehicles can leave the freeway, causing a somewhat lower density. This happens several times after each other with waves as a result. The plot of the speed, Figure 3(b), also shows the effect of the vehicles driving onto the freeway at the beginning. When the off-ramp gets full, the higher density occurs, which leads to a lower speed. Moreover, since the density varies near the filled off-ramp, the speed also varies during the rest of the simulation.

The flow on the off-ramps is shown in Figure 4. One of the off-ramps is full after 1000 s. Then the variations in the flow can clearly be seen.

In the urban network there are four interesting kinds of queues, which are shown in Figure 5. The first two are origin queues. The first origin queue clears during the green time, the second queue does not clear, resulting in a longer queue building up. The sinusoidal variations in the arriving traffic can be seen clearly. The third and fourth plots show queue lengths of the queues in some links in the network. The traffic comes from another intersection and arrives in platoons. The queue in Figure 5(c) can in fact clear during the green time. After 1400 s this queue becomes shorter. This happens because there are three sub-queues waiting in this link, which together cannot be longer than the total link length. When one of these sub-queues becomes very long there is less space for the other two sub-queues so they stay shorter. Plot 5(d) shows a queue in a link that cannot clear during the green time. The traffic comes from two other intersections, and arrives in two platoons. This can be seen by the two horizontal parts when the queues build up. After 1200 s the queue is full, and no traffic can enter the link until some vehicles leave the link at the front side.

The TTS in this situation is 642.3 veh·h.

7.2 Model Predictive Control

When we apply MPC to the network of the case study, the change due to the control scheme can be seen the best in the urban network. The traffic is held waiting at the origins, so all the traffic in the network itself can drive faster. The queues at the origins build up, while the queues on the link are shorter than before. When MPC is used, the queues vary more because the control scheme changes during the simulation. This results in varying sizes of platoons, and different timing between the intersections.

When MPC is applied the TTS becomes 593.1 veh·h, which implies a performance improvement of about 8 % compared to the fixed-time control scheme.

8 CONCLUSIONS

We have proposed a new, extended model for urban traffic that is based on Kashani's model, but that has the following additional features: horizontal queues, a shorter time step, and destination-dependent queues, which results in a more accurate description of the urban traffic. The urban model was combined with the METANET freeway traffic flow model, and with a model that described the interaction between the urban and the freeway model. This resulted in an overall model for mixed urban and freeway traffic networks. Next, we have used this model as a basis for a model predictive control (MPC) approach for coordinated traffic control of mixed urban and freeway traffic networks. The model and the control approach have been illustrated via a simple case study, for which MPC control resulted in a reduction with about 8 % of the total time spent with respect to fixed-time control.

Topics for future research include: thorough assessment of the trade-off between accuracy and computational complexity of the model for various set-ups and choices of simulation steps and control parameters, calibration of the model parameters based on measured traffic data (a first step could be a comparison to available integrated microscopic models such as CORSIM), refinement of the models, development of a decentralized MPC approach for larger networks, and application of the proposed approach to real-life case-studies.

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LIST OF FIGURES

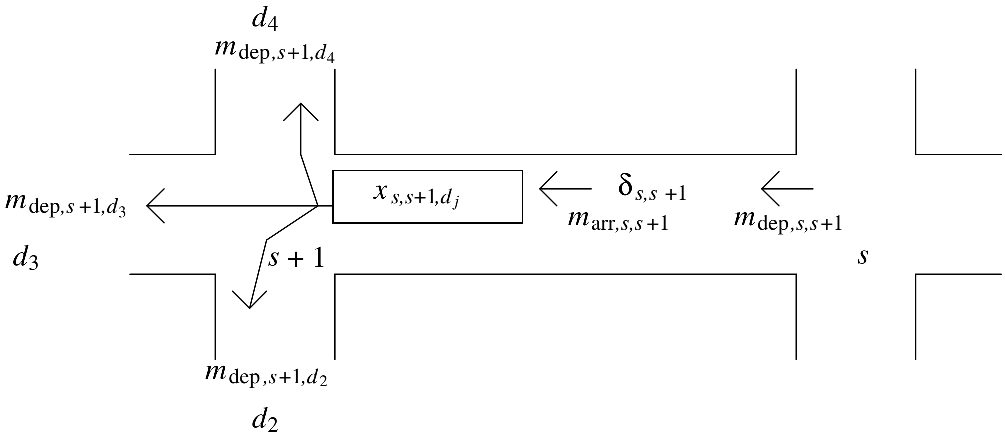


FIGURE 1 Variables used in the urban model.

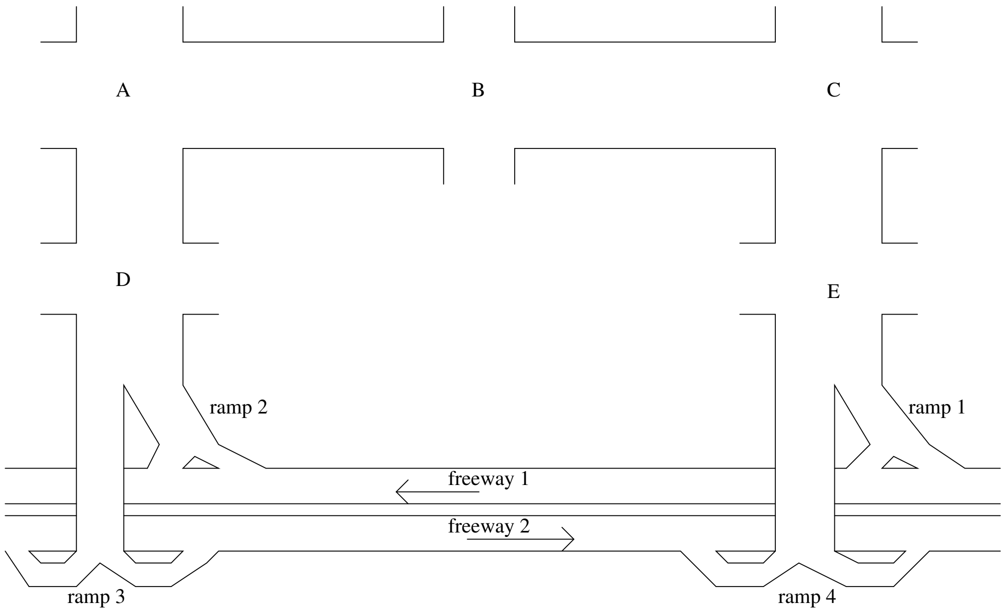
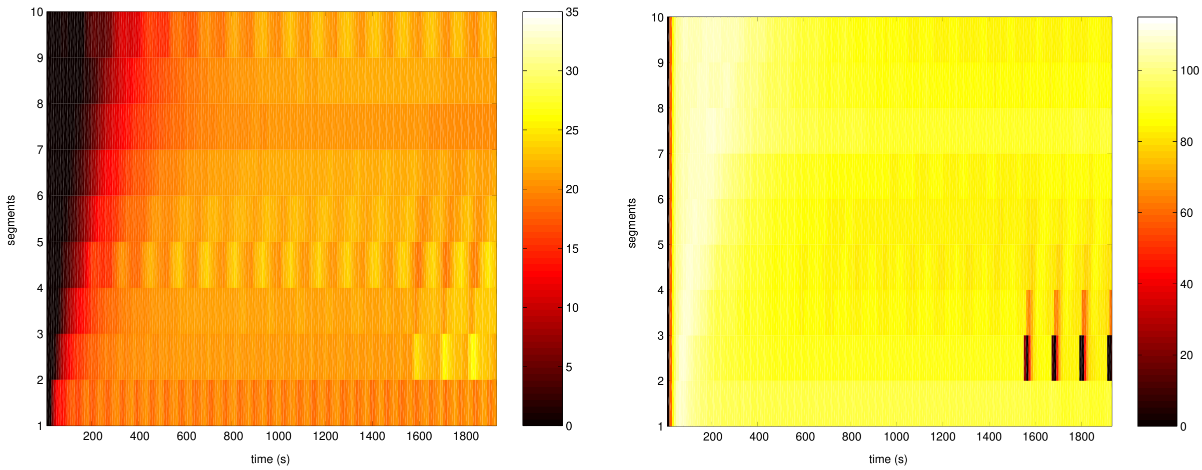


FIGURE 2 Mixed urban/freeway network used in the case study.



(a) Density (veh/km) on the freeway.

(b) Speed (km/h) on the freeway.

FIGURE 3 Freeway density and speed for fixed-time control.

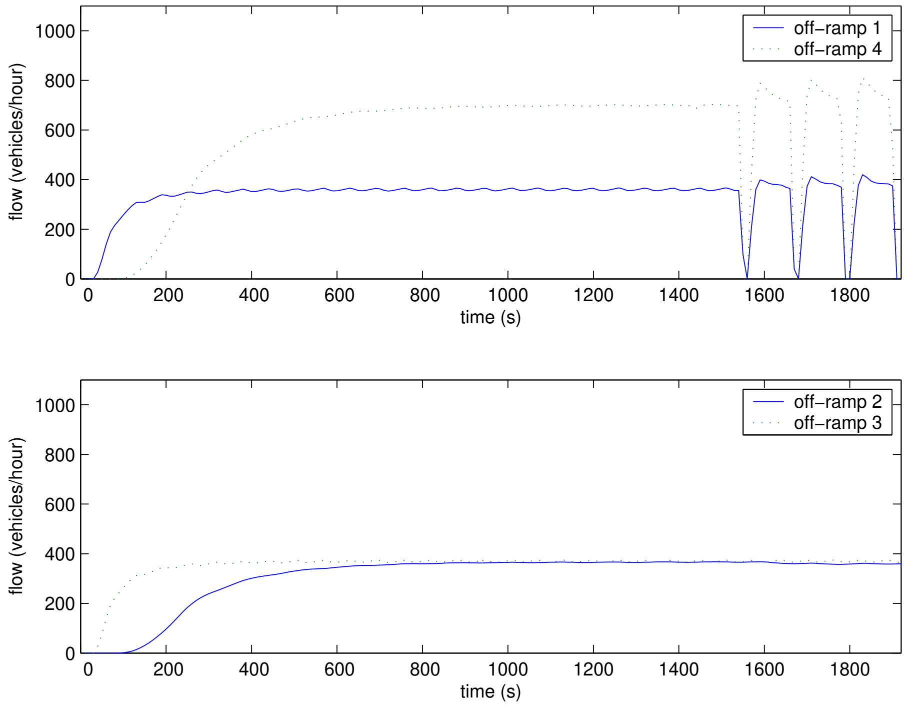
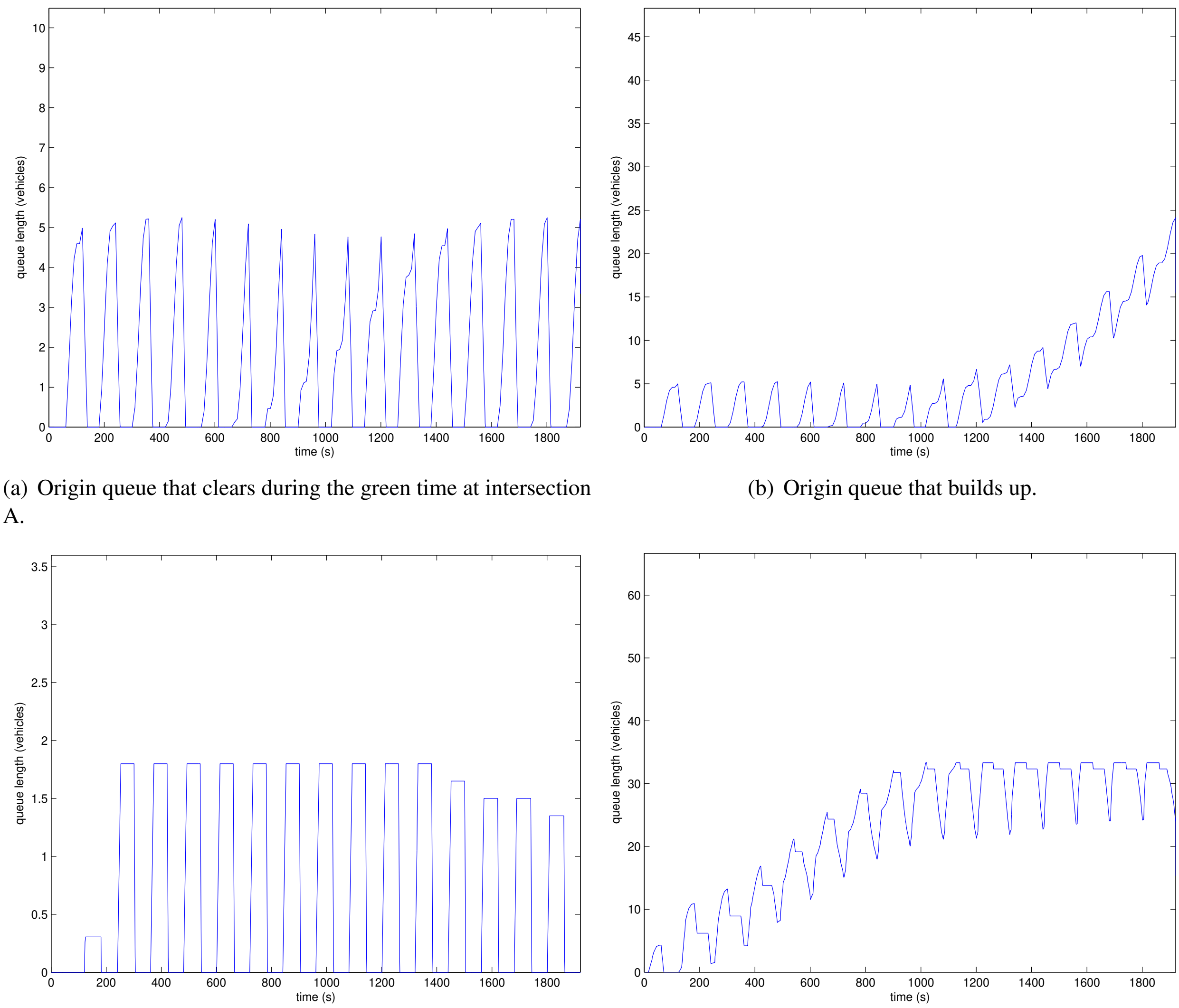


FIGURE 4 Flow entering the off-ramps for fixed-time control.



(a) Origin queue that clears during the green time at intersection A.

(b) Origin queue that builds up.

(c) Queue that clears during the green time at intersection C.

(d) Queue that reaches maximum queue length on link C–B.

FIGURE 5 Urban queue lengths for fixed-time control.