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Technical report 04-008

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If you want to cite this report, please use the following reference instead:
G. Schullerus, V. Krebs, B. De Schutter, and T. van den Boom, “Input signal design for identification of max-plus-linear systems,” *Automatica*, vol. 42, no. 6, pp. 937–943, June 2006. doi:10.1016/j.automatica.2006.01.025

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Input signal design for identification of max–plus–linear systems ⋆

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Abstract

Max–plus–linear systems efficiently describe the dynamics of event time sequences of a class of discrete event systems. The present contribution addresses the problem of designing adequate input signals for state space identification of max–plus–linear systems. It is shown that the input signal design problem can be rewritten as a set of upper bound constraints and therefore solved using an existing algorithm. This input signal design method allows to incorporate additional objectives and constraints, e.g. minimum or maximum input event separation, time order constraints, etc., which are desirable or even required for the input signals and the resulting process behavior.

Key words: Max–plus–linear systems, Parameter estimation, Input signal design, Identification, Discrete event systems.

1 Introduction

Max–plus–linear systems describe the dynamics of event time sequences of a class of discrete event systems by linear equations in a particular algebra, the so called max–plus algebra (Cuninghame-Green 1979, Baccelli *et al.* 1992). The structural equivalence of these equations to conventional discrete time systems has been used to adapt various well known concepts from control engineering to this system class (Cofer and Garg 1996, Menguy *et al.* 2000*a*, Menguy *et al.* 2000*b*, De Schutter and van den Boom 2001, Cottenceau *et al.* 2001). The application of any of these methods requires a process model obtained by theoretical or physical modeling and / or identification algorithms.

To identify a parametric model of a system, first an algorithm for parameter estimation and second, a method for input signal design is needed. The parameter estimation problem for max–plus–linear systems has been solved by estimating the parameters of an ARMA model (Gallot *et al.* 1997) or the impulse response (Menguy 1997, Menguy *et al.* 2000*a*, Caileanu 2001), by determining state space models using either the system's Markov parameters (De Schutter and De Moor 1995) or minimizing a prediction error based on input–output data (De Schutter *et al.* 2002) or the measurement of the complete state vector (Caileanu 2001). The second problem, i.e. the input signal design, is not addressed in these publications but it is shown in (Gallot *et al.* 1997, Menguy *et al.* 2000*a*), that the system parameters are in general overestimated. The input signal design problem for max–plus–linear systems is addressed in (Schullerus *et al.* 2003, Schullerus 2004) where it is also shown that the problem of overestimation can be overcome by ad-

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equately designed input signals. However, the methods introduced so far either do not allow to incorporate additional constraints on these signals and the resulting system behavior or they provide a general solution with a high computational burden (Schullerus *et al.* 2003). Therefore, we propose a new solution to the input signal design problem where additional constraints due to safety or performance requirements on the input and output signals can be incorporated and the computational burden is much less than in the approach first introduced in (Schullerus *et al.* 2003).

The next section briefly reviews the basic notions of max–plus–linear systems (for details see (Cuninghame-Green 1979, Baccelli *et al.* 1992)). The parameter estimation algorithm and the question on whether and under which circumstances an estimated parameter is equal to its corresponding true value is then discussed. Section 4 introduces the input signal design problem, discusses new sufficient conditions for the existence of a solution, and presents a new method to solve this problem. Finally, this method is illustrated by identifying a max–plus–linear model for a manufacturing cell.

2 Max–plus–linear systems

We consider discrete event systems where the evolution of the events is governed by synchronization effects and where no structural alternatives occur. To describe the behavior of these systems the time instant when event e_i occurs for the k–th time is denoted by the "dater" $x_i(k)$ and the input event times are given by $u_j(k)$. The evolution of the event times $x(k) \in \mathbb{R}^n_{\max}$ and the output event times $y(k) \in \mathbb{R}^{n_y}_{\max}$ depending on the input event times $u(k) \in \mathbb{R}^{n_u}_{\max}$, where $\mathbb{R}_{\max} = \mathbb{R} \cup \{-\infty\}$, can be modeled by the equations

$$x(k+1) = A \otimes x(k) \oplus B \otimes u(k+1) \; , \qquad (1)$$

$$y(k) = C \otimes x(k) \; , \qquad (2)$$

where $A \in \mathbb{R}^{n\times n}_{\max}$, $B \in \mathbb{R}^{n\times n_u}_{\max}$, and $C \in \mathbb{R}^{n_y\times n}_{\max}$. The operators $\oplus$ and $\otimes$ are called the addition and multiplication operators of the max–plus algebra and are defined by $x \oplus y = \max(x, y)$, $x \otimes y = x + y$, $\forall\, x, y \in \mathbb{R}_{\max}$. The max–plus power will be denoted in this paper by $a^{\otimes^k}$. The neutral elements of max–plus addition and max–plus multiplication are $\varepsilon \overset{\text{def}}{=} -\infty$ and 0, respectively. Note that ε is absorbing with respect to $\otimes$. The max–plus algebraic matrix addition and multiplication are defined similar to the conventional algebra: $\forall\, P, Q \in \mathbb{R}^{n\times p}_{\max}$: $(P \oplus Q)_{ij} = P_{ij} \oplus Q_{ij}$, $\forall\, P \in \mathbb{R}^{n\times p}_{\max}$ and $\forall\, Q \in \mathbb{R}^{p\times q}_{\max}$: $(P \otimes Q)_{ij} = \bigoplus_{k=1}^{p} (P_{ik} \otimes Q_{kj})$.

3 Parameter identification

The solution of a parameter identification problem consists of two parts: First, an algorithm to determine estimates for the parameters from given measurements and second a method for experiment design. The aim of experiment or input signal design is to influence the measurements by choosing input signals in such a way that the measurements contain as much information as possible about the parameters of the system under consideration (Goodwin and Payne 1977, Ljung 1999). In the sequel, we will formulate the parameter estimation problem and illustrate its solution using already existing results and concepts from the literature. Since the obtained estimated values are in general only upper bounds for the true parameters, we then establish a link between the equality of an estimated parameter to its corresponding true value and the properties of the measurement set. This is then the starting point for the input signal design method presented in Section 4.

The parameter estimation problem described subsequently is based on the assumption that the internal structure of the system is known. It can be described by structure matrices defined as follows (Lunze 1997):

Definition 1 *The structure matrix S_P of a matrix $P \in \mathbb{R}^{n\times m}_{max}$ is an element of $\mathbb{R}^{n\times m}_{max}$. For all $i = 1, \ldots, n$, and $j = 1, \ldots, m$, if $P_{ij} = \varepsilon$ then $S_{Pij} = \varepsilon$ and otherwise if $P_{ij} \neq \varepsilon$ then $S_{Pij} = 0$.*

Obviously, a structure matrix describes a class of matrices with the same ε–structure but different finite elements. We will denote by $\mathcal{S}(S_P)$ the set of all matrices P with the same structure matrix S_P. We consider the following parameter estimation problem:

Problem 2 (Parameter estimation) *Given the structure matrices S_A and S_B of a max–plus–linear system (1), input event times $u(k)$, $k = 1, \ldots, N$, the measurements $x(k)$, $k = 0, \ldots, N$, and the prediction error*

$$\xi(k+1) = x(k+1) - \Big(\hat{A} \otimes x(k) \oplus \hat{B} \otimes u(k+1)\Big) \; , \quad (3)$$

determine estimates $\hat{A} \in \mathcal{S}(S_A)$ and $\hat{B} \in \mathcal{S}(S_B)$ for the system matrices A and B such that $\bigoplus_{k=1}^{N} |\xi_i(k)|$, the maximum absolute value of each element in the prediction error vector, is minimized.

Remark 3 *The structure matrices of the system are determined by the layout and the internal connection between different subparts of the system (see, e.g., (Baccelli* et al. *1992)). For most discrete event systems the internal structure is known. Hence, we may without loss of generality assume that all entries of $\hat{A}$ and $\hat{B}$ are different from ε (so we can remove the ε entries from $\hat{A}$ and $\hat{B}$ or put $\hat{A}_{ij} = \varepsilon$ and $\hat{B}_{i'j'} = \varepsilon$ for all index pairs (i, j) and (i', j') where $S_{Aij} = \varepsilon$ and $S_{Bi'j'} = \varepsilon$, respectively and only consider the finite entries).*

Remark 4 *For the sake of brevity and simplicity of notation we only consider the state update equation (1) here* *and omit the output equation (2). Note however that the results derived below can easily be extended to also include the output equation since this equation can be dealt with in the same way as the state update equation. Furthermore, for most discrete event systems such as, e.g., manufacturing systems, one can usually measure all the state variables as they correspond to the starting times of the various production units (i.e. we have $y(k) = x(k)$).*

In order to simplify the notation in the following discussion, we introduce the parameter matrix $\Theta = [A\ B]$ and the measurement vector $m^T(k+1) = [x^T(k)\ u^T(k+1)]$. The prediction error then reads:

$$\xi(k+1) = x(k+1) - \widehat{\Theta} \otimes m(k+1) .$$

The solution to the parameter estimation problem (Problem 2) is now given by (Cuninghame-Green 1979):

$$\widehat{\Theta}_{ij}^{(N)} = \widetilde{\Theta}_{ij}^{(N)} + \frac{1}{2}\,\delta_i \qquad \text{with} \tag{4}$$

$$\widetilde{\Theta}_{ij}^{(N)} = \min_{k=1,\ldots,N} \big(x_i(k) - m_j(k)\big) \qquad \text{and} \tag{5}$$

$$\delta_i = \bigoplus_{k=1}^{N} \left(x_i(k) - \bigoplus_{r=1}^{n+n_u} \widetilde{\Theta}_{ir}^{(N)} \otimes m_r(k) \right) . \tag{6}$$

The variable δ_i is the maximum value of the prediction error elements $\xi_i(k)$ for $k = 1,\ldots,N$. As shown in (Menguy *et al.* 2000*a*), in the absence of noise and for time–invariant parameters $\widetilde{\Theta}_{ij}^{(N)}$ has two particular properties for any set of measurements:

$$x_i(k) = \bigoplus_{r=1}^{n+n_u} \widetilde{\Theta}_{ir}^{(N)} \otimes m_r(k),\ i = 1,\ldots,n,\ k = 1,\ldots,N, \tag{7}$$

$$\widetilde{\Theta}^{(N)} \geq \Theta . \tag{8}$$

From (7) it immediately follows that $\delta_i = 0\ \forall\ i = 1,\ldots,n$. Thus, $\widetilde{\Theta}^{(N)} = \widehat{\Theta}^{(N)}$ such that the solutions given by (4) and (5) are identical. If the system parameters are not time–invariant or noise is present, then

$$x_i(k) \geq \bigoplus_{r=1}^{n+n_u} \widetilde{\Theta}_{ir}^{(N)} \otimes m_r(k),\ i = 1,\ldots,n,\ k = 1,\ldots,N, \tag{9}$$

holds (Baccelli *et al.* 1992, Cuninghame-Green 1979). So we obtain $\delta_i \geq 0$ and therefore from (4) and (8)

$$\widehat{\Theta}^{(N)} \geq \widetilde{\Theta}^{(N)} \ \geq\ \Theta . \tag{10}$$

Remark 5 *The equations (7) and (8) show that even in the absence of noise and for time–invariant parameters although the prediction error is minimized an estimated parameter $\widehat{\Theta}_{ij}^{(N)}$ is always greater or equal than the true parameter value Θ_{ij}. If noise is present or the parameters are not time–invariant then the estimate may increase even more as illustrated by (10).*

We will now address this issue and show that for particular properties of a noise–free measurement set and time–invariant systems, the estimated value is equal to the corresponding true value. The following theorem (Schullerus 2004) gives a necessary and sufficient condition:

Theorem 6 *Assume that the system parameters are time–invariant and the measurements are noise–free. Then, for a given index pair (i,j), $i \in \{1,\ldots,n\}$ and $j \in \{1,\ldots,n+n_u\}$ we have $\widehat{\Theta}_{ij}^{(N)} = \Theta_{ij}$ if and only if $\exists\, \tilde{k} \in \{1,\ldots,N\}$, such that*

$$\Theta_{ij} \otimes m_j(\tilde{k}) \geq \bigoplus_{\substack{r=1\\ r\neq j}}^{n+n_u} \Theta_{ir} \otimes m_r(\tilde{k}) . \tag{11}$$

The condition (11) for the parameter Θ_{ij} is equivalent to $x_i(\tilde{k}) = \Theta_{ij} \otimes m_j(\tilde{k})$ which is similar to a condition given in (Menguy 1997) for the identification of the impulse response of a max–plus–linear system, where such a parameter is called "active". In order to obtain an estimate which is equal to the corresponding true value of the parameter Θ_{ij} we have to influence the measurements in such a way that this parameter becomes active by increasing m_j with respect to m_r, $r = 1,\ldots,n+n_u$, $r \neq j$ such that Θ_{ij} and m_j determine the value of x_i. In general, the trajectories generated by the system under the action of arbitrary input signals will not satisfy (11). In the next section, a new method will be presented to design a set of input signals such that the resulting measurements satisfy (11), that is the parameter Θ_{ij} is active and as a result $\widehat{\Theta}_{ij}^{(N)} = \Theta_{ij}$.

4 Input signal design

As illustrated in equation (4) and Theorem 6 parameter estimation as well as the condition for equality between true and estimated parameter value are formulated for each parameter Θ_{ij} separately. We will now present an input signal design method that determines input signals based on a model of the system such that the condition (11) is satisfied for a given index pair (i,j), $i \in \{1,\ldots,n\}$ and $j \in \{1,\ldots,n+n_u\}$. For the sake of brevity, we will assume in the sequel that the system parameters are time–invariant and the measurements are not corrupted by noise. As the true parameter values are unknown at this time, we use an estimate $\widehat{\Theta}^{(k_0)} \in \mathcal{S}(S_\Theta)$ obtained from the first k_0 measurements and arbitrary input signals. Since we only consider finite entries in Θ (cf. Remark 3) and since these finite entries correspond to processing times, transportation times, and so on, they are always nonnegative (Baccelli *et al.* 1992) (cf. also the example in Section 5). In addition, the property (10) holds, so the parameters of the system are known to stay within certain bounds given by $0 \leq \Theta_{ir} \leq \widehat{\Theta}_{ir}^{(k_0)}$. Introducing the lower bound 0 on the left hand side and the upper bound $\widehat{\Theta}_{ir}^{(k_0)}$ on the right hand side of (11), we obtain with $\tilde{k} = k_0 + \ell$, $\ell \geq 1$ a *sufficient condition* for (11):

$$m_j(k_0+\ell) \geq \bigoplus_{\substack{r=1\\ r\neq j}}^{n+n_u} \widehat{\Theta}_{ir}^{(k_0)} \otimes m_r(k_0+\ell)\ . \tag{12}$$

Since the values of m are not available during the input signal design process we will use estimates $\widehat{m}(k_0+\ell) = [\hat{x}^T(k_0+\ell-1)\ \ u^T(k_0+\ell)]^T$ for these values obtained with the model parameters given by $\widehat{\Theta}^{(k_0)}$.

Problem 7 (Input signal design) *Given the vector $x(k_0)$ and an estimate $\widehat{\Theta}^{(k_0)} = \begin{bmatrix}\hat{A}^{(k_0)} & \hat{B}^{(k_0)}\end{bmatrix}$ obtained from the previous identification, determine an input trajectory $u(k_0+1),\dots,u(k_0+N_\mathrm{i})$, such that for a given index pair (i,j) the state trajectory $\hat{x}(k_0),\dots,\hat{x}(k_0+N_\mathrm{i}-1)$, determined with the prediction model equation*

$$\begin{aligned}&\hat{x}(k+1) = \hat{A}^{(k_0)} \otimes \hat{x}(k) \oplus \hat{B}^{(k_0)} \otimes u(k+1)\\ &\text{for } k = k_0,\dots,k_0+N_\mathrm{i}-2,\ \ \hat{x}(k_0) = x(k_0)\end{aligned} \tag{13}$$

satisfies $\quad \widehat{m}_j(k_0+N_\mathrm{i}) \geq \bigoplus_{\substack{r=1\\ r\neq j}}^{n+n_u} \Delta \otimes \widehat{\Theta}_{ir}^{(k_0)} \otimes \widehat{m}_r(k_0+N_\mathrm{i})$

or equivalently

(1) if $j \in \{n+1,\dots,n+n_u\}$, then

$$\begin{aligned}u_{j-n}(k_0+N_\mathrm{i}) \geq &\bigoplus_{r=1}^{n} \Delta \otimes \hat{A}_{ir}^{(k_0)} \otimes \hat{x}_r(k_0+N_\mathrm{i}-1)\\ &\oplus \bigoplus_{\substack{r=1\\ r\neq j-n}}^{n_u} \Delta \otimes \hat{B}_{ir}^{(k_0)} \otimes u_r(k_0+N_\mathrm{i}),\end{aligned} \tag{14}$$

(2) if $j \in \{1,\dots,n\}$, then

$$\begin{aligned}\hat{x}_j(k_0+N_\mathrm{i}-1) \geq &\bigoplus_{\substack{r=1\\ r\neq j}}^{n} \Delta \otimes \hat{A}_{ir}^{(k_0)} \otimes \hat{x}_r(k_0+N_\mathrm{i}-1)\\ &\oplus \bigoplus_{r=1}^{n_u} \Delta \otimes \hat{B}_{ir}^{(k_0)} \otimes u_r(k_0+N_\mathrm{i})\ ,\end{aligned} \tag{15}$$

where Δ is an appropriately chosen constant that accounts for the differences between the model parameters and their corresponding true values.

Remark 8 *In contrast to (Schullerus* et al. *2003) we search an input sequence to satisfy the sufficient condition (11) for $\ell = N_\mathrm{i}$ and not for at least one $\ell \leq N_\mathrm{i}$ as in (Schullerus* et al. *2003). Thus, the complexity of the problem and the computational burden is reduced.*

The condition (14) can easily be satisfied even for $N_\mathrm{i} = 1$ by selecting an input event time u_{j-n} which is "late enough". In contrast, the determination of input signals such that (15) is satisfied requires the consideration of the system dynamics. The following theorem gives a sufficient condition on the existence of such input signals (Schullerus 2004):

Theorem 9 *If $\forall\, r \in \{1,\dots,n \,|\, r \neq j\ ,$ and $(S_A)_{ir} \neq \varepsilon\}$ there exist $s \in \{1,\dots,n_u\}$ and $K \in \mathbb{N}$ such that*

$$(S_B)_{is} = \varepsilon\ , \tag{16}$$

$$\bigoplus_{p=1}^{n} ({S_A}^{\otimes^{K-1}})_{jp} \otimes (S_B)_{ps} \neq \varepsilon\ , \tag{17}$$

$$\bigoplus_{p=1}^{n} ({S_A}^{\otimes^{k-1}})_{rp} \otimes (S_B)_{ps} = \varepsilon \qquad \forall\ k = 1,\dots,K\ , \tag{18}$$

then, for any $x(k_0)$ and any matrices $\hat{A}^{(k_0)} \in \mathcal{S}(S_A)$ and $\hat{B}^{(k_0)} \in \mathcal{S}(S_B)$ there exists an input signal sequence such that (15) is satisfied.

PROOF. Starting from $x(k_0)$ the solution $\hat{x}(k_0+k)$, $k \geq 1$ is given by

$$\begin{aligned}\hat{x}(k_0+k) = &\hat{A}^{(k_0)\otimes^{k}} \otimes x(k_0)\\ &\oplus \hat{A}^{(k_0)\otimes^{k-1}} \otimes \hat{B}^{(k_0)} \otimes u(k_0+1)\\ &\oplus \dots \oplus \hat{A}^{(k_0)} \otimes \hat{B}^{(k_0)} \otimes u(k_0+k-1)\\ &\oplus \hat{B}^{(k_0)} \otimes u(k_0+k)\ ,\end{aligned}$$

so we have for $q = 1,\dots,n$ and $K = k$

$$\begin{aligned}\hat{x}_q(k_0+K) = &\bigoplus_{p=1}^{n} (\hat{A}^{(k_0)\otimes^{K}})_{qp} \otimes x_p(k_0)\\ &\oplus \bigoplus_{\substack{h=1\\ h\neq s}}^{n_u} \left(\bigoplus_{p=1}^{n} (\hat{A}^{(k_0)\otimes^{K-1}})_{qp} \otimes (\hat{B}^{(k_0)})_{ph} \right) \otimes u_h(k_0+1)\\ &\oplus \dots \oplus \bigoplus_{\substack{h=1\\ h\neq s}}^{n_u} (\hat{B}^{(k_0)})_{qh} \otimes u_h(k_0+K)\\ &\oplus \bigoplus_{p=1}^{n} (\hat{A}^{(k_0)\otimes^{K-1}})_{qp} \otimes (\hat{B}^{(k_0)})_{ps} \otimes u_s(k_0+1)\\ &\oplus \dots \oplus (\hat{B}^{(k_0)})_{qs} \otimes u_s(k_0+K)\ .\end{aligned}$$

As the equations (17) and (18) describe structural properties of the system, they hold for any $\hat{A}^{(k_0)} \in \mathcal{S}(S_A)$ and

$\hat{B}^{(k_0)} \in \mathcal{S}(S_B)$. Thus, for $q = r, r \neq j$, $\hat{x}_r(k_0+K)$ does not depend on the input values $u_s(k_0+1), \dots, u_s(k_0+K)$, while $\hat{x}_j(k_0+K)$ is influenced by this input sequence. So we can increase $\hat{x}_j(k_0+K)$ without increasing $\hat{x}_r(k_0+K)$. Due to (16) the choice of $u_s(k_0+K+1)$ does not influence the right hand side of (15). So we can find an input sequence $u(k_0+1), \dots, u(k_0+K+1)$, such that (15) is satisfied. □

Remark 10 *The proof of Theorem 9 shows that if the sufficient conditions (16), (17) and (18) are satisfied, there exists an input signal sequence of length $K+1$ such that (15) holds. Thus, we can choose $N_\mathrm{i} \geq K+1$. The theorem also illustrates that controllability is a necessary but not sufficient condition for the existence of an input signal sequence.*

Due to (10) we have $\hat{A}^{(k_0)} \geq A$ and $\hat{B}^{(k_0)} \geq B$. So for any given input signal sequence $u(k_0+1), \dots, u(k_0+K)$, $\hat{x}(k_0+K) \geq x(k_0+K)$ holds. Therefore, an input signal sequence that satisfies the condition (15) does not necessarily result in measurements that satisfy (11). The following theorem shows that an appropriate choice of Δ in Problem 7 leads to a solution that also satisfies (11).

Theorem 11 *If the assumptions of Theorem 9 are satisfied, then the solution to the input signal design problem (Problem 7), where*

$$\Delta \geq \bigoplus_{r=1}^{n} \left((\hat{A}^{(k_0)\otimes^{K}})_{jr} \right) \oplus \bigoplus_{k=1}^{K} \bigoplus_{h=1}^{n_u} \left(\bigoplus_{p=1}^{n} (\hat{A}^{(k_0)\otimes^{K-k}})_{jp} \otimes (\hat{B}^{(k_0)})_{ph} \right)$$

also satisfies (11).

PROOF. As the assumptions of Theorem 9 are satisfied, there always exists an input signal sequence such that (15) holds. Since we have $\bigoplus_r \alpha_r - \bigoplus_r \beta_r \leq \bigoplus_r (\alpha_r - \beta_r)$ for $\alpha_r, \beta_r \in \mathbb{R}$ using the same reasoning than at the beginning of the proof of Theorem 9 and recalling that the operator $\otimes$ represents conventional addition, we have for $K = N_\mathrm{i} - 1$ due to (10)

$$\hat{x}_j(k_0+K) - x_j(k_0+K) \leq \bigoplus_{r=1}^{n} \left((\hat{A}^{(k_0)\otimes^{K}})_{jr} - (A^{\otimes^{K}})_{jr} \right) \oplus \bigoplus_{k=1}^{K} \bigoplus_{h=1}^{n_u} \left(\bigoplus_{p=1}^{n} (\hat{A}^{(k_0)\otimes^{K-k}})_{jp} \otimes (\hat{B}^{(k_0)})_{ph} - \bigoplus_{p=1}^{n} (A^{\otimes^{K-k}})_{jp} \otimes (B)_{ph} \right) \leq \Delta \, .$$

Since $\hat{x}_j(k_0+K)$ satisfies (15), we also have

$$x_j(k_0+K) \geq \hat{x}_j(k_0+K) - \Delta \geq \bigoplus_{\substack{r=1\\r\neq j}}^{n} \hat{A}_{ir}^{(k_0)} \otimes \hat{x}_r(k_0+K) \oplus \bigoplus_{r=1}^{n_u} \hat{B}_{ir}^{(k_0)} \otimes u_r(k_0+K+1) \geq \bigoplus_{\substack{r=1\\r\neq j}}^{n} A_{ir} \otimes x_r(k_0+K) \oplus \bigoplus_{r=1}^{n_u} B_{ir} \otimes u_r(k_0+K+1) \, .$$

Thus, (11) holds as well. □

We will now use the upper bound constraint (UBC) algorithm introduced and discussed in detail in (Walkup 1995, Walkup and Borriello 1998) to determine an input signal sequence such that (15) is satisfied for a given index pair (i, j). An upper bound constraint (UBC) in the unknowns $z_1, \dots, z_N$, is an inequality of the form

$$z_k \leq \alpha_1 \otimes z_1 \oplus \alpha_2 \otimes z_2 \oplus \ldots \oplus \alpha_N \otimes z_N \, , \tag{19}$$

where the variables $\alpha_i \in \mathbb{R}$ are constants. In (Walkup 1995, Walkup and Borriello 1998) it is shown that a max–plus–linear equation of the form

$$R \otimes z \oplus s = T \otimes z \oplus v \tag{20}$$

for $z \in \mathbb{R}_{\max}^{N}$ and with constants $R, T \in \mathbb{R}_{\max}^{M \times N}$, $s, v \in \mathbb{R}_{\max}^{M}$ can also be written as a system of UBCs.

Rewriting the condition (15) and recalling that $\oplus$ and $\otimes$ represent conventional maximization and addition, respectively, yields

$$\hat{x}_j(k_0+N_\mathrm{i}-1) - \hat{x}_r(k_0+N_\mathrm{i}-1) \geq \hat{A}_{ir}^{(k_0)} \otimes \Delta \quad \text{for } r = 1, \dots, n, \ r \neq j, \quad \text{and} \tag{21}$$

$$\hat{x}_j(k_0+N_\mathrm{i}-1) - u_r(k_0+N_\mathrm{i}) \geq \hat{B}_{ir}^{(k_0)} \otimes \Delta \quad \text{for } r = 1, \dots, n_u \, , \tag{22}$$

which is equivalent to the following UBCs:

$$\hat{x}_r(k_0+N_\mathrm{i}-1) \leq \big(-\hat{A}_{ir}^{(k_0)} \big) \otimes (-\Delta) \otimes \hat{x}_j(k_0+N_\mathrm{i}-1) \quad \text{for } r = 1, \dots, n, \ r \neq j \, , \quad \text{and}$$
$$u_r(k_0+N_\mathrm{i}) \leq \big(-\hat{B}_{ir}^{(k_0)} \big) \otimes (-\Delta) \otimes \hat{x}_j(k_0+N_\mathrm{i}-1) \quad \text{for } r = 1, \dots, n_u \, . \tag{23}$$

In a similar manner we can formulate additional event separation and time order constraints on the input signals such as e.g. $\delta_{u_{\min}} \leq u(k+1) - u(k) \leq \delta_{u_{\max}}$ or $u_r(k) \geq u_s(k) + \delta_{\mathrm{sep}_{rs}}$. Such constraints resulting from technical requirements on the process behavior can also be recast as a system of UBCs. As the prediction equations (13) for $k = k_0, \dots, k_0+N_\mathrm{i}-2$, are of the form (20), they can also be written as a system of UBCs. Hence, we can use the UBCsolv algorithm (Walkup 1995, Walkup and Borriello 1998) to compute an adequate input sequence for identification. If the solution of the UBC system is bounded from above, the UBC algorithm iteratively solves a series of square subsystems of the original UBC system [1]. The sequence of solutions of these subsystems converges monotonously to the solution of the original UBC system. To ensure a finite solution we can always add an upper bound for u and x which is far enough in the future so that it bounds the solution from above but does not interfere with the other constraints in any way. Then, in a finite number of steps the algorithm either converges to the solution that satisfies all UBCs or finds that the system is infeasible. Note however, that the UBC method is not polynomial–time.

Using the measurements resulting from the computed input sequence for a new parameter estimation yields an estimate $\widehat{\Theta}_{ij}^{(k_0+N_i)} = \Theta_{ij}$ for a given index pair (i, j). To determine all unknown parameters of the system, we proceed iteratively. In each iteration one input signal sequence is determined for one unknown parameter.

Remark 12 *The discussion presented in this section assumes time–invariant parameters and measurements which are not corrupted by noise. Note however, that the principle of designing input signals such that particular parameters are made active is valid even if these assumptions are not satisfied. Some results are given in (Schullerus 2004); however, the issue is still a matter of research.*

5 Application example

To illustrate the application and some results of the method introduced in the previous section on a practical example, we now consider the hardware model of a manufacturing cell shown in Figure 1. It is a part of a small–scale–model of a flexible manufacturing system controlled by a PLC available at the Institut für Regelungs- und Steuerungssysteme.

Let $x_1(k)$ be the time instant when the crane (not shown in Figure 1) starts for the k–th time to transport a part to conveyor 1. After its transportation time the crane places the part onto this conveyor at time $x_2(k)$. Then, at time $x_3(k)$ the conveyors 1 and 2 start transporting the part to the machine and the machining starts. Both operations are finalized at time $x_4(k)$. Afterwards the k–th part is transported by the conveyors 2 and 3 to its final position on conveyor 3 and picked up by the crane at time $x_5(k)$. To pick up the k–th part from conveyor 3, the crane started for the k–th time to proceed to conveyor 3 at time $x_6(k)$. The input signals $u_1(k)$ and $u_2(k)$ represent the time instants when the crane receives the k–th command to proceed to conveyor 1 or 3, respectively. This behavior can be described by the max–plus–linear system (1) (a detailed description is given in (Schullerus 2004)). The aim is to determine estimates for the unknown finite parameters $A_{ij} \neq \varepsilon$ and $B_{ij} \neq \varepsilon$ from event time measurements by applying the procedure described in Section 4. For technical reasons, there are constraints on the input signals, requiring particular separation intervals between x_2, x_5 and u_1, u_2. These constraints can be formulated as a set of inequalities.

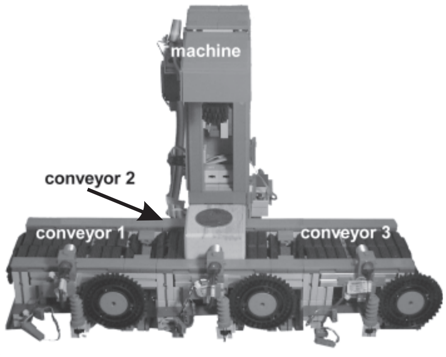

Fig. 1. Manufacturing cell showing a part on conveyor 2.

k	1	2	3	4	5	6	7
$x(k)$	0	66	131	196	260	323	387
	12	75	140	205	269	334	398
	15	78	143	209	273	338	418
	30	93	158	223	287	411	467
	48	113	167	232	410	467	494
	40	105	146	212	402	456	484
$u(k)$	0	65	130	196	260	323	387
	40	105	146	212	402	456	482

Table 1
$x(k)$, $u(k)$ used in the example, given in seconds.

First, the estimation is carried out using a set of arbitrary input signals for $k = 1, \ldots, 4$, listed in Table 1 along with the resulting measurements. This yields the following estimates (upper bounds) for the system parameters (where we have not included the initial state in this estimation as it is unknown):

$$\hat{A}^{(4)} = \begin{bmatrix} \varepsilon & 54.5 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 63.5 & 60.5 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 66 & 63 & \mathbf{48} & \varepsilon & \varepsilon \\ \varepsilon & 81 & 78 & 63 & 45 & \varepsilon \\ \varepsilon & 92 & 89 & 74 & 54 & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & 33 & \varepsilon \end{bmatrix}, \quad \hat{B}^{(4)} = \begin{bmatrix} 0.5 & \varepsilon \\ 9.5 & \varepsilon \\ 13 & \varepsilon \\ 27 & \varepsilon \\ 36 & 8 \\ \varepsilon & 0 \end{bmatrix}.$$

[1] The UBCsolv algorithm does not require the system to be square. So the number of UBCs does not have to be equal to the number of variables.

To illustrate the results of the signal design procedure, consider the estimation of the parameter A_{34} (indicated in bold in the matrix given above). The condition (15) for this particular parameter

$$\begin{aligned}\hat{x}_4(k_0+N_\mathrm{i}-1) \geq{}& \hat{A}_{32}^{(k_0)} \otimes \Delta \otimes \hat{x}_2(k_0+N_\mathrm{i}-1)\\ &\oplus \hat{A}_{33}^{(k_0)} \otimes \Delta \otimes \hat{x}_3(k_0+N_\mathrm{i}-1)\\ &\oplus \hat{B}_{31}^{(k_0)} \otimes \Delta \otimes u_1(k_0+N_\mathrm{i}) \qquad (24)\end{aligned}$$

must be satisfied for $k_0 = 4$ and $N_\mathrm{i} = 3$. We choose $\Delta = 0$ smaller than indicated in Theorem 11 in order to illustrate that even in this case we can obtain the desired result. Then, an adequate input signal sequence for A_{34} is determined based on the estimated values in $\hat{A}^{(4)}$ and $\hat{B}^{(4)}$ using the UBC approach. This yields the input and the corresponding state trajectory of Table 1. Since we have chosen Δ smaller than indicated by Theorem 11, we need to verify whether or not the measurements satisfy (24). As this is the case, we now obtain the correct estimate $\hat{A}_{34}^{(7)} = 9$ from the total input–state sequence.

6 Conclusions

The focus of the present contribution is on an input signal design method that is required for an accurate parameter estimation of max–plus–linear systems. Based on an already existing parameter estimation method and a condition for the determination of the true system parameters, the input signal design problem was formulated, new sufficient conditions for the existence of a solution were given and a new input signal design method was developed and illustrated in an example. The method constitutes an improvement compared to the already existing approaches in the sense that the computational burden is reduced with respect to an already existing general solution while keeping the capability of considering additional constraints in the input signal design procedure. Future research on this method will focus in more detail on the influence of noise and parameter variations and will establish necessary and sufficient conditions for the existence of a solution.

Acknowledgements

This work has been supported by the Deutsche Forschungsgemeinschaft (DFG) under contract number Kr 949/9, by the Dutch Technology Foundation STW projects "MPC for hybrid systems" (DMR.5675), "Multi–agent control of large–scale hybrid systems" (DWV.8188) and by the European IST project "SICONOS"(IST–2001–37172).

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