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MODEL PREDICTIVE CONTROL FOR PERTURBED PIECEWISE AFFINE SYSTEMS

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Abstract
We give an overview of our recent results on model predictive control (MPC) for a class of non-smooth hybrid systems subject to perturbations. More specifically, we consider continuous and non-continuous piecewise-affine (PWA) systems. Most results on MPC for PWA systems involve non-perturbed systems. In this paper we consider perturbed PWA systems with bounded disturbances, i.e., PWA systems whose system equations contain a disturbance term with a bounded support. We show that for both continuous and non-continuous PWA systems an MPC approach consisting of an off-line step and an (efficient) on-line step can be developed.

Key words
Piecewise affine systems, model predictive control, max-min-plus-scaling systems, hybrid systems.

1 Introduction

In this paper we give an overview of our recent results on model predictive control (MPC) for continuous and non-continuous piecewise-affine (PWA) systems. MPC (Maciejowski, 2002) is a model-based, receding-horizon control approach that uses on-line or off-line optimization to determine appropriate control inputs. Although MPC has its origins in the process industry and was developed for linear and non-linear continuous-variable systems, it has recently been extended to hybrid systems.

PWA systems are a class of hybrid systems that are characterized by an input/state space that is subdivided in several polyhedral regions, in each of which the behavior of the system is described by an affine state space model. PWA models are often used to describe the behavior of hybrid systems since they form the "simplest" extension of linear systems that can still model nonlinear and non-smooth processes with arbitrary accuracy and since they can deal with hybrid phenomena. PWA systems have been studied by many authors (Bemporad *et al.*, 2000*b*; Bemporad and Morari, 1999; Chua and Deng, 1988; Chua and Ying, 1983; Johansson, 2003; Kevenaar and Leenaerts, 1992; Leenaerts and van Bokhoven, 1998; Sontag, 1981; Veliov and Krastanov, 1986). In particular, Sontag has considered PWA systems from a classical control perspective (Sontag, 1981). He has also studied specific properties like representation, realization, observability, and decidability questions. Furthermore, recently, several authors (Bemporad *et al.*, 2000*a*; Bemporad *et al.*, 2000*b*; Bemporad and Morari, 1999; Borrelli, 2003; Ferrari-Trecate *et al.*, 2001; Kerrigan and Mayne, 2002; Lazar *et al.*, 2004*b*; Mayne and Raković, 2003; Raković *et al.*, 2004; Rantzer and Johansson, 2000) have developed an MPC approach for PWA systems. Furthermore, we have recently developed an MPC approach for continuous PWA systems (De Schutter and van den Boom, 2004) using the equivalence between continuous PWA systems and max-min-plus-scaling (MMPS) systems, i.e., systems the behavior of which can be described by the operations maximization, minimization, addition and scalar multiplication. This equivalence allows to compute the MPC input via the on-line solution of several linear programming problems.

Most results on MPC for PWA systems involve non-perturbed systems. We now consider perturbed PWA systems with bounded disturbances, i.e., PWA systems whose system equations contain a disturbance term with a bounded support. This disturbance term could represent bounded modeling errors, noise and/or disturbances. In this paper we give an overview of our results on MPC for perturbed PWA systems (Necoara *et al.*, 2004; Necoara *et al.*, 2005*b*; Necoara *et al.*, 2005*a*).

First, we consider MPC for perturbed continuous PWA systems (or MMPS systems) with a PWA cost criterion. The proposed MPC method for this class of systems is based on minimizing the worst-case cost criterion, i.e., a min-max approach. We show that the resulting MPC optimization problem can be computed efficiently using a two-level optimization approach consisting of an off-line and an on-line step. In the off-line step we have to solve a multi-parametric linear programming problem and to compute a canonical expression of the cost criterion. On-line, we have to solve a set of linear programming problems, for which efficient optimization algorithms exist.

Next, we consider general PWA systems. Here, we also use an MPC scheme consisting of an off-line step and an on-line step. In the first step, we compute off-line the set of states that can be steered to a certain convex target set using a semi-feedback controller. This local controller can be computed using linear matrix inequalities (LMIs). The second step consists in solving on-line, at each sample step a mixed-integer linear programming problem. This results in a min-max feedback MPC scheme based on a dual-mode approach that stabilizes the system.

This paper is organized as follows. In Section 2 we present PWA and MMPS functions and systems, and we establish the equivalence between continuous PWA systems and MMPS systems. Next, we give a short introduction to MPC in Section 3. In Section 4 and Section 5 we then discuss MPC for perturbed MMPS systems and for perturbed PWA systems respectively.

2 Piecewise affine systems and max-min-plus-scaling systems

In this section we present PWA and MMPS functions and systems, and we discuss the link between continuous PWA systems and MMPS systems. We also present canonical forms for MMPS functions.

2.1 Piecewise affine (PWA) systems

Definition 2.1. (Chua and Deng, 1988) *A scalar-valued function $f : \mathbb{R}^n \to \mathbb{R}$ is said to be a PWA function if and only if the following conditions hold:*

1. *The domain space $\mathbb{R}^n$ is divided into a finite number of polyhedral regions $R_{(1)}, \ldots, R_{(N)}$.*
2. *For each $i \in \{1, \ldots, N\}$, f can be expressed as $f(x) = \alpha_{(i)}^T x + \beta_{(i)}$ for any $x \in R_{(i)}$ with $\alpha_{(i)} \in \mathbb{R}^n$ and $\beta_{(i)} \in \mathbb{R}$.*

If f is continuous on any boundary between two regions, then we say that f is a continuous PWA function. A vector-valued function is continuous PWA if each of its components is continuous PWA.

A *PWA system* is a system of the form

$$x(k+1) = \mathscr{P}_x(x(k), u(k)) \tag{1}$$

$$y(k) = \mathscr{P}_y(x(k), u(k)) \ , \tag{2}$$

with $\mathscr{P}_x, \mathscr{P}_y$ vector-valued PWA functions and where u, y, and x represent the input, output and state of the system respectively. If $\mathscr{P}_x, \mathscr{P}_y$ are continuous, we say that the system is continuous PWA. For more information on PWA functions and PWA systems we refer to (Bemporad *et al.*, 2000*b*; Chua and Deng, 1988; Chua and Ying, 1983; Johansson, 2003; Kevenaar and Leenaerts, 1992; Leenaerts and van Bokhoven, 1998; Sontag, 1981) and the references therein.

2.2 Max-min-plus-scaling (MMPS) systems

Definition 2.2. *An MMPS function $f : \mathbb{R}^n \to \mathbb{R}^m$ is defined by the recursive grammar*

$$f(x) := x_i \,|\, \alpha \,|\, \max(f_k(x), f_l(x)) \,| \\ \min(f_k(x), f_l(x)) \,|\, f_k(x) + f_l(x) \,|\, \beta f_k(x) \ , \tag{3}$$

with $i \in \{1, \ldots, n\}$, $\alpha, \beta \in \mathbb{R}$, and where $f_k, f_l : \mathbb{R}^n \to \mathbb{R}^m$, are again MMPS functions; the symbol $|$ stands for "or", and max *and* min *are performed entrywise.*

Systems described by a state space model of the form

$$x(k+1) = \mathscr{M}_x(x(k), u(k)) \tag{4}$$

$$y(k) = \mathscr{M}_y(x(k), u(k)) \ , \tag{5}$$

where $\mathscr{M}_x, \mathscr{M}_y$ are (vector-valued) MMPS functions, are called *MMPS systems*.

2.3 Equivalence of continuous PWA and MMPS systems

Theorem 2.3. *If f is a continuous PWA function of the form given in Definition 2.1, then there exist index sets $I_1, \ldots, I_\ell \subseteq \{1, \ldots, N\}$ such that*

$$f = \max_{j=1,\ldots,\ell} \min_{i \in I_j} (\alpha_{(i)}^T x + \beta_{(i)}) \ . \tag{6}$$

Proof. See (Gorokhovik and Zorko, 1994; Ovchinnikov, 2002).

From the definition of MMPS functions it follows that any MMPS function is also a continuous PWA function. Hence, continuous PWA systems and MMPS systems are equivalent, i.e., for a given continuous PWA model there exists an MMPS model (and vice versa) such that the input-output behavior of both models coincides. So we have:

Proposition 2.4. *Continuous PWA systems and MMPS systems are equivalent.*

2.4 Canonical forms of MMPS functions

Each MMPS function can be rewritten in a max-min or min-max canonical form as follows:

Theorem 2.5. *A scalar-valued MMPS function f can be rewritten into the min-max canonical form*

$$f = \min_{i=1,\ldots,K} \max_{j=1,\ldots,n_i} (\alpha_{(i,j)}^T x + \beta_{(i,j)}) \tag{7}$$

or into the max-min canonical form

$$f = \max_{i=1,\ldots,L} \min_{j=1,\ldots,m_i} (\gamma_{(i,j)}^T x + \delta_{(i,j)}) \tag{8}$$

for some integers K, L, $n_1,\ldots,n_K$, $m_1,\ldots,m_L$, vectors $\alpha_{(i,j)}, \gamma_{(i,j)}$, and real numbers $\beta_{(i,j)}, \delta_{(i,j)}$.
For vector-valued MMPS functions the above statements hold componentwise.

Proof. See (De Schutter and van den Boom, 2004).

3 Model predictive control (MPC)

In this section we give a short introduction to the basic ideas behind MPC. For more detailed information on MPC we refer the interested reader to (Camacho and Bordons, 1995; Clarke *et al.*, 1987; García *et al.*, 1989; Maciejowski, 2002) and the references therein.

MPC is a model-based control approach that allows constraints on the inputs and outputs. In MPC at each sample step the optimal control inputs that minimize a given objective function over a given prediction horizon are computed, and applied using a receding horizon approach.

More specifically, in MPC we compute at each sample step k an optimal control input that minimizes a cost criterion over the period $[k, k+N_\mathrm{p}-1]$ where N_p is the prediction horizon. The cost criterion $J(k) = J_\mathrm{out}(k) + \lambda J_\mathrm{in}(k)$ used in MPC reflects the reference tracking error (J_out) and the control effort (J_in), where λ is a nonnegative weight parameter. We usually also include (linear) constraints on the inputs, states and output that should be satisfied over the prediction period.

If we have a prediction model for the system (e.g., a model of the form (1)–(2) or (4)–(5), and if we assume that at sample step k the current state can be measured, estimated or predicted using previous measurements, we can make an estimate $\hat{y}(k+j|k)$ of the output of the model at sample step $k+j$ based on the state $x(k)$ and the future inputs $u(k+i)$, $i = 0,\ldots,j-1$. After computation of the optimal control sequence $u(k), u(k+1),\ldots,u(k+N_\mathrm{p}-1)$, only the first control sample $u(k)$ will be implemented, subsequently the horizon is shifted one sample; next, the model and the state are updated using new information from the measurements, and a new MPC optimization is performed for sample step $k+1$. This is the receding horizon approach used in MPC.

4 MPC for perturbed MMPS systems and continuous PWA systems

In this section we discuss MPC for perturbed MMPS systems, or equivalently for continuous PWA systems. This section is based on (Necoara *et al.*, 2004; Necoara *et al.*, 2005*b*).

4.1 Multi-parametric linear programming

A multi-parametric linear programming problem (MPLP) is defined as follows (Borrelli *et al.*, 2003).

Consider matrices and vectors $S \in \mathbb{R}^{m\times n}$, $c \in \mathbb{R}^n$, $q \in \mathbb{R}^m$, $U \in \mathbb{R}^{m\times s}$, and let $\Theta \subseteq \mathbb{R}^s$ be the set in which the parameter variable θ of the MPLP lives. We assume that Θ is a polyhedral set, i.e., it can be written as $\Theta = \{\theta \in \mathbb{R}^s \,|\, W\theta \leqslant \omega\}$ for some matrix W and vector ω. Now the MPLP is defined as

$$\max_{x\in\mathbb{R}^n} c^T x \qquad \text{subject to } Sx \leqslant q + U\theta, \tag{9}$$

where x is the optimization variable of the MPLP, and θ the parameter variable of the MPLP. For simplicity, we will assume in this paper that for any $\theta \in \Theta$ (where Θ is a bounded polyhedron), the MPLP problem (9) has a finite optimal solution.

Let $V^*(\theta)$ denote the maximum value of the objective function in the MPLP problem (9), and $x^*(\theta)$ the optimizer related to $V^*(\theta)$ for any $\theta \in \Theta$. Note that in general, $x^*(\theta)$ is set-valued. The following proposition characterizes the solution of an MPLP:

Proposition 4.1. *With the above notations, the function $V^* : \Theta \to \mathbb{R}$ is a concave MMPS function, i.e., an MMPS function that can be written as a* min*-plus-scaling expression (so no* max *is required). Furthermore, there exists an MMPS function $X^* : \Theta \to \mathbb{R}^n$ such that $X^*(\theta) \in x^*(\theta)$ for all $\theta \in \Theta$.*

Proof. See (Necoara *et al.*, 2004).

The reader is referred to (Borrelli *et al.*, 2003) for an algorithm for computing the solution of an MPLP.

4.2 The worst-case MPC problem for perturbed continuous PWA or MMPS systems

Now we extend the *deterministic* continuous PWA model (1)–(2), or equivalently the MMPS model (4)–(5) to take also the uncertainty into account. Recall that the MPC method is based on a prediction model of the system and that this prediction model is used to determine the optimal control inputs for the system. Therefore, we must also take into account the uncertainty when we implement MPC. If we ignore the disturbance in the plant, this can lead to errors in the system equations and even an unstable closed-loop behavior.

As in conventional linear systems, we model the disturbances by including an additive term in the system equations for continuous PWA systems. Hence, we consider the *perturbed continuous PWA* model:

$$x(k+1) = \mathscr{P}_x(x(k), u(k), e(k)) \tag{10}$$
$$y(k) = \mathscr{P}_y(x(k), u(k), e(k)), \tag{11}$$

where $\mathscr{P}_x$ and $\mathscr{P}_y$ are continuous vector-valued PWA functions, and the uncertainty caused by disturbances in the estimation of the real system is gathered in the uncertainty vector $e(k)$. We assume that this uncertainty is included in a bounded polyhedral set $\mathscr{E} = \{e \in \mathbb{R}^s \,|\, Se \leqslant q\}$.

Using the equivalence between continuous PWA and MMPS systems, the perturbed continuous PWA model (10)–(11) can be also written as a perturbed MMPS system:

$$x(k+1) = \mathscr{M}_x(x(k),u(k),e(k)) \tag{12}$$
$$y(k) = \mathscr{M}_y(x(k),u(k),e(k)), \tag{13}$$

where $\mathscr{M}_x$, $\mathscr{M}_y$ are vector-valued MMPS functions.

We assume that at each step k of MPC, the state $x(k)$ is available (can be measured or estimated) and we gather the uncertainty over the interval $[k,k+N_\mathrm{p}-1]$ in the vector $\tilde{e}(k) = [e^T(k),\ldots,e^T(k+N_\mathrm{p}-1)]^T \in \tilde{\mathscr{E}}$, where $\tilde{\mathscr{E}}$, according to our assumption, is a bounded polyhedral set. Then it is easy to see that the prediction $\hat{y}(k+j|k)$ of the future output for the system (12)–(13) can be written in MMPS form, for $j=1,\ldots,N_\mathrm{p}$.

Let r denote the reference signal, and define

$$\tilde{u}(k) = \left[u^T(k) \ \ldots \ u^T(k+N_\mathrm{p}-1)\right]^T, \tag{14}$$
$$\tilde{y}(k) = \left[\hat{y}^T(k+1|k) \ \ldots \ \hat{y}^T(k+N_\mathrm{p}|k)\right]^T, \tag{15}$$
$$\tilde{r}(k) = \left[r^T(k+1) \ \ldots \ r^T(k+N_\mathrm{p})\right]^T \ . \tag{16}$$

Just as in (De Schutter and van den Boom, 2004) we consider the following output and input cost functions:

$$J_{\mathrm{out},1}(k) = \|\tilde{y}(k)-\tilde{r}(k)\|_1, \quad J_{\mathrm{in},1}(k) = \|\tilde{u}(k)\|_1, \tag{17}$$
$$J_{\mathrm{out},\infty}(k) = \|\tilde{y}(k)-\tilde{r}(k)\|_\infty, \quad J_{\mathrm{in},\infty}(k) = \|\tilde{u}(k)\|_\infty \ . \tag{18}$$

Note that these cost functions are also MMPS functions since $|x| = \max(x,-x)$ for $x \in \mathbb{R}$. Hence, after computing the $\tilde{y}(k)$ using successive substitution (note that this yields an MMPS function), we can compute $J(k)$ as a function of $x(k)$, $\tilde{u}(k)$ and $\tilde{e}(k)$. It is easy to verify that with a combination of the output and input cost functions of (17)–(18) this also yields an MMPS function. Hence, we can write $J(k)$ in max-min canonical form:

$$J(\tilde{e}(k),\tilde{u}(k),x(k)) = \max_{j=1,\ldots,l} \min_{i\in S_j}(\bar{\alpha}^T_{(i,j)}x(k) + \bar{\beta}^T_{(i,j)}\tilde{u}(k) + \bar{\gamma}^T_{(i,j)}\tilde{e}(k) + \bar{\delta}_{(i,j)}). \tag{19}$$

Note that if the reference signal r depends on k then $\delta_{(i,j)}, \bar{\delta}_{(i,j)}$ will depend also on k.

Just as in (De Schutter and van den Boom, 2004) we consider only linear constraints on the input, i.e., constraints of the form $P(k)\tilde{u}(k)+q(k) \leqslant 0$.

The *worst-case MMPS-MPC problem* at step k can now be defined as

$$\min_{\tilde{u}(k)} \max_{\tilde{e}(k)\in\tilde{\mathscr{E}}} J(\tilde{e}(k),\tilde{u}(k),x(k)) \quad \text{subject to } P(k)\tilde{u}(k)+q(k) \leqslant 0 \ . \tag{20}$$

4.3 An algorithm for the worst-case MPC problem for perturbed MMPS systems

For a given $\tilde{u}(k)$, $x(k)$ we define the *inner* worst-case MMPS-MPC problem as follows:

$$\max_{\tilde{e}(k)\in\tilde{\mathscr{E}}} J(\tilde{e}(k),\tilde{u}(k),x(k)). \tag{21}$$

We denote

$$\tilde{e}^*(\tilde{u}(k),x(k)) = \arg\max_{\tilde{e}(k)\in\tilde{\mathscr{E}}} J(\tilde{e}(k),\tilde{u}(k),x(k)), \tag{22}$$
$$J^*(\tilde{u}(k),x(k)) = J(\tilde{e}^*(\tilde{u}(k),x(k)),\tilde{u}(k),x(k)). \tag{23}$$

Now we have:

Proposition 4.2. *For a given $\tilde{u}(k)$ and $x(k)$, $\tilde{e}^*(\tilde{u}(k),x(k))$ given by (22) can be computed using a sequence of linear programming (LP) problems.*

Proof. This proof can also be found in (Necoara *et al.*, 2004) but we include it for the sake of completeness.

Let the bounded polyhedral set $\tilde{\mathscr{E}}$ in which $\tilde{e}(k)$ lives be given by $\tilde{\mathscr{E}} = \{\tilde{e}(k) \,|\, \tilde{S}\tilde{e}(k) \leqslant \tilde{q}\}$. Now we determine for any fixed $[\tilde{u}^T(k)\, x^T(k)]^T$ the optimal $\tilde{e}^*(\tilde{u}(k),x(k))$, using the *max-min canonical form* (19) of $J(\cdot)$, by solving the following optimization problem:

$$\max_{\tilde{e}(k)} \max_{j=1,\ldots,l} \min_{i\in S_j}(\bar{\alpha}^T_{(i,j)}x(k) + \bar{\beta}^T_{(i,j)}\tilde{u}(k) + \bar{\gamma}^T_{(i,j)}\tilde{e}(k) + \bar{\delta}_{(i,j)}) \quad \text{subject to } \tilde{S}\tilde{e}(k) \leqslant \tilde{q} \ . \tag{24}$$

This problem is equivalent to

$$\max_{j=1,\ldots,l} \max_{\tilde{e}(k)} \min_{i\in S_j}(\bar{\alpha}^T_{(i,j)}x(k) + \bar{\beta}^T_{(i,j)}\tilde{u}(k) + \bar{\gamma}^T_{(i,j)}\tilde{e}(k) + \bar{\delta}_{(i,j)}) \quad \text{subject to } \tilde{S}\tilde{e}(k) \leqslant \tilde{q} \ . \tag{25}$$

Now for each $j=1,\ldots,l$ we have to solve

$$\max_{\tilde{e}(k)} \min_{i\in S_j}(\bar{\alpha}^T_{(i,j)}x(k) + \bar{\beta}^T_{(i,j)}\tilde{u}(k) + \bar{\gamma}^T_{(i,j)}\tilde{e}(k) + \bar{\delta}_{(i,j)}) \quad \text{subject to } \tilde{S}\tilde{e}(k) \leqslant \tilde{q}, \tag{26}$$

which is equivalent to the following LP problem:

$$\max_{\tilde{e}(k),t_{(j)}(k)} t_{(j)}(k) \tag{27}$$

subject to

$$t_{(j)}(k) \leqslant \bar{\alpha}^T_{(i,j)}x(k) + \bar{\beta}^T_{(i,j)}\tilde{u}(k) + \bar{\gamma}^T_{(i,j)}\tilde{e}(k) + \bar{\delta}_{(i,j)} \quad \text{for each } i \in S_j$$
$$\tilde{S}\tilde{e}(k) \leqslant \tilde{q} \ .$$

To obtain the solution of (24) we solve (27) for each $j = 1,\dots,l$, with the optimal solution $[t^*_{(j)}(\tilde{u}(k),x(k))\ \tilde{e}^{*T}_{(j)}(\tilde{u}(k),x(k))]^T$ and then we select as $\tilde{e}^*(\tilde{u}(k),x(k))$, the optimal solution $\tilde{e}^*_{(j)}(\tilde{u}(k),x(k))$ for which $\min_{i\in S_j}(\bar{\alpha}^T_{(i,j)}x(k) + \bar{\beta}^T_{(i,j)}\tilde{u}(k) + \bar{\gamma}^T_{(i,j)} \tilde{e}^*_{(j)}(\tilde{u}(k),x(k)) + \bar{\delta}_{(i,j)})$ is the largest.

Remark 4.3 We define $U = \{\tilde{u}(k)\,|\,P(k)\tilde{u}(k)+q(k) \leqslant 0\}$ and we assume U to be bounded. This assumption is not restrictive, because in practice the input $\tilde{u}(k)$ will always be bounded. Furthermore, the feasible set of the states X is assumed to be also a bounded polyhedron. This implies that for any $[\tilde{u}^T(k)\ x^T(k)] \in U\times X$ (a bounded polyhedron), the MPLP (21) has a finite optimal solution. ◊

Furthermore, from Proposition 4.1 it follows that (see also (Necoara *et al.*, 2004)):

Proposition 4.4. *With the notations (22)–(23), $J^*: U\times X \to \mathbb{R}$ is an MMPS function and $\tilde{e}^*: U\times X \to \mathbb{R}^s$ is a PWA function.*

The *outer* worst-case MMPS-MPC problem is now defined as

$$\begin{aligned}&\min_{\tilde{u}(k)} J^*(\tilde{u}(k),x(k)) \\ &\text{subject to }\ P(k)\tilde{u}(k)+q(k)\leqslant 0\ .\end{aligned} \tag{28}$$

Proposition 4.5. *Given $x(k)$, the outer worst-case MMPS-MPC problem can be solved using a sequence of LP problems.*

Proof. This proof can also be found in (Necoara *et al.*, 2004) but we include it for the sake of completeness.
From Proposition 4.4 we know that $J^*: U\times X \to \mathbb{R}$ is an MMPS function. Therefore, it can be written in the following min-max canonical form $J^*(\tilde{u}(k),x(k)) = \min_{j=1,\dots,\hat{l}}\max_{i\in T_j}(\mu^T_{(i,j)}x(k) + \nu^T_{(i,j)}\tilde{u}(k)+\xi_{(i,j)})$. Then, the outer worst-case MMPS-MPC problem (28) can be written as

$$\begin{aligned}&\min_{\tilde{u}(k)}\min_{j=1,..,\hat{l}}\max_{i\in T_j}(\mu^T_{(i,j)}x(k)+\nu^T_{(i,j)}\tilde{u}(k)+\xi_{(i,j)}) \\ &\text{subject to }\ P(k)\tilde{u}(k)+q(k)\leqslant 0\ .\end{aligned} \tag{29}$$

For each $j = 1,\dots,\hat{l}$ we must thus solve the following linear programming problem:

$$\begin{aligned}&\min_{\tilde{u}(k),t_{(j)}} t_{(j)} \\ &\text{subject to} \\ &\quad t_{(j)} \geqslant \mu^T_{(i,j)}x(k)+\nu^T_{(i,j)}\tilde{u}(k)+\xi_{(i,j)} \\ &\qquad\qquad \text{for each } i\in T_j \\ &\quad P(k)\tilde{u}(k)+q(k)\leqslant 0\ .\end{aligned} \tag{30}$$

In order to obtain the solution of (28), we solve (30), obtaining the optimal solution $[t^*_{(j)}(x(k))\ \tilde{u}^{*T}_{(j)}(x(k))]^T$ for each $j = 1,\dots,\hat{l}$, and then we select the optimal $\tilde{u}^*(x(k))$ as the optimal solution $\tilde{u}^*_{(j)}(x(k))$ for which $\max_{i\in T_j}(\mu^T_{(i,j)}x(k) + \nu^T_{(i,j)}\tilde{u}^*_{(j)}(x(k)) + \xi_{(i,j)})$ is the smallest.

Based on the above results we now present an algorithm to solve the worst-case MMPS-MPC problem:

Algorithm 1

Step 1: Solve *off-line* the inner worst-case MMPS-MPC problem (21) via the MPLP approach. According to Proposition 4.4 $J^*(x,u)$ is an MMPS function. Also compute the min-max canonical form of this function off-line.
Step 2: Compute *on-line* (at each step k) the solution of the outer worst-case MMPS-MPC problem (28) according to Proposition 4.5.

The solution of the MPLP of Step 1 can be obtained with different algorithms such as the geometric algorithm of (Borrelli *et al.*, 2003), or the dual-based vertex enumeration approach of (Diehl and Björnberg, 2004), which was proposed in the context of robust MPC.

Note that when r in not a constant reference signal the function $J^*(x,u)$ will depend also on $r(k+1),\dots,r(k+N_{\mathrm{p}})$ at sample step k, so in that case we will have to include $\tilde{r}$ as an additional argument of J in Step 1 when we compute the min-max canonical form.

Corollary 4.6. *Using Algorithm 1, the worst-case MMPS-MPC problem can be solved using a sequence of LP problems. Moreover the associated controller is a PWA function of the state $x(k)$ (and of $\tilde{r}(k)$ if the reference signal is not constant).*

5 MPC for perturbed PWA systems

Now we derive an MPC approach for general perturbed — thus not necessarily continuous — PWA systems. The aim of this approach is to design a robustly stable MPC controller. This will be done using a two-step approach. In the first step we design a piecewise linear feedback controller, that stabilizes the nominal system. Next, we construct a polyhedral robustly positively invariant set for the system, and we design MPC schemes that steer the perturbed system to the invariant set, resulting in an overall stabilizing MPC scheme. This section is based on (Necoara *et al.*, 2005*a*).

5.1 Notations and definitions

We use the following notations: a PWA system with additive disturbance is defined as

$$x(k+1) = A_ix(k)+B_iu(k)+a_i+w(k),\ \text{if } x(k)\in\mathscr{P}_i\,, \tag{31}$$

where w denotes disturbance and $\{\mathscr{P}_i\}_{i\in\mathscr{I}}$ is a finite partition of $\mathbb{R}^n$. The closure $\mathrm{cl}(\mathscr{P}_i)$ is given by

$\mathrm{cl}(\mathscr{P}_i) = \{x \,|\, E_i x \geqslant e_i\}$. When $a_i = 0$, $e_i = 0$ for all $i \in \mathscr{I}$, we get a piecewise linear (PWL) system:

$$x(k+1) = A_i x(k) + B_i u(k) + w(k), \text{ if } x(k) \in \mathscr{P}_i, \quad (32)$$

It is assumed that the disturbance belongs to a bounded polyhedron $w \in W$, and that the control and state are required to satisfy the constraints $u \in U_c$ and $x \in X_c$; X_c, U_c and W are all polytopes, with $0 \in U_c, W$ and $0 \in \mathrm{int}(X_c)$.

Given two sets $Y, Z \subset \mathbb{R}^n$, the Minkowski sum of Y and Z is defined as $Y \oplus Z = \{y + z \,|\, y \in Y, z \in Z\}$.

5.2 Stabilizing feedback controller for the nominal PWL system

First we design a local stabilizing feedback controller for the nominal PWL system associated with the perturbed PWL system (32), viz.

$$x(k+1) = A_i x(k) + B_i u(k), \text{ if } x(k) \in \mathscr{P}_i \quad (33)$$

Now we determine a PWL state feedback controller $u(k) = F_i x(k)$, if $x(k) \in \mathscr{P}_i$ such that the nominal system (33) in closed-loop with this controller is stable. We search for a piecewise quadratic Lyapunov function (Mignone *et al.*, 2000; Rantzer and Johansson, 2000) $V(x) = x^T P_i x$, if $x \in \mathscr{P}_i$, such that

$$x^T (A_i + B_i F_i)^T P_j (A_i + B_i F_i) x - x^T P_i x < 0 \quad (34)$$
$$x^T P_i x > 0 \quad (35)$$
$$\text{for all } x \in \mathscr{P}_i \text{ and for all } (i, j) \in \mathscr{I} \times \mathscr{I}$$
$$\text{that satisfy } x \in \mathscr{P}_i \text{ and } (A_i + B_i F_i) x \in \mathscr{P}_j.$$

Since (34)–(35) has to be valid only for $x \in \mathscr{P}_i$, we can use the S-procedure (Boyd *et al.*, 1994), which results in: Find F_i, P_i, U_{ij}, V_i for all $(i, j) \in \mathscr{I} \times \mathscr{I}$, where U_{ij}, V_i have all entries non-negative that satisfy the following matrix inequalities:

$$(A_i + B_i F_i)^T P_j (A_i + B_i F_i)^T - P_i + E_i^T U_{ij} E_i < 0 \quad (36)$$
$$P_i > E_i^T V_i E_i \quad (37)$$

We have the following solution for (36)–(37) where the symbol * is used to induce symmetry:

Theorem 5.1. *Equations* (36)–(37) *have a solution if and only if the following matrix inequalities have a solution*

$$\begin{bmatrix} B_i^T P_j B_i & B_i^T P_j A_i \\ * & A_i^T P_j A_i - P_i + E_i^T U_{ij} E_i \end{bmatrix} < \begin{bmatrix} I & -F_i \\ * & F_i^T F_i \end{bmatrix} \quad (38)$$
$$P_i > E_i^T V_i E_i \quad (39)$$

where U_{ij}, V_i have all entries non-negative for all (i, j).

Proof. See (Necoara *et al.*, 2005*a*).

Next, we discuss some relaxations for (36)–(37). The first relaxation is to replace (37) with $P_i > 0$. In this case we can apply the Schur complement to (36) (see also (Necoara *et al.*, 2005*a*)), and then we obtain that for $P_i > 0$, the matrix inequalities (36) are equivalent to

$$\begin{bmatrix} P_i - E_i^T U_{ij} E_i & * \\ A_i + B_i F_i & S_j \end{bmatrix} > 0 \quad (40)$$
$$0 < P_j \leqslant S_j^{-1}, \text{ for all } (i, j) \in \mathscr{I} \times \mathscr{I}. \quad (41)$$

Now we discuss a second relaxation. If we do not apply the S-procedure for (34)–(35), i.e., we replace the condition "$x \in \mathscr{P}_i$" with the condition "$x \in \mathbb{R}^n$", then (34)–(35) becomes

$$(A_i + B_i F_i)^T P_j (A_i + B_i F_i) - P_i < 0, \; P_i > 0 \quad (42)$$

for all $(i, j) \in \mathscr{I} \times \mathscr{I}$. We can linearize (42) using the well-known linearizing change of variable $S_i = P_i^{-1}, Y_i = F_i S_i$ (this type of linearization was used also in (Mignone *et al.*, 2000; Lazar *et al.*, 2004*a*; Lazar *et al.*, 2004*b*)). This then results in a linear matrix inequality (LMI) (see also (Necoara *et al.*, 2005*a*)). If we can solve this LMI and obtain P_i, F_i, then the piecewise feedback controller $u(k) = F_i x(k)$ if $x(k) \in \mathscr{P}_i$ asymptotically stabilizes the origin of system (33).

5.3 Convex robustly positively invariant sets

Now assume that we have determined a state feedback controller $u(k) = F_i x(k)$ if $x(k) \in \mathscr{P}_i$ that stabilizes the nominal system (33). If we define $A_{F_i} = A_i + B_i F_i$, then the closed-loop perturbed PWL system becomes:

$$x(k+1) = A_{F_i} x(k) + w(k), \text{ if } x(k) \in \mathscr{P}_i. \quad (43)$$

We define $X_F = \cup_{i \in \mathscr{I}} \{x \in \mathscr{P}_i \,|\, x \in X_c, F_i x \in U_c\}$.

Definition 5.2. (Kolmanovsky and Gilbert, 1998) *A set $\Omega \subseteq X_F$ is a* robustly positively invariant *(RPI) set for system (43) if for any $x \in \Omega \cap \mathscr{P}_i$ with $i \in \mathscr{I}$, we have $A_{F_i} x + w \in \Omega$ for all $w \in W$. The maximal RPI set for system (43) is defined as the largest RPI set for (43).*

It can be easily seen that the maximal RPI set associated with system (43) is in general a non-convex set. For system (43) the evolution of the mode $i = i(k)$ depends on the state $x(k)$. Nevertheless, for ease of computation of a convex (polyhedral) RPI set for (43), we will disregard this relation mode-state and we will consider that $i(k)$ evolves independently of $x(k)$ (i.e., $i(k+1) \in \mathscr{I}$ for all $k \geqslant 0$). This type of relaxation was also used also in (Chisci *et al.*, 2003; Lazar *et al.*, 2004*b*) in order to obtain a convex invariant set for deterministic PWL systems. So, we replace the PWL system (43) with the following time-varying system

$$x_{k+1} = A_{F_{i(k)}} x_k + w_k, \qquad \text{with } i(k+1) \in \mathscr{I} \tag{44}$$

where $i(\cdot)$ is a switching signal in $\mathscr{I}^{\mathbb{N}}$.

Definition 5.3. *A set Ω is an RPI set for (44) if for any $x \in \Omega$ we have $A_{F_i}x + w \in \Omega$ for any possible switching $i \in \mathscr{I}$ and any admissible disturbance $w \in W$.*

Now we construct an RPI set for (44). We define the following set recursion:

$$\begin{aligned} \mathscr{O}_0^i &= X_0^i = \{x \,|\, x \in X_\mathrm{c}, F_i x \in U_\mathrm{c}\}, \\ \mathscr{O}_t^i &= \{x \in X_{F_i} \,|\, A_{F_i}x \oplus W \subseteq \cap_{j \in \mathscr{I}} \mathscr{O}_{t-1}^j\} \end{aligned} \tag{45}$$

for any $i \in \mathscr{I}$ and $t = 1, 2, \ldots$. It is clear from (45) that $\mathscr{O}_{t+1}^i \subseteq \mathscr{O}_t^i$, and therefore $\mathscr{O}_t^i$ converges to $\mathscr{O}_\infty^i$. We define: $\mathscr{O}_\infty^i = \lim_{t \to \infty} \mathscr{O}_t^i = \cap_{t \geqslant 0} \mathscr{O}_t^i$ and $\mathscr{O}_\infty = \cap_{i \in \mathscr{I}} \mathscr{O}_\infty^i$. We have

Theorem 5.4. *(i) The maximal RPI set included in $\cap_{i \in \mathscr{I}} X_{F_i}$ for the system (44) is the* convex *set $\mathscr{O}_\infty$.*
(ii) The set $\mathscr{O}_\infty$ is an RPI set for the PWL system (43).

Proof. See (Necoara *et al.*, 2005*a*).

Because the sets $\mathscr{O}_t^i$ are described by a finite number of linear inequalities, it is important to know whether $\mathscr{O}_\infty$ can be *finitely determined*, i.e. , whether there exists a finite t^* such that $\mathscr{O}_{t^*}^i = \mathscr{O}_{t^*+1}^i$ for all $i \in \mathscr{I}$. In (Necoara *et al.*, 2005*a*) we some give necessary conditions for finite determination. Note that finite determination implies that $\mathscr{O}_\infty = \cap_{i \in \mathscr{I}} \mathscr{O}_{t^*}^i$ is a polyhedral set.

In the sequel we propose a robustly stabilizing MPC scheme for the PWL system (32). We consider the case where the mode at each sample step is known (i.e., can be determined or measured independently of the uncertainty about the state, e.g., the mode is only determined by the unperturbed part of the state; an example could be a gear box in a car where the gear position determines the mode).

5.4 Robust MPC with known mode: Feedback min-max MPC scheme

In this section we develop a stable MPC scheme for the PWL system (32), with known mode despite the presence of disturbances, based on a feedback min-max approach. For deterministic systems, almost all MPC schemes contain two ingredients: a terminal set and a terminal cost (see (Mayne *et al.*, 2000) for a survey). If the system is uncertain, the stability and also the feasibility may be lost. In order to achieve robustness, the controller must stabilize the system for all possible realizations of the disturbance along the prediction horizon. In this section we use a dual-mode MPC formulation.

In order to determine a suitable control law, an optimal control problem $\mathscr{V}_N(.)$ with horizon N is solved. Let $\mathbf{w} = (w(0), \ldots, w(N-1))$ be a possible realization of the disturbance over the interval 0 to $N-1$. Efficient control in the presence of disturbances requires state feedback; so, the decision variable (for a given initial state x) in the optimal control problem is a control policy defined as

$$\pi = (u(x), \mu_1(\cdot), \ldots, \mu_{N-1}(\cdot)) \ , \tag{46}$$

where $u(x) \in U_\mathrm{c}$ and $\mu_k : X_\mathrm{c} \to U_\mathrm{c}$, $k = 1, \ldots, N-1$ is a state feedback control law. Let $x(k; x, \pi, \mathbf{w})$ denote the solution to (32) at step k. The *feedback min-max optimization problem* is defined as

$$\mathscr{V}_N(x) : \min_{\pi} \max_{\mathbf{w} \in W^N} \sum_{k=0}^{N-1} l(x_k, u_k) \tag{47}$$

$$\begin{aligned} &\text{subject to} \\ &\quad x_k = x(k; x, \pi, \mathbf{w}) \in X_\mathrm{c} \\ &\quad u_k = \mu_k(x(k; x, \pi, \mathbf{w})) \in U_\mathrm{c} \\ &\quad x_N = x(N; x, \pi, \mathbf{w}) \in \mathscr{O}_\infty \\ &\qquad \forall k \in \{1, \ldots, N-1\} \text{ and } \forall \mathbf{w} \in W^N \ , \end{aligned}$$

where $l(x, u)$ is convex and such that $l(x, u) \geqslant \alpha(d(x, \mathscr{O}_\infty))$ if $x \notin \mathscr{O}_\infty$ and $l(x, u) = 0$ if $x \in \mathscr{O}_\infty$ with α a $K-$function (Scokaert and Mayne, 1998). The distance of a point x to the closed, convex set $\mathscr{O}_\infty$ is defined as $d(x, \mathscr{O}_\infty) = \min_{x^o \in \mathscr{O}_\infty} \|x - x^o\|$. In the sequel we consider $\|X\|$ as the p-norm ($\|X\|_p$, $p \geqslant 1$) for vectors and matrices.

For linear systems problem (47) can be solved using the approaches proposed in (Bemporad *et al.*, 2003; Diehl and Björnberg, 2004; Pluymers *et al.*, 2004; Muñoz de la Peña *et al.*, 2004; Scokaert and Mayne, 1998; Kerrigan and Maciejowski, 2004). In our setting, due to the nonlinearities of the system, these approaches cannot be applied directly. To overcome this problem, we propose to restrict the admissible control policies π to only those that guarantee that, for every value of the disturbance, the mode of the system $i(k)$ is unique at each sample step k:

$$x(k; x, \pi, \mathbf{w}) \in \mathscr{P}_{i(k)}, \ \forall \mathbf{w} \in W^N. \tag{48}$$

Therefore, we restrict the system to the admissible control policies only that guarantee the mode of the system is "certain" at sample step k, but the state is not known (recall that this could, e.g., hold when the mode is only determined by the unperturbed part of the state; an example could be a gear box in a car where the gear position determines the mode). It can be easily observed that imposing (48) to the system (32) the state set generated by the disturbance at each sample step k is a *con-*

vex set:

$$x(k;x,\pi,W^k) = x(k;x,\pi,\mathbf{0}) + X(k;i(0),\ldots,i(k-1),W^k) \tag{49}$$

where the first term expresses the nominal trajectory corresponding to the system (33) and the second term represents a convex uncertainty set associated with the state, which depends on the switching mode sequence $i(0),\ldots,i(k-1)$ and on the set W^k.

Using the constraint (48) and the fact that W is a bounded polyhedron with v vertices, let $\mathscr{L}_v^N$ denote the set of indexes ℓ such that $\mathbf{w}^\ell = (w(0)^\ell,\ldots,w(N-1)^\ell)$ takes values only on the vertices of W. It is clear that $\mathscr{L}_v^N$ is a finite set with the cardinality $V_N = v^N$. Further, let $\mathbf{u}^\ell = (u_0^\ell,\ldots,u_{N-1}^\ell)$ denote a control sequence associated with the ℓth disturbance realization $\mathbf{w}^\ell$ and let $x_k^\ell = x(k;x_0,\mathbf{u}^\ell,\mathbf{w}^\ell)$ be the solution of the PWL model (32) with the additional constraint (48). Therefore, given the current state x_k, the MPC optimization to be solved at sample step k becomes

$$\mathscr{V}_{N-k}(x_k) : \min_{\mathbf{u}} \max_{\ell\in\mathscr{L}_v^{N-k}} \sum_{j=0}^{N-k-1} l(x_{k+j|k}^\ell, u_{k+j|k}^\ell) \tag{50}$$

subject to

$$\begin{aligned}
&\text{constraint (48)},\ x_{k|k}^\ell = x_k, \qquad \forall \ell\in\mathscr{L}_v^{N-k}\\
&x_{k+j|k}^\ell \in X_c,\ \ j=1,\ldots,N-k-1,\ \forall\ell\in\mathscr{L}_v^{N-k}\\
&u_{k+j|k}^\ell \in U_c,\ \ j=0,\ldots,N-k-1,\ \forall\ell\in\mathscr{L}_v^{N-k}\\
&x_{N|k}^\ell \in \mathscr{O}_\infty,\ \forall\ell\in\mathscr{L}_v^{N-k}\\
&x_{k+j|k}^{\ell_1} = x_{k+j|k}^{\ell_2} \Rightarrow u_{k+j|k}^{\ell_1} = u_{k+j|k}^{\ell_2}, \forall \ell_1,\ell_2\in\mathscr{L}_v^{N-k}\ .
\end{aligned}$$

where $x_{k+j|k}^\ell$ is the prediction of the state at step $k+j$ given by the model (32), corresponding to the ℓth disturbance realization $(w(0)^\ell,\ldots,w(N-k-1)^\ell)$ and applying the input sequence $u_{k|k}^\ell,\ldots,u_{N-1|k}^\ell$. The constraint (48) is imposed only to the states $x_{k+j|k}^\ell$ with $j=1,\ldots,N-k-1$ and not to $x_{N|k}^\ell$. The only constraint on the state $x_{N|k}^\ell$ is the terminal constraint: $x_{N|k}^\ell \in \mathscr{O}_\infty$. We use a variable horizon scheme as in (Scokaert and Mayne, 1998).

The feedback min-max MPC controller is based on a dual-mode approach. For any $k \geqslant 0$, given the current state x_k, the algorithm is formulated as follows where $u^{\mathrm{RH}}(x)$ denotes the control input applied to the system according to the receding horizon strategy.

Feedback min-max MPC algorithm (Algorithm I)

Off-line step: Compute $\mathscr{O}_\infty$

On-line step: For each k:

Step 1: if $x_k \in \mathscr{O}_\infty \cap \mathscr{P}_i$ then take $u^{\mathrm{RH}}(x_k) = F_i x_k,\ \ \forall i \in \mathscr{I}$

Step 2: otherwise, solve (50) and set $u^{\mathrm{RH}}(x_k)$ to the first control in the optimal solution computed: $u_{k|k}^\ell$.

As regards the stability of this algorithm we have the following result:

Theorem 5.5. *The feedback min-max MPC law $u^{\mathrm{RH}}(.)$ given by Algorithm I makes $\mathscr{O}_\infty$ robustly finite-time stable for the system* (32) *in closed-loop with $u^{\mathrm{RH}}(x)$ with a region of attraction $\bar{X}_N$.*

Proof. See (Necoara *et al.*, 2005*a*).

Note the optimization problem (50) can be recast as a mixed-integer linear programming (MILP) problem when the p-norm used is either $\|\cdot\|_1$ or $\|\cdot\|_\infty$.

6 Discussion

In this paper we have given an overview of our recent results in connection with MPC for PWA systems. More specifically, we have presented an MPC method for MMPS (or equivalently for continuous PWA) systems, and an MPC method for (general) PWA systems. We have considered the disturbances as an extra additive term on the system equations. For each class the proposed approach consists of an on-line step and an off-line step.

For the MMPS and continuous PWA systems we can compute the optimal MPC input using a two-level optimization approach. In first step we have to solve off-line an MP-LP or to compute the vertices of some polyhedral cones and then to write the min-max expression of the worst-case performance criterion. On-line we solve only a sequence of LP problems.

For the general PWA systems the proposed approach was also a two-step approach. First we have use LMIs to find a PWL controller that stabilizes the nominal system. Next, we compute a *convex* robustly positively invariant set for the perturbed PWL system and we steer the system towards this set. To this aim we have proposed a robustly stable feedback min-max MPC scheme that uses the fact that the mode of the system is certain at each step. This MPC scheme is based on solving at each step an MILP problem.

Topics for future research include: a thorough assessment and comparison of the proposed methods including their computational complexity, extension to stochastic disturbances, further improvement of the efficiency of the proposed approaches.

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