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Robust hybrid MPC applied to the design of an adaptive cruise controller for a road vehicle

Daniele Corona, Ion Necoara, Bart De Schutter and Ton van den Boom

***Abstract*— Hybrid Model Predictive Control (MPC) for piecewise affine (PWA) systems is used to solve a control problem for the tracking of a road vehicle. The study originates from the design of an adaptive cruise controller (ACC), that aims to track the velocity transmitted by a leading vehicle. In addition we consider disturbances due to model mismatch and to state measurements. The design specifications and corresponding constraints related to safety and comfort lead to nontrivial control problem. We present the results of a method that merges the use of hybrid MPC for tracking and regulation to a final equilibrium state, for which we compute the terminal cost and constraint set both in the deterministic and the perturbed case.**

I. Introduction

Applying the control techniques developed for linear or smooth nonlinear systems to control hybrid systems is prohibited by the presence of integer variables, which yield complex numerical problems. Nevertheless, a considerable number of control methods were proposed in the literature. The results obtained by Bemporad and Morari in [2] showed that, under general conditions, a *Model Predictive Control* (MPC) based approach can be successfully used to control hybrid systems, by solving *on-line* a mixed integer optimization problem. The control law may also be obtained *off-line* [3] by solving a parameterized mixed integer programming problem. Properties like robustness [12], [17] or stability [8], [14], [16] were also investigated. In the survey [15] an overview on the approaches for guaranteeing stability in nonlinear MPC is presented. The usual technique is based on the computation of a terminal cost and constraint set. The optimal cost is chosen as a candidate Lyapunov function for the closed-loop system.

In this paper we design an on-line hybrid MPC controller applied to tracking during transient and regulation to a stationary state. This study was motivated by the design of an adaptive cruise controller (ACC) for a Smart. The target is to follow a leading vehicle in a highway environment. In order to meet realistic conditions several constraints are introduced, fulfilling safety, comfort and environmental issues. We assume that the reference trajectory, transmitted by the leading vehicle, will eventually reach a stationary state, around which we guarantee stability and feasibility of the controller, even in presence of bounded disturbances. In general this stationary value is arbitrary, in the hybrid framework we choose it along the switching manifold. The main added value of this paper lies in the combination and application of several existing hybrid techniques for a relatively simple but challenging case study. Furthermore the results of this study point to interesting issues that merit further investigation.

We first describe the model, the optimal control problem and the goals. Then we present the procedure used to construct the terminal cost and constraint set to guarantee stability even in presence of disturbances. Finally we show simulation results on the considered application, highlighting the necessity of a robust stabilizing hybrid control, at the price of a higher computational effort.

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II. Model and problem statement

Model We consider the following nonlinear system

$$\dot{x} = f(x) + Bu + w \tag{1}$$

$$\text{s.t.} \qquad \bar{e}_x \vartheta + \bar{e}_u u + \bar{e}_w w \leq \bar{g} \tag{2}$$

where $f(x)$ is a continuous function, B is a constant matrix, w is a bounded disturbance, ϑ is a set of parameters. Equation (2) defines the feasible set of the control problem.

Problem We address the issue of designing the control input $u(t)$, $t > 0$ for system (1) that tracks a time-varying reference signal $[r(t)^{\mathrm{T}}, u_{\mathrm{ref}}(t)^{\mathrm{T}}]^{\mathrm{T}}$. We assume the property that $r(t) \to x_{\mathrm{e}}$, $u_{\mathrm{ref}}(t) \to u_{\mathrm{e}}$, as $t \to +\infty$ and that $[x_{\mathrm{e}}^{\mathrm{T}}, u_{\mathrm{e}}^{\mathrm{T}}]^{\mathrm{T}}$ satisfy $f(x_{\mathrm{e}}) + Bu_{\mathrm{e}} = 0$ and the constraints (2).

Approach This framework is suitable for MPC-based techniques. In fact MPC handles constrained problems, affected by disturbances and/or measurements errors. MPC is commonly used in discrete-time (DT) and is in general an on-line tool. Thus, a DT representation of system (1) with a sampling time T adequate to trade off system behavior with on-line optimization fulfillments. We tackle the problem above in two steps. First we provide as prediction model a PWA approximation of vector field $f(x)$. To this purpose we construct m affine systems (A_i, F_i), $i = 1, \ldots, m$ and a partition $\mathcal{P}$ of the state space such that $f(x) \approx \mathcal{A}_i x + \mathcal{F}_i$, $x \in \mathcal{P}_i$, where $\mathcal{P}_i$'s are polyhedral sets, and then we derive the corresponding DT representation [4]. Hence we consider

$$\begin{aligned} &x(k+1) = A_i x(k) + B_i u(k) + F_i + w(k), \ x \in \mathcal{P}_i \\ &\text{s.t. } e_x \vartheta(k) + e_u u(k) + e_w w(k) \leq g, \end{aligned} \tag{3}$$

where $\vartheta(k)$ is a vector of parameters given at time k. Secondly, we consider the following constrained optimization problem.

Problem 1: Let the prediction horizon N_{p} ($1 \leq N_{\mathrm{p}} < \infty$) be given and $Q, Q_{N_{\mathrm{p}}}, R > 0$ be matrices of appropriate

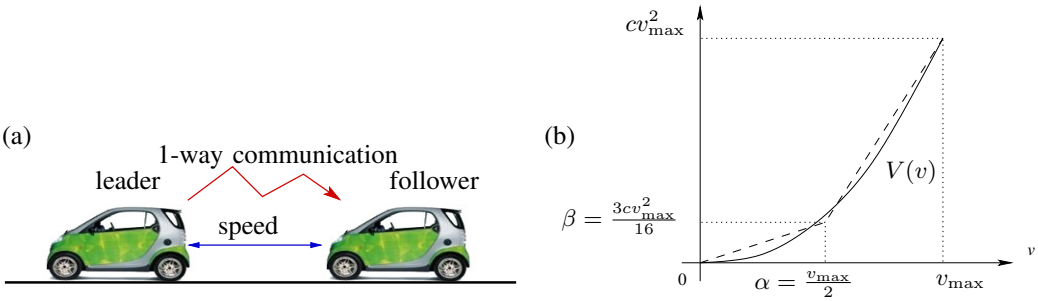

Fig. 1. *(a) ACC set up and (b) nonlinear to PWA approximation.*

dimensions. We define the problem

$$\min_{\mathbf{u}\in\Pi_{N_\mathrm{p}}(\vartheta(k))} \Big\{ V_{N_\mathrm{p}}(\vartheta(k),\mathbf{u}) \triangleq \sum_{j=0}^{N_\mathrm{p}-1} L(\varepsilon(k+j),\eta(k+j)) + F(\varepsilon(k+N_\mathrm{p})), \Big\} \tag{4}$$

s.t. (3), where $\varepsilon(k) = x(k) - r(k)$ is the *tracking error*, $\eta(k) = u(k) - u_{\mathrm{ref}}(k)$ and $\mathbf{u}(k) = [u(k)^{\mathrm{T}}, \ldots, u(k+N_\mathrm{p}-1)^{\mathrm{T}}]^{\mathrm{T}}$ is the sequence of N_p control inputs constrained to the feasible set $\Pi_{N_\mathrm{p}}(\vartheta)$, see (5). We define $F(\varepsilon(k)) \triangleq ||Q_{N_\mathrm{p}}\varepsilon(k)||_p$ and $L(\varepsilon(k),\eta(k)) \triangleq ||Q\varepsilon(k)||_p + ||R\eta(k)||_p$. ■

At each time step N_p samples of the reference trajectory are known (predicted or pre-scheduled). Suppose that Problem 1 is feasible and let $\mathbf{u}^0(\vartheta(k))$ denote its optimal solution. Then, the control action $u^0(k) \triangleq \left(\mathbf{u}^0(\vartheta(k))\right)_1$ (i.e. the first block element of the vector $\mathbf{u}^0(\vartheta(k))$ is applied to the nonlinear model (1), in a *receding horizon* manner. Next, the set of parameters $\vartheta(k+1)$ is updated at time step $k+1$ and, a new optimal control problem is solved to obtain the new control action $u^0(k+1)$.

We study problem (1) for norms $p = 1$ and $p = 2$, to allow the conversion into a constrained mixed integer linear and quadratic programming (MILP and MIQP) [1], [7], [13], for which we use the solver `Cplex`.

The design of the control law is split into two phases: during the transient of the reference trajectory we consider *tracking*. When the reference has reached its steady state we enforce *stability* of the hybrid MPC (which is not guaranteed a priori [2]) in the regulation. To this aim we compute a terminal cost and a constraint set, as in [16]. The disturbance $w(k)$, due to the PWA approximation and to the error measurements, may give infeasibilities. For this reason we also implement *robust min max* MPC [15], and use the algorithm in [17] to guarantee robust stability.

So far it is not yet possible to ensure stability and robustness for time-varying reference signals, thus we compute the invariant set and the corresponding terminal constraint in stationary condition.

III. MPC FOR PWA SYSTEMS

In this section the methods used to compute the invariant set and the terminal cost and constraint set in the deterministic and robust case will be briefly summarized. In particular the following subsection deals with model (3) considered unperturbed, i.e., $w = 0$. The reader is referred to [16], [17] for detailed descriptions and the proofs. It is relevant to note that this study is significant when the equilibrium pair $[x_\mathrm{e}^{\mathrm{T}}, u_\mathrm{e}^{\mathrm{T}}]^{\mathrm{T}}$ belongs to the switching manifold. To simplify notation we will omit the dependencies of k from ϑ and u.

A. Deterministic MPC

Given an equilibrium pair $[x_\mathrm{e}^{\mathrm{T}}, u_\mathrm{e}^{\mathrm{T}}]^{\mathrm{T}}$ for the system, i.e. $x_\mathrm{e} = A_i x_\mathrm{e} + B_i u_\mathrm{e} + F_i$ for all $i \in \mathcal{I}_0 \subseteq \mathcal{I}$ we want to determine a stabilizing PWA feedback controller $u(x) \triangleq \varphi_i x + \gamma_i$ and the corresponding *common* Lyapunov function: $V(x) \triangleq (x - x_\mathrm{e})^{\mathrm{T}} P (x - x_\mathrm{e})$, respectively. In [16] we provide a detailed discussion about the computations (based on the *S-procedure*) of φ_i, γ_i and P using the structure of the system. For the closed loop system $x(k+1) = (A_i + B_i\varphi_i)x(k) + B_i\gamma_i + F_i$, $x \in \mathcal{P}_i$, we construct a *positively invariant set* X_f [16], [17]. The control objectives are stability and performance within the constraints in (3) with $w = 0$ (i.e., unperturbed). This is achieved as follows: at time step k the control is determined by solving problem (1) with $r(k) = x_\mathrm{e}$ and $u_{\mathrm{ref}}(k) = u_\mathrm{e}$. For each ϑ at time k we define the feasible control set:

$$\Pi_{N_\mathrm{p}}(\vartheta) \triangleq \{\mathbf{u} : \xi(\vartheta_i, u_i) \leq g, \forall i \in \underline{N_\mathrm{p}-1}, x_{N_\mathrm{p}} \in X_\mathrm{f}\}, \tag{5}$$

where $\xi(\vartheta_i, u_i) = e_x\vartheta_i + e_u u_i$ and $\underline{N} \triangleq \{1, \ldots, N\}$. We define an MPC scheme based on a terminal cost and constraint set approach [15]:

$$V_{N_\mathrm{p}}^0(\vartheta) = \min_{\mathbf{u}\in\Pi_{N_\mathrm{p}}(\vartheta)} V_{N_\mathrm{p}}(\vartheta,\mathbf{u}) \tag{6}$$

MPC is an iterative approach in which at each event (ϑ, k) the optimal control sequence $\mathbf{u}^0(\vartheta) \triangleq \arg\min_{\mathbf{u}\in\Pi(\vartheta)} V_N(\vartheta,\mathbf{u})$ is computed but only the first component $u_0^0(x)$ is applied to the system.

Proposition 3.1: [16] Using the 2-norm (i.e., $p = 2$), the unperturbed PWA system (3) in closed loop with the MPC controller $\left(\mathbf{u}^0(\vartheta(k))\right)_1$ is asymptotically stable.

B. Robust MPC

The difference between the model of the plant (1) and the prediction model (3) and the presence of disturbances may cause poor performance of the MPC controller, leading to bad tracking or even to infeasibility. Therefore, it is natural to consider robust MPC, which takes care of this mismatch. We now consider (3) as prediction model, where the disturbance w lies in a given polytope: $W = \{w \in \mathbb{R}^q : Sw \leq s\}$. With the feedback controller $u(x) = \varphi_i x + \gamma_i$ derived in Section III-A, we construct a *robustly positively invariant set*

TABLE I

Definitions and values of the entries of equation (8).

m	Mass of vehicle	800 kg
c	Viscous coefficient	0.5 kg/m
μ	Coulomb friction coefficient (dry asphalt)	0.01
b	Traction force	3700 N
g	Gravity acceleration	9.8 m/s^2

TABLE II

Values of the parameters specifying the constraints.

T	Sampling time	1 s
$x_{\min}$	Minimum velocity	5.0 m/s
$x_{\max}$	Maximum velocity	37.5 m/s
a_{acc}	Comfort acceleration	2.5 m/s^2
a_{dec}	Comfort deceleration	-1 m/s^2
$u_{\max}$	Maximum throttle/brake	1
Δ_u	Maximum throttle/brake variation	0.2
α	Switching velocity	18.75 m/s

$\bar{X}_{\text{f}}$ for the closed loop system $x(k+1) = f(x(k), u(x(k))) + w(k)$, using the procedure in [17].

Effective control in the presence of disturbance requires to optimize over feedback policies [15] rather than open-loop input sequences. Let $\mathbf{w} = [w_0^T, ..., w_{N_{\text{p}}-1}^T]^T$ be a possible realization of the disturbance over the interval 0 to $N_{\text{p}}-1$, i.e. $\mathbf{w} \in \mathcal{W} = W_{\text{p}}^N$. Also, let $\phi(k; x, \pi, \mathbf{w})$ denote the solution of (3) at step k when the initial state is x at step 0, the control is determined by the policy $\pi = (\mu_0(\cdot) \dots \mu_{N_{\text{p}}-1}(\cdot))$ and the disturbance sequence is $\mathbf{w}$. For the stage cost we impose that $L(x, u) = 0$ if $x \in \bar{X}_{\text{f}}$. We restrict the admissible control policies π to those that guarantee that for every value of the disturbance the mode of the system $i(k)$ is unique at each time sample k: $\phi(k; x, \pi, \mathbf{w}) \in \mathcal{P}_{i(k)}$, $\forall \mathbf{w} \in \mathcal{W}$. Since W is a bounded polyhedron with v vertexes let $\mathcal{L}_v^{N_{\text{p}}}$ denote the set of indexes ν such that $\mathbf{w}^\nu = [w_0^{\nu,T} \dots w_{N_{\text{p}}-1}^{\nu,T}]^T$ takes values only on the vertexes of $\mathcal{W}$. It is clear that $\mathcal{L}_v^{N_{\text{p}}}$ is a finite set with the cardinality $V_{N_{\text{p}}} = v^{N_{\text{p}}}$. Further, let $\mathbf{u}^\nu = [u_0^{\nu,T} \dots u_{N_{\text{p}}-1}^{\nu,T}]^T$ denote a control sequence associated with the ν-th disturbance realization $\mathbf{w}^\nu$ and let $x_i^\nu = \phi(i; x, \mathbf{u}^\nu, \mathbf{w}^\nu)$. For each event $\vartheta(k)$ the set of feasible policies is:

$$\bar{\Pi}_{N_{\text{p}}}(\vartheta) \triangleq \{\mathbf{u}^\nu : \xi(\vartheta_i^\nu, u_i^\nu) \leq g, \forall \nu \in \mathcal{L}_v^{N_{\text{p}}}, \forall i \in \underline{N_{\text{p}} - 1}, \\ \vartheta_i^{\nu_1} = \vartheta_i^{\nu_2} \Rightarrow u_i^{\nu_1} = u_i^{\nu_2}, \forall \nu_1, \nu_2 \in \mathcal{L}_v^{N_{\text{p}}}, x_{N_{\text{p}}} \in \bar{X}_{\text{f}}\}.$$

Given a maximum prediction horizon $N_{\text{p,max}}$, the robust feedback min-max optimization problem that we solve at event $\vartheta(k)$ is the following:

$$\bar{V}_{N_{\text{p}}}^0(\vartheta) \triangleq \min_{\substack{\mathbf{u}^\nu \in \bar{\Pi}_{N_{\text{p}}}(\vartheta) \\ N_{\text{p}} \in \underline{N_{\text{p,max}}}}} \max_{\nu \in \mathcal{L}_v^{N_{\text{p}}}} V_{N_{\text{p}}}(\vartheta, \mathbf{u}^\nu) \tag{7}$$

The feedback min-max MPC controller is based on a dual-mode approach. For any $k \geq 0$, given the current state x, the algorithm is formulated as follows:

1) if $x \in \bar{X}_{\text{f}} \cap \mathcal{P}_i$ then $u(x) = \varphi_i x + \gamma_i$, $\forall i \in \mathcal{I}$;

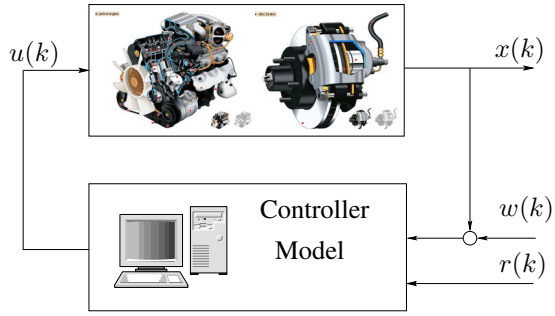

Fig. 2. *MPC setup: prediction model vs. simulation model and disturbance injection.*

2) otherwise, solve (7) and set $u(k)$ equal to the first sample in the optimal solution computed.

Proposition 3.2: [17] The uncertain PWA system (3) in closed loop with the feedback min-max MPC law given by the *Algorithm 1* makes $\bar{X}_{\text{f}}$ *robustly finite-time stable [11].*

From computational point of view, the optimization problem (7) can be recast as an MILP if $p \in \{1, \infty\}$ or as an MIQP if $p = 2$.

IV. Cruise setup and simulations

The goal of a cruise controller is to track the velocity of the vehicle in front, to guarantee secure driving, smoothness of platoons traffic [10], comfort and optimal usage of the engine/brake system. The descriptive scenario is shown in Figure 1.a, where two cars are driving after another. We consider here platoons of two vehicles, but the extension to general case is also possible. The speed measurements/predictions of the leading vehicle are collected and transmitted to the follower that will consider it as a reference. We first describe the general setup, then we implement a *deterministic* MPC in both 1 and 2-norm. We observe that due to disturbances, infeasibilities occur, motivating thus the use of *robust* MPC. When the reference reaches a stationary value, we implement, for both cases, the stabilizing methods in Section III, by plugging into the MPC scheme the terminal cost and constraint set.

Model We consider a nonlinear *viscous* friction and a *road-tire* static friction, proportional to the mass m of the vehicle. Braking will be simulated by applying a negative input. The model for positive velocity of the rear vehicle is:

$$m\dot{x}(t) + cx^2(t) + \mu\, mg = bu(t) \tag{8}$$

where $x(t)$ is the speed of the vehicle, $bu(t)$ is the traction/brake force, proportional to the input $u(t)$. Numerical values are listed in Table I. An approximation (Figure 1.b) of the nonlinear friction curve $V = cv^2$ leads to the PWA system:

$$\begin{cases} m\dot{x}(t) + c_1 x(t) + f_1 = bu(t), & x < \alpha \\ m\dot{x}(t) + c_2 x(t) + f_2 = bu(t), & x \geq \alpha, \end{cases}$$

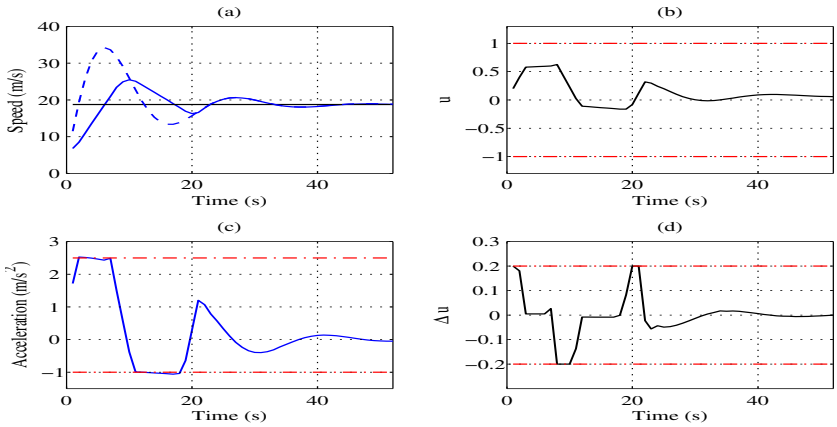

Fig. 3. *Results for 1-norm deterministic MPC without external noise. (a) System (solid) and reference (dashed) velocities, (b) Control input and constraints, (c) Acceleration and constraint (note the violations), (d) Δu and constraints.*

where the coefficients c_1, c_2, f_1, f_2 are derived via least square[1]. The DT ($T = 1s$) model is

$$\begin{cases} x(k+1) = A_1 x(k) + B_1 u(k) + F_1 + w(k), x < \alpha \\ x(k+1) = A_2 x(k) + B_2 u(k) + F_2 + w(k), x \geq \alpha \end{cases} \tag{9}$$

with $A_1 = 0.9912$, $B_1 = 4.6047$, $F_1 = -0.0976$, $A_2 = 0.9626$, $B_2 = 4.5381$, $F_2 = 0.44284$, and $w(k) \in [-0.5, 0.5]$ is a bounded disturbance on the measurement on the speed.

Constraints Safety, comfort and economy or environmental issues constrain the state x and the control input u. In particular we consider limitations on the velocity, acceleration, control input $u(k)$ and its variation $u(k+1) - u(k)$. We require, for all k,

$$\begin{aligned} x_{\min} \leq x(k) \leq x_{\max} \\ -u_{\max} \leq u(k) \leq u_{\max} \\ a_{\text{dec}} T \leq x(k+1) - x(k) \leq a_{\text{acc}} T \\ -\Delta_u T \leq u(k+1) - u(k) \leq \Delta_u T. \end{aligned} \tag{10}$$

Numerical values are listed in Table II. Although some of these constraints may be violated without causing major damages, i.e., collision or engine breakdown, we consider all of them as *hard*. For the sake of simplicity only velocity was modeled. A more general setup, with position constraints, is studied in [5].

Prediction model The PWA model (9) is transformed into a *mixed logical dynamics* (MLD) model using a binary variable $\delta(k)$ [2] that equals 0(1) when the active mode is system 1(2). More precisely the MLD model is $x(k+1) = A_1 x(k) + Lv(k) + F_1 + w(k)$, where $L = [A_2 - A_1 | B_2 - B_1 | F_2 - F_1 | B_1]$ and $v(k) = [z(k), y(k), \delta(k), u(k)]^{\text{T}}$ (with $z(k) = x(k)\delta(k)$, $y(k) = u(k)\delta(k)$, $\delta(k) \in \{0, 1\}$) is the auxiliary *mixed logical* vector. To overcome the nonlinearity of $z(k), y(k)$ we introduce the inequalities [2]:

$$\begin{aligned} x_{\min}\delta(k) \leq z(k) \leq x_{\max}\delta(k) \\ -x_{\max}(1 - \delta(k)) \leq z(k) - x(k) \leq -x_{\min}(1 - \delta(k)) \\ |y(k)| \leq u_{\max}\delta(k) \\ |y(k) - u(k)| \leq u_{\max}(1 - \delta(k)), \end{aligned}$$

[1]A finer approximation is possible by setting more breakpoints.

and the switching condition $-\delta(k)(v_{\min} - \alpha) \leq x(k) - v_{\min}$ and $\delta(k)(\alpha - v_{\max}) \leq -x(k) + \alpha$. Finally, we have the following prediction model:

$$x(k+1) = A_1 x(k) + Lv(k) + F_1 + w(k) \tag{11}$$

$$\text{s.t. } \bar{e}_x \vartheta(k) + \bar{e}_u u(k) + \bar{e}_w w(k) \leq \bar{g}, \tag{12}$$

$k \geq 0$, where $\vartheta(k) = [x(k), u(k-1)]^{\text{T}}$ is the vector of parameters.

Tracking and regulation The weight matrices in problem (1) are $Q = 1$ and $R = 0.01$. The length of the prediction horizon is $N_{\text{p}} = 4$. The goal is to tune an on-line controller $u(k)$ that tracks the reference velocity within the constraints (12). The initial velocity is $x_0 = 6$ m/s. As depicted in Figure 2, at each k the controller receives the N_{p} steps ahead prediction of the reference speed of the front vehicle. Specific scenarios, such as intelligent platooning or automatically driven/supervised vehicles, allow to predict *exactly* the future actions of the front vehicle. In road traffic scenario the intentions of the driver may be modeled statistically, or taken constant, or averaged on past actions.

By measuring the current speed, it computes the best control action using the prediction model (11)-(12) and feeds the first element to the car actuators, modeled here by equation (8). In the framework of the hybrid system it is relevant to study the behavior of the stabilizing controller around the switching velocity $x_{\text{e}} = \alpha$. Without loss of generality, our reference signal has the steady state $x_{\text{e}} = \alpha$.

Simulations using deterministic MPC: The stabilizing controller obtained via the Lyapunov arguments described in Section III-A is $u(x) = -0.072x + 1.411$, and we observe that it is common for both subsystems. Additionally we obtain the terminal cost $Q_{N_{\text{p}}} = 1.766$ and the terminal constraint set $X_{\text{f}} = \{x : 16.011 \leq x \leq 21.488\}$ to be used only when the reference signal of the front vehicle has reached its stationary value.

In Figure 3.(a-d) we show Velocity, Control, Acceleration and Δu, obtained using the 1-norm in the unperturbed case. The same simulation with the 2-norm is depicted in Figure 4. From both figures we point out that even in the absence

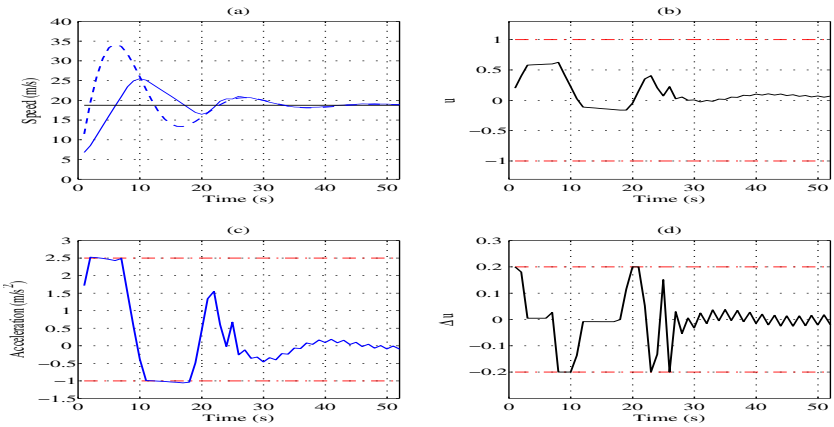

Fig. 4. *Results for the 2-norm deterministic MPC without external noise. (a) System (solid) and reference (dashed) velocities, (b) Control input and constraints, (c) Acceleration and constraint (note the violations), (d) Δu and constraints.*

TABLE III

Number of variables, constraints and on-line time.

MPC Problem	n_c	n_v	n_d	Time (s)
1-norm deterministic (MILP)	20	4	82	0.063
2-norm deterministic (MIQP)	16	4	66	0.059
1-norm robust (MILP)	310	4	1336	0.6

of external disturbances there is a minor violation of the acceleration constraints, due to model mismatch. We also highlight a numerical aspect of the solution obtained using the 2-norm criterion. In the vicinity of the equilibrium point the solver presents some chattering, that is not present in the 1-norm solution. The MIQP problem related to the MPC cost minimization is positive semidefinite because of the unweighted auxiliary variables of the MLD system. This may provoke numerical inaccuracy around the global minimum. Although stability is guaranteed *a priori* only in the 2-norm case (Proposition 3.1), in this case the 1-norm is also stable.

In Figure 5 we show the solution offered by the deterministic MPC when the disturbance signal is active[2]. The large infeasibilities motivate the use of the robust MPC.

In Table III we give data of the on-line problems that affect the computational effort: the number of continuous and discrete variables n_c and n_d, of constraints n_v and the maximum computation time during the simulation. In both deterministic cases the latter is approximately 0.06 s, smaller than the sample time T.

Simulations using robust MPC: Figure 6 shows the results obtained with the use of robust MPC described in Section III-B. This robust MPC scheme is implemented only in 1-norm, leading to a MILP problem, of size given in Table III. We observe the benefits of robust MPC which eliminates the infeasibilities due to disturbances. For this robust rejection we use the terminal cost $Q_{N_p} = 0$ for all simulation time. Using the arguments of Section III-B, we construct, around the reference stationary state, a *robust positively invariant* set to which it corresponds the terminal constraint set $\bar{X}_f = \{x : 11.903 \leq x \leq 22.634\}$, that guarantees stability of the hybrid MPC despite the disturbances. Note that both sets $(X_f, \bar{X}_f)$ are not the maximal, thus they may be different. The robust MPC requires a significantly higher computation effort, as remarked in Table III. In fact the number of variables of the MILP corresponding to the $\min \max$ optimal control problem (7) grows exponentially with N_p, making the method unpractical in fast applications. The 2-norm case was omitted here because computationally very demanding.

[2]The signal $w(k)$ is random, but common in all simulations.

V. Computational Complexity

The mixed-integer programs are widely solved with *branch and bound* methods. The general paradigm of branch and bound deals with optimization problems over a search space that can be presented as the nodes of a tree. New subproblems are created by restricting the range of the integer variables. The whole subtree below a specific node i can be pruned as long as the corresponding LP/QP problem is either infeasible or its lower bound is greater than the lower bound of all *pending* nodes [9]. Improved lower bounds are obtained in the context of the *active set methods*. Despite the numerous techniques that, in practice, speed up the search considerably, none of the branch and bound algorithms guarantees to lower the worst-case exponential complexity.

VI. Conclusions

We have studied the problem of designing an adaptive cruise controller for a road vehicle by means of hybrid MPC, which enabled us to tackle the control problem in an MILP/MIQP formulation. Once the reference has reached a steady state value we have also considered the problem of guaranteeing stability, via the construction of terminal cost and invariant set. Moreover we have considered the robust MPC that allowed us to remove infeasibilities due to disturbances, at the cost of higher computational effort. It is not yet clear whether the hybrid MPC with feasibility and stability properties may be extended to tracking problems, opening a possible area of future investigation [6].

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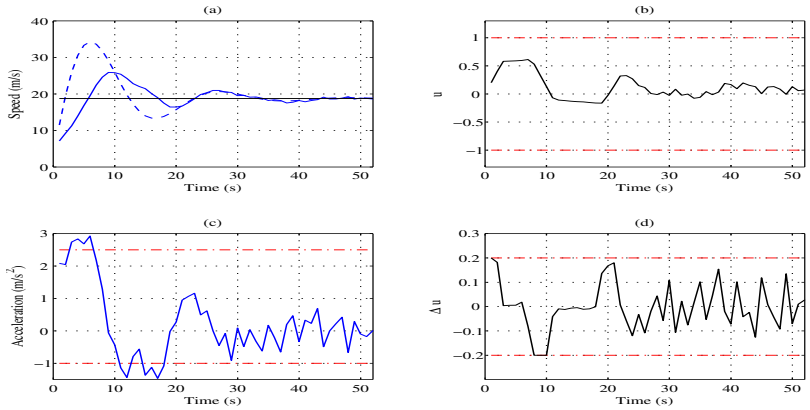

Fig. 5. *Results for 1-norm deterministic MPC with external noise. (a) System (solid) and reference (dashed) velocities, (b) Control input and constraints, (c) Acceleration and constraints (note the violations), (d) Δu and constraints.*

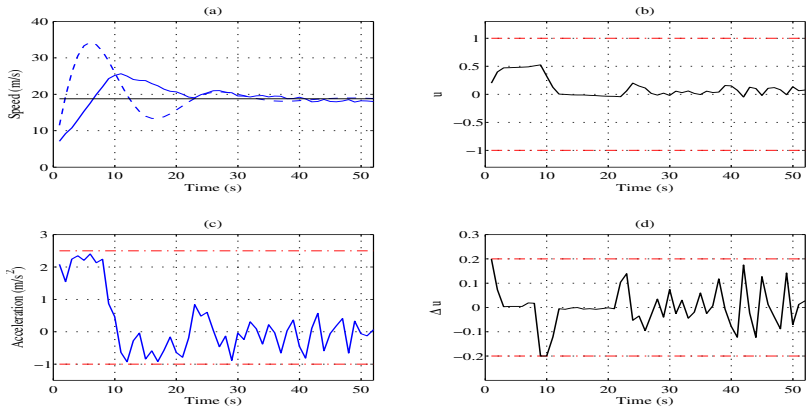

Fig. 6. *Results for robust MPC with external noise. (a) System (solid) and reference (dashed) velocities, (b) Control input and constraints, (c) Acceleration and constraints, (d) Δu and constraints. No infeasibilities occur.*

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