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Delft Center for Systems and Control
Delft University of Technology
Mekelweg 2, 2628 CD Delft
The Netherlands
phone: +31-15-278.24.73 (secretary)
URL: https://www.dcsc.tudelft.nl

Stabilization of switched affine systems: An application to the buck-boost converter

Daniele Corona, Jean Buisson, Bart De Schutter, Alessandro Giua

***Abstract*— In this paper we extend a technique developed to design a feedback stabilizing control law for autonomous switched systems all modes of which are unstable. More specifically, we extend the *switching table procedure* to the class of affine switched systems, the dynamics of which either do not have an equilibrium point or, if they do, it is not common. This method is then applied to the DC-DC *buck-boost converter*. The design of the control law is based on dynamic programming and it results in a partition of the state space into switching look-up tables. A comparison with a Lyapunov based technique is also discussed.**

I. INTRODUCTION

Switched systems [19] form a particular class of hybrid systems where the occurrence of a discrete event, controlled or uncontrolled, triggers the change in the mode of the system. As a consequence of the highly sophisticated technology in electronics observed in the last decades, countless physical plants, machines and devices integrate discrete and continuous behavior and they can be modeled in the switched system framework. An important class of switched systems, called *autonomous* [19], is characterized by the sole control action of the switching signal. One of the milestone papers in the field is [2], where the author, through a simple example, highlights some paradoxical behaviors of this class.

Among the many application fields of switched systems we consider power converters (Boost, Buck, multilevel converters), that are widespread used in industry, and in particular in variable speed DC motor drives, in computer power supplies, in cell phones and in cameras. They are electrical circuits controlled by switches (transistors, diodes), used to adapt the energy supplied by a power source to a load. Aiming at reducing switching losses and EMI (Electromagnetic Interference) of power converters, a lot of soft switching techniques are developed so that high efficiency, small size and low weight can be achieved. In nominal conditions, these circuits have been designed so that the switching action does not provoke discontinuity.

Practically, these devices are controlled through a Pulse-Width-Modulation (PWM) where the switching behavior of the closed loop system is averaged with a nonlinear model [18]. Continuous control approaches are then used, among which passivity based control [18] and sliding mode control [20].

D. Corona and B. De Schutter are with the Delft Center for Systems and Control, Delft University of Technology, Delft, The Netherlands. B. De Schutter is also with the Marine and Transport Technology department, Delft University of Technology. d.corona@tudelft.nl,b@deschutter.info

J. Buisson is with Supélec-IETR, Cesson-Sévigné, Rennes, France. jean.buisson@supelec.fr

A. Giua is with the Department of Electrical and Electronic Engineering, University of Cagliari, Italy. giua@diee.unica.it

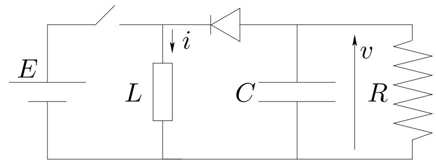

Fig. 1: *Circuit scheme of the buck-boost converter.*

Alternatively these devices are good candidate for hybrid modeling and they can be modeled by switched systems (without jumps). For a complete, general study on analysis and design of switched system we refer to the recent books [16], [19], where stability, robustness, controllability and optimal control are studied. In the context of stability analysis and control design a standard technique is to investigate the conditions of existence of a common [17] or multiple Lyapunov function [2], or to use geometric [13] approaches. The stabilization of a switched system can be transformed into a non-convex problem, for which LMIs [8], [10] or iterative methods [14] may be used. Properties of uniform stability for a switched system were studied in [12].

A possible technique used to stabilize switched systems is described in [5] and it is based on an optimal control approach. As explained in [7], this method, the *Switching Table Procedure* (STP), is viable when all dynamics admit a common equilibrium point. Here we provide an extension to the case where the system is affine and the dynamics have no common equilibrium or no equilibrium.

II. THE BUCK-BOOST CONVERTER

In order to derive models for DC-DC converters, different energy based approaches, such as circuit theory, bond graphs, Euler Lagrange, Hamiltonian approach can be used. For switching systems, extensions have been proposed for the Hamiltonian approach [9] or for the bond graph approach [3]. In most of these systems, one physical switch is controlled (e.g. a transistor), while the other one may be not (e.g. a diode).

A simple circuit representation of the *ideal* buck-boost converter is depicted in Figure 1. The continuous source E has a negligible internal resistance and infinite power. No energy is lost in the inductor L nor in the capacitor C. The diode has no voltage drop in conducting mode and switches exactly at zero voltage level. In a normal operating mode of an ideal converter both the controlled and uncontrolled switches occur simultaneously.

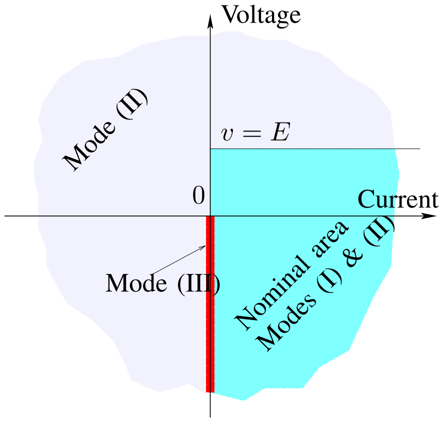

Fig. 2: *Partition of the state space for the different modes of the converter.*

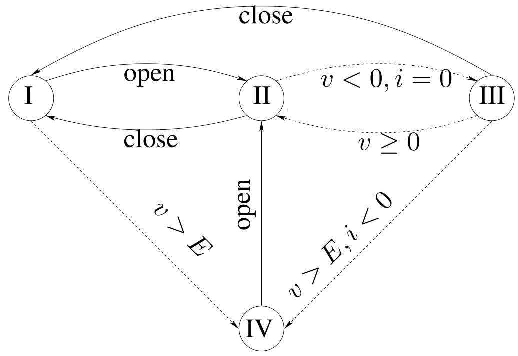

Fig. 3: *Oriented graph of the switching behavior of the converter. Solid lines: controlled switches, dashed lines: diode state-based switches.*

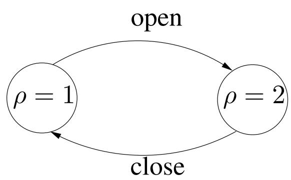

Fig. 4: *Oriented graph with only the 2 nominal operating modes.*

The converter *theoretically* has four possible operating modes. We label them with the variable ρ and we denote, as in Figure 1, by v the voltage on the capacitor and by i the inductor current. The four modes are: (I) switch closed, diode blocked ($\rho = 1$), (II) switch open, diode conducting ($\rho = 2$), (III) switch open, diode blocked ($\rho = 3$), (IV) switch closed, diode conducting ($\rho = 4$).

In nominal behavior only modes (I) and (II) are involved. The nominal working area of the space state is $\mathcal{N} \equiv \{(i, v) \in \mathbb{R}^2 \; : \; i \geq 0, v \leq E\}$ depicted in Figure 2 in the dark-shaded area (right-bottom area of the figure). The four modes are represented by the nodes of the oriented graph in Figure 3. The arcs indicate the discrete transitions from one mode to another; the controlled switches are solid lines, while the diode switches, depending on the state of the system, are dashed.

In state (I) the battery transfers energy into the inductor while, on the load side, the capacitor is feeding the load. After some time the switch is opened and the system goes to mode (II) where the energy stored in the inductor can now flow towards the load and the capacitor. Then the controller may close the switch again to mode (I) and so on. If the duration in mode (II) is protracted all the magnetic energy is transferred to the load and the buck-boost converter switches to the *discontinuous* mode (III) [11]. This state is reached when the condition $i = 0, v < 0$ is attained. In this mode the current remains equals to zero and the capacitor is feeding the load. From (III) it is possible to switch to (I) by closing the switch. Finally let us observe that mode (IV) is in fact critical, because it imposes two different voltage levels in the same point (v on the anode and E on the cathode of the diode in conducting mode). If for some reason the voltage v overtakes E when the switch is closed, a *safe* controller must immediately open the switch leading to mode (II), in order to prevent harmful current peaks across the diode. Mode (IV) is, to some extent, a *fault* mode.

Let us denote by $x = [i, v]^{\mathrm{T}}$ the state. The differential equations corresponding to each location of the graph in Figure 3, are the following. In location (I) $\dot{x} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix} x + \begin{bmatrix} \frac{E}{L} \\ 0 \end{bmatrix}$, in location (II) $\dot{x} = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix} x$, in location (III) $\dot{x} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix} x$.

In the ideal case, it is not possible to provide a model for location (IV), as it violates the laws of the electrical networks. Under such assumption it is allowed to remove location (IV) from the model. The attention may thus be focused on a model that only contains three states and whose dynamics are given above. Furthermore we also assume that the controller of the switch is fast enough to prevent the complete discharge of the inductor during the evolution in location (II), allowing to disregard the third dynamics. This framework is relevant when the working point of the converter admits an *invariant region*[1] completely included in the nominal working area in Figure 2.

These considerations lead to restrict the model in Figure 3 to the one depicted in Figure 4 with dynamics [4]:

$$\dot{x} = \begin{bmatrix} 0 & \frac{\rho-1}{L} \\ \frac{1-\rho}{C} & -\frac{1}{RC} \end{bmatrix} x + \begin{bmatrix} \frac{2-\rho}{L} E \\ 0 \end{bmatrix} = A_\rho x + F_\rho, \quad (1)$$

where $\rho \in \{1, 2\}$ is the switching signal. The system is described in terms of the state variables and a control signal $\rho(t)$, which switches among the possible modes in order to stabilize to a specific operating point x_{p}.

III. THE SWITCHING TABLE PROCEDURE

The method, based on the switching table procedure (STP), used to obtain the control law is described in [7] and [5], Chapter 7. It can be applied to the class of switched autonomous systems, $\dot{x} = A_\rho x$, denoted by $\{A_\rho\}_{\rho \in \mathcal{S}}$, where $\mathcal{S}$ is a set of s modes indexed by ρ. It consists in

[1]For each initial point taken in such region there exists a controlled switching sequence that keeps the state within the invariant region.

determining a partition of the state space that indicates what mode A_ρ should be active for the current state value. The partition can be obtained by solving

$$\begin{array}{rl} J(x_0,\rho_0) = \min\limits_{\rho(t)} \displaystyle\int_0^{+\infty} x^{\mathrm{T}} Q_{\rho(t)} x dt \\ \text{s.t.} \quad \dot{x}(t) = A_{\rho(t)} x(t) \\ (x(0),\rho(0)) = (x_0,\rho_0) \end{array} \tag{2}$$

parameterized on the initial point (x_0, ρ_0), where Q_ρ are appropriate weight matrices.

We now briefly sketch how the STP can be used for stabilizing purposes. This will be done in three steps. A complete description and proofs can be found in [6], [7]. Initially we consider only a finite number N of switches, assuming that at least one dynamics of $\{A_\rho\}_{\rho\in\mathcal{S}}$ is stable. Then we show how the procedure can be extended to the case of $N = \infty$. Finally, we relax the assumption on the stability of at least one dynamics and show how the STP can serve to design a stabilizing control law.

Step 1 In the first step we show that the optimal control law for the optimization problem (2) takes the form of a state feedback. When k out of N switches are available the current *hybrid system state* (x, ρ) indicates, via a look-up table $\mathcal{C}_k^\rho$, whether a switch from the current dynamics A_{ρ_k} to $A_{\rho_{k-1}}$, should occur. The look-up table $\mathcal{C}_k^\rho$ is a partition of the state space into different regions $\mathcal{R}_\rho$, where $\rho \in \mathcal{S}$ is the mode to switch to if $x \in \mathcal{R}_\rho$. For autonomous systems and quadratic cost these partitions are *homogeneous*, i.e., if a strategy is valid for a specific $\bar{x}$, then it is also valid for any point $\lambda\bar{x}$, $\lambda \in \mathbb{R}^+$, allowing to restrict the interest to a unitary semi-sphere Σ_n.

The tables are constructed *recursively*, on the increasing number k of remaining switches, using the information already computed when $k-1$ switches are available. The procedure is iterated until $k = N$. Briefly, assume that k switches are remaining and the current hybrid state is (y, ρ), with $y \in \Sigma_n$. The residual cost consists of two terms: one due to the evolution in the current location ρ. The other is the cost from the hybrid state $(z(t), \sigma)$, where: (i) $z(t) = e^{A_\rho t} y$ is the point reached in time t in mode ρ and (ii) $\sigma \in \mathcal{S}$ is the mode where the system switches to at time t^+. For each $(z(t), \sigma)$, this cost has been already computed at the previous stage $k-1$.

The residual cost function is minimized over the couple (t, σ), using continuous optimization over $t \geq 0$ and enumeration over the single discrete variable $\sigma \in \mathcal{S}$. The optimal arguments $t^*(y,\rho), \sigma^*(y,\rho)$ allow to build the table $\mathcal{C}_k^\rho$. A complete description of this algorithm is given in [5]. The procedure is initialized by computing the residual cost with 0 switches as:

$$J_0^*(y,\rho) \triangleq \begin{cases} y^{\mathrm{T}} Z_\rho y & \text{if } A_\rho \text{ is stable} \\ +\infty & \text{else,} \end{cases} \tag{3}$$

where Z_ρ solves the equation $A_\rho^{\mathrm{T}} Z_\rho + Z_\rho A_\rho = -Q_\rho$.

Step 2 If the system is allowed to switch indefinitely there exists a sufficiently big $\bar{N}$ such that for all $N > \bar{N}+1$ it holds [6] $\mathcal{C}_N^\rho \equiv \mathcal{C}_{\bar{N}+1}^\rho$.

The proof lies on the fact that for every initial hybrid state (y, ρ) the value of the cost converges with the increasing number of switches. As a consequence it holds that, for all $\rho \in \mathcal{S}$, $\mathcal{C}_\infty^\rho \triangleq \lim_{N\to\infty} \mathcal{C}_N^\rho \equiv \mathcal{C}_{\bar{N}+1}^\rho$.

Furthermore, if the switched system automaton graph is *totally connected*, i.e., for all $\rho, \sigma \in \mathcal{S}$, with $\rho \neq \sigma$, there exists an oriented arc of the automaton graph from node ρ to node σ, it holds for all $\rho, \sigma \in \mathcal{S}$, $\mathcal{C}_\infty^\rho \equiv \mathcal{C}_\infty^\sigma \equiv \mathcal{C}_\infty$, meaning a unique table for all modes.

To construct the table $\mathcal{C}_\infty$ the value of $\bar{N}$ is needed. We leave to further investigation a method to compute $\bar{N}$ in advance; so far the approach consists in constructing tables until a convergence criterion[2] is met.

Step 3 We show how the STP can be used to obtain an optimal stabilizing switching signal in the case when all dynamics of $\{A_\rho\}_{\rho\in\mathcal{S}}$ are unstable. To this purpose we add to $\{A_\rho\}_{\rho\in\mathcal{S}}$ a stable *dummy* dynamics A_{s+1}, that serves to give a finite value to the function $J_0^*(y,\rho)$, obtaining an *augmented* system, $\{A_\rho\}_{\rho\in\tilde{\mathcal{S}}}$ with $|\tilde{\mathcal{S}}| = |\mathcal{S}| + 1$.

If the partition $\tilde{\mathcal{C}}_\infty$, solution of the same optimal control problem for the *augmented* system, does not contain the label relative to A_{s+1}, then the table $\tilde{\mathcal{C}}_\infty$ is also a solution for $\{A_\rho\}_{\rho\in\mathcal{S}}$. A particular care must be taken in the choice of the weight matrix for the dummy dynamics.

Theorem 3.1 ([7]*):* Consider a switched system $\{A_\rho\}_{\rho\in\mathcal{S}}$, and an optimal control problem with $N = \infty$ and weight matrices $Q_\rho > 0$, $\rho \in \mathcal{S}$. Define an augmented $\{A_\rho\}_{\rho\in\tilde{\mathcal{S}}}$ and a corresponding optimal control problem, with $Q_{s+1} = qQ$, $q \in \mathbb{R}^+$, $Q > 0$. We have that:

1) If the switched system $\{A_\rho\}_{\rho\in\mathcal{S}}$ is *globally exponentially stabilizable* [15], then there exists a $q \in \mathbb{R}^+$ such that the table $\mathcal{C}_\infty$ does not contain the label associated to A_{s+1}.
2) If there exists a $q \in \mathbb{R}^+$ such that the table $\mathcal{C}_\infty$, computed by solving an optimal control problem on $\{A_\rho\}_{\rho\in\tilde{\mathcal{S}}}$, does not contain the label associated to A_{s+1}, then the switched system $\{A_\rho\}_{\rho\in\mathcal{S}}$ is asymptotically stabilizable. □

The above theorem provides a constructive way to design an asymptotic stabilizing switching law for a switched system $\{A_\rho\}_{\rho\in\mathcal{S}}$. The method can be summarized in four points:

1) Associate to the switched system an optimal control problem with $N = \infty$;
2) Define an augmented system $\{A_\rho\}_{\rho\in\tilde{\mathcal{S}}}$ with a stable dynamics and weight $Q_{s+1} = qQ$, where $q \gg 0$ and $Q > 0$;
3) Construct the table $\tilde{\mathcal{C}}_\infty$ solving an optimal control problem on $\{A_\rho\}_{\rho\in\tilde{\mathcal{S}}}$;
4) If this table does not contain the label associated to the stable mode A_{s+1}, then $\mathcal{C}_\infty \equiv \tilde{\mathcal{C}}_\infty$.

We do not provide an a priori rule to establish whether the switched system is stabilizable and an analytical way to compute an appropriate value of q. Knowing whether the system is stabilizable remains in the general case

[2]Typically a threshold on the improvements in the value of cost function.

undecidable [1]. The numerical complexity of this algorithm is extensively discussed in [6].

IV. Extension of the STP procedure

In this section an extension of the STP as described previously is considered. In particular we study the possibility of using the STP as a design tool to regulate a switched affine system to a desired point of the state space $x_\mathrm{p} \in \mathbb{R}^n$.

We can define the following problem:

Problem 1: Given a switched affine system of the form

$$\dot{x} = \tilde{A}_\rho x + \tilde{F}_\rho, \tag{4}$$

$\rho \in \mathcal{S}$, with totally connected automaton graph, design the switching signal $\rho(t)$ so that the state x is steered to a desired value x_p. □

In the particular case when x_p is a stable equilibrium point of one of the modes of the system (4), let us say $\bar{\rho}$, Problem 1 has a straightforward solution: execute any finite switching sequence with final element $\bar{\rho}$. Once in location $\bar{\rho}$ the system will autonomously reach the stable equilibrium point and no further control action is needed. This scenario is however very particular, because it requires that the specific point x_p solves the strong condition $\tilde{A}_\rho x_\mathrm{p} + \tilde{F}_\rho = 0$ for at least one $\rho \in \mathcal{S}$.

We study now the case of designing a feedback control law for the switching signal $\rho(t)$ that regulates the state to a generic desired value x_p, assuming that this point is *not* an equilibrium for any mode of the system.

In order to apply the STP to this framework we associate to the system above an LQ criterion to minimize. As explained above we consider a set of positive definite weight matrices $\tilde{Q}_\rho$ for each mode, that penalizes the offset from the target x_p. That is

$$\begin{aligned} J(x_0,\rho_0) = \min_{\rho(t)} & \int_0^{+\infty} (x - x_\mathrm{p})^\mathrm{T} \tilde{Q}_{\rho(t)} (x - x_\mathrm{p}) dt \\ \text{s.t.} \quad & \dot{x}(t) = \tilde{A}_{\rho(t)} x(t) + \tilde{F}_{\rho(t)} \\ & (x(0),\rho(0)) = (x_0,\rho_0). \end{aligned} \tag{5}$$

It is convenient to perform a shift of the state space centered in x_p, thus $\tilde{y} \in \mathbb{R}^n$ with $\tilde{y} = x - x_\mathrm{p}$. In this new set of coordinates the affine switched system becomes $\dot{\tilde{y}} = \tilde{A}_\rho \tilde{y} + F_\rho$, where $F = A_\rho x_\mathrm{p} + \tilde{F}_\rho$ and problem (5) becomes

$$\begin{aligned} J(\tilde{y}_0,\rho_0) = \min_{\rho(t)} & \int_0^{+\infty} \tilde{y}^\mathrm{T} \tilde{Q}_{\rho(t)} \tilde{y} dt \\ \text{s.t.} \quad & \dot{\tilde{y}}(t) = \tilde{A}_{\rho(t)} \tilde{y}(t) + F_{\rho(t)} \\ & (\tilde{y}(0),\rho(0)) = (\tilde{y}_0,\rho_0). \end{aligned} \tag{6}$$

The next step is to reformulate the switched affine system $\dot{\tilde{y}} = \tilde{A}_\rho \tilde{y} + F_\rho$ as a switched system $\{A_\rho\}_{\rho\in\mathcal{S}}$. To this purpose we consider an augmented space variable $y \in \mathbb{R}^{n+1}$ obtained by extending the original state space vector with an additional variable y_{n+1} and governed by the dynamics $\dot{y} = A_\rho y$, where $A_\rho = \begin{bmatrix} \tilde{A}_\rho & F_\rho \\ 0 & 0 \end{bmatrix}$ and weight matrix $Q_\rho = \begin{bmatrix} \tilde{Q}_\rho & 0 \\ 0 & 0 \end{bmatrix}$. The dummy variable y_{n+1} remains constant for any initial state, thus, $y(t) = [\tilde{y}(t), y_{n+1}(0)]^\mathrm{T}$. If $y_{n+1}(0) = 1$ the problem

$$\begin{aligned} J(y,\rho_0) = \min_{\rho(t)} & \int_0^{+\infty} y^\mathrm{T} Q_{\rho(t)} y dt \\ \text{s.t.} \quad & \dot{y} = A_{\rho(t)} y \\ & (y(0),\rho(0)) = ([\tilde{y}_0^\mathrm{T}, 1]^\mathrm{T}, \rho_0) \end{aligned} \tag{7}$$

is equivalent to (6).

The matrices A_ρ of the switched system $\{A_\rho\}_{\rho\in\mathcal{S}}$ are *all* unstable, because $\tilde{y} = 0$ is not an equilibrium point for any of the modes. The objective of the switching control law for the new switched system is to steer the vector field y towards $y_\mathrm{eq} = [0,\ldots,0,1]^\mathrm{T}$. In this case in fact the original system has reached the target x_p.

Consider now an augmented switched system $\{A_\rho\}_{\rho\in\tilde{\mathcal{S}}}$ of $\{A_\rho\}_{\rho\in\mathcal{S}}$ as described in Section III, step 3, with $A_{s+1} = \begin{bmatrix} \tilde{A}_{s+1} & 0 \\ 0 & 0 \end{bmatrix}$ and $Q_{s+1} = \begin{bmatrix} \tilde{Q}_{s+1} & 0 \\ 0 & 0 \end{bmatrix}$, and the following assumption:

Assumption 4.1: The matrix $\tilde{A}_{s+1}$ is Hurwitz and the matrix $\tilde{Q}_{s+1}$ is positive definite. □

We can now prove the following proposition:

Proposition 4.2: Under Assumption 4.1 above Theorem 3.1 holds despite the fact that matrix Q_{s+1} is not strictly positive definite and dynamics A_{s+1} is not strictly stable.

Proof: This is an immediate consequence of the fact that for every possible initial state y_0, the cost function $J(y_0, s+1) = \int_0^{+\infty} y^\mathrm{T} Q_{s+1} y dt$, for the mode $s+1$ is finite. In fact, by construction and assumption, the first n components of y are integrable, and the component $n+1$ is constant but it has a null weight. □

This result allows one to use the STP to stabilize an all-unstable-modes switched affine system to a desired specific point of the state space. This is done by minimizing a quadratic criterion that penalizes the distance of the current state from the desired target point. Note however that, as stated in Theorem 3.1, the STP is guaranteed to find a state feedback switching signal in the particular case that the switched system is globally stabilizable. As an application of this new result we consider the case study of the buck-boost converter.

V. Numerical experiments

Consider the buck-boost converter in Figure 1. The numerical values of the physical system are normalized, hence we chose $E = 1$, $L = 1$, $C = 1$ and $R = 1$. The initial step of the implementation is to adapt the physical system in Section II to the method in Sections III and IV.

First we select the set-point $x_\mathrm{p} = [2,-1]^\mathrm{T}$ and we define a dummy couple A_3, F_3, that steers the state to the set point x_p. A choice [4] of this dynamics can be obtained by solving on ρ (see (1)) $A_\rho x_\mathrm{p} + F_\rho = 0$, yielding $A_3 = \begin{bmatrix} 0 & 0.5 \\ -0.5 & -1 \end{bmatrix}$ and $F_3 = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$. In this case the obtained dynamics has also a physical interpretation: in the neighborhood of its equilibrium point x_p it approximates the sliding surface of the system with infinite switching rate.

TABLE I: *Critical tuning parameters of STP applied to the buck-boost converter.*

Number of switches	$N = 25$
Number of samples	$N_s = 2000$
Time horizon	$\tau_{\max} = 500$
Number of points (zenith)	$N_\varphi = 15$
Weight of stable mode	$q = 1000$

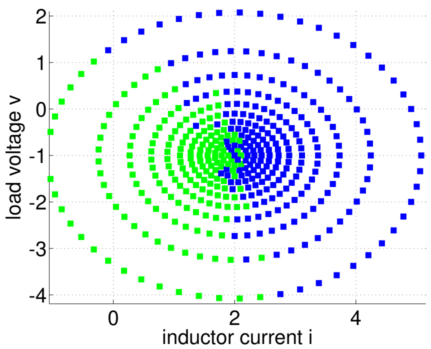

Fig. 5: *Table $\mathcal{C}_\infty$ obtained for buck-boost converter and parameters in Table I. The left side (green) imposes a closed switch (A_1), the right side (blue) open (A_2).*

Then we transform the affine equations (1) into the form $\dot{y} = A_\rho y$, by the introduction of an additional state variable. This leads us to work in $\mathbb{R}^3$, and more precisely along the affine plane $y_3 = 1$. This transformation allows to preserve important properties of the STP.

The weight matrices for modes one and two are chosen as the identity. The dummy dynamics (Section III, step 3) is penalized with a factor of $q = 10^3$.

We observed for $N = 25$ the convergence of the switching tables. The discretization of the $\mathbb{R}^3$ unitary semi-sphere is obtained in polar coordinates by sampling the zenith angle φ with $N_\varphi = 15$ and the azimuth angle with $N_\vartheta = 60$ samples[3], for a total number of 574 points. In Table I we provide the parameters of the STP we have used.

The obtained partition $\mathcal{C}_\infty$ is computed in $\mathbb{R}^3$, but the meaningful part is the intersection with the affine plane $y_3 = 1$, *projecting* the $\mathbb{R}^3$ solution along the affine plane, with an imaginary light point in the origin, as illustrated by Figure 6. We chose this particular distribution of points because we want to have a higher degree of precision around the origin, which corresponds to the working point. Note that only the labels (colors in this case) associated to dynamics A_1 and A_2 appear.

In Figure 5 we depict the table $\mathcal{C}_\infty$ restricted to the subspace $y_3 = 1$, to be used during the simulation. When the state $y = [y_1, y_2]^{\mathrm{T}}$ is in the green area, then the mode $\rho = 1$, (closed switch in Figure 1) is active. On the contrary, when $y = [y_1, y_2]^{\mathrm{T}}$ is in the blue area, then the mode $\rho = 2$ (open switch) is active.

VI. Simulation Results

The synthesis of the control law was obtained, with the tuning parameters of the algorithm listed in Table I.

[3]For the semi-sphere, the range of the zenith angle is $\pi/2$, 4 times less than the range of the azimuth angle, which is 2π.

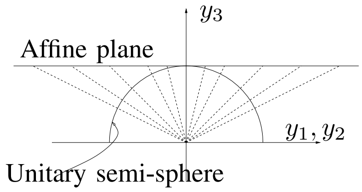

Fig. 6: *Side view of the projection on the affine plane $y_3 = 1$ of the table $\mathcal{C}_\infty$ obtained on the unitary semi-sphere.*

The computations were carried out with Matlab 7, on a 2 GHz Pentium, requiring a total off-line computation time of about 8.95×10^4 seconds. The resulting control law, represented in Figure 5, is affected by numerical error along the switching surface. This is often observed in those examples where the solution for the switching sequence collapses into a sliding surface. Note that it is possible to smoothen the solution by means of 2-dimensional filtering algorithms. We decided not to follow this way because it results into a suboptimal solution. It is intuitive that the sub-optimality of the numerical solution decreases with higher granularity of the state space discretization, but the proof is not straightforward and is of interest for future research.

The on-line controller decides the best strategy by choosing the information contained in the *closest neighbor* point to the current state value. Other policies, for example based on averaging the indication contained in a surrounding of points are also possible.

The table $\mathcal{C}_\infty$ was tested on 8 different initial points, listed in Table II. The corresponding trajectories are plotted in Figure 7.a. The optimal strategy is to remain in the initial location until the switching surface is hit. From there on a chattering behavior is activated and the state is steered towards the equilibrium point along the sliding surface. Theoretically this occurs at an infinite frequency. In practice the switching occurs at the same sampling time of the on-line simulator. Remark that the required on-line time is merely reduced to table browsing. It is also possible to impose a minimum permanence time in each location, provided that the delay is smaller than the discharge period of the inductor, as discussed in Section II.

Another possible control law can be designed with a Lyapunov based method [4]. In this approach, based on physical considerations, a unique Lyapunov function, which is directly derived from the model, is proposed. It allows to stabilize a physical switched affine system around a non common equilibrium point using different strategies such as maximum descent or minimum switching. The obtained control law is reported in Figure 7.b, where in addition the trajectories from the 8 different initial points are plotted.

In Table II we report the performances for the trajectories obtained by using the two methods. It can be seen that both laws are stabilizing and it is relevant to observe that in addition the STP provides minimization of a performance.

VII. Conclusions

A case study, the buck-boost converter, was considered as an example to extend the *switching table procedure*,

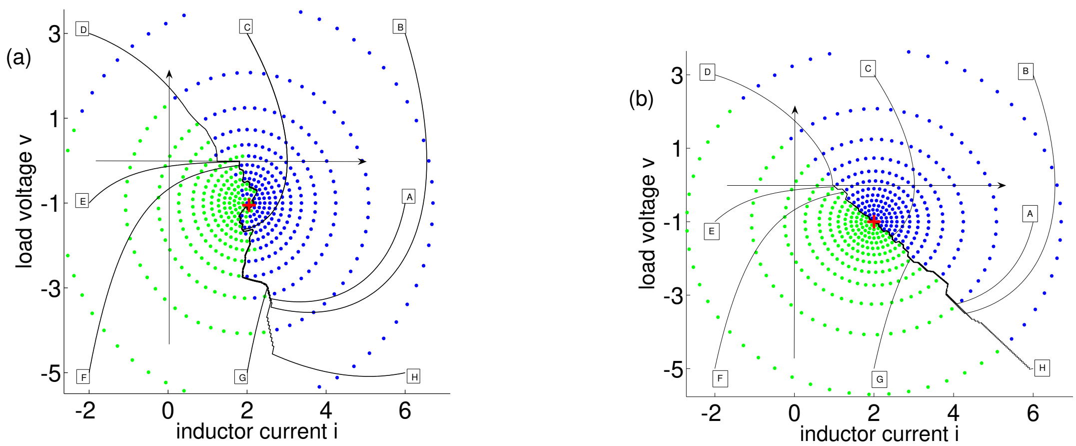

Fig. 7: *Trajectories from different initial points resulting from (a) STP solution, (b) solution based on common Lyapunov function.*

TABLE II: *Comparison of the solution obtained with the STP and with Lyapunov-based method for different initial points (Figure 7).*

Label (Fig. 7)	Point	Cost (STP)	Cost (Lyap)
A	$[4, 0]^T$	16.2	19.8
B	$[4, 4]^T$	34.1	38.3
C	$[0, 4]^T$	6.0	6.1
D	$[-4, 4]^T$	25.4	25.8
E	$[-4, 0]^T$	24.2	24.6
F	$[-4, -4]^T$	28.3	28.7
G	$[0, -4]^T$	6.6	7.6
H	$[4, -4]^T$	20.9	31.4

presented in [5]. We have shown how to regulate to a generic point a switched system composed of dynamics with different or no equilibrium. The procedure is based on dynamic programming and principle of convergence for infinite time horizon methodologies. We have shown how the STP can be applied to regulate the system state to a desired target point. A limitation of the method is its long, although offline, computation time. Further investigation could involve improvements that allow to speed up the calculations and the tuning of the parameters. Despite these difficulties, the obtained solution proved to be efficient for the considered application.

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